Multiscale Intertemporal Capital Asset Pricing Model

Ryuta Sakemoto*

November 21st, 2020

Forthcoming, *International Journal of Finance and Economics*

Abstract

This study investigates the multiscale intertemporal capital asset pricing model. We focus upon differences across timescales since they represent heterogeneities of investors in markets. This study employs a wavelet approach to decompose return data into multiple timescales. Furthermore, we impose a same risk-aversion parameter constraint into all portfolios, which is proposed by Engel and Bali (2010) who show that the constraint provides a reasonable equity risk premium at a daily frequency.

We observe positive relations between the expected returns on portfolios and the covariance of the market at a daily frequency, while these relations change as timescales increase. We find that a negative risk-return relation, which might be related to a correction process of overreaction at an approximately weekly frequency (2 days to 16 days). The strongest positive relation is observed at an approximately monthly frequency (16 to 32 days). Monthly portfolio rebalances are widely used and might impact stock market return patterns. The equity risk premium in the longer frequency ranges from 8.64% to 11.10%. Our results are robust after controlling for macroeconomic variables, market implied volatility and test portfolios. Moreover, we investigate size and value factors and reveal that the risk premia disappear in the longer frequency, which suggests that Intertemporal CAPM is satisfied.

Keywords: ICAPM, wavelet, risk factor, investment horizon, risk-aversion
JEL codes: C32, G11, G12

* Okayma University, Okayama-ken, Japan and Keio Economic Observatory, Keio University, Tokyo, Japan. Email:ryuta.sakemoto@gmail.com

1.Introduction

The theoretical literature highlights that an asset that is highly correlated with the market portfolio bears a high expected return. Merton (1973) proposes the Intertemporal Capital Asset Pricing Model (ICAPM), and a conditional expected return is dependent upon its conditional variance. The relation between conditional volatility and expected returns on equities has been explored in the literature (e.g. French et al., 1987; Campbell, 1987; Glosten et al. 1993; Ghysels et al., 2005, Guo and Whitelaw, 2006). Furthermore, Merton (1980) provides the theoretical background that expected returns for any assets should be associated with covariance risk with the market. Bali (2008), and Bali and Engle (2010) conduct empirical tests in order to explore Merton's (1980) prediction. Bali (2008) tests whether conditional covariance between the market and test portfolios predicts future returns on the test portfolios. He imposes the same relative risk-aversion parameter upon investors across all portfolios. He reveals that this restriction derives the reasonable value of the relative risk-aversion parameter. Bali and Engle (2010) adopt a different covariance estimation method and observe results that are consistent with those of Bali (2008). Our study motivates the intertemporal relation and investigates whether a high conditional correlation leads to a high expected return.

The contribution of this study is to extend the ICAPM test to multiple scales, since the choice of time intervals is important when we estimate market betas. For instance, Hand et al. (1989) point out that a spread between high and low betas increases as a return interval increases. Handa et al. (1993) conduct the multivariate CAPM test and find an inconsistent result when monthly and annual returns are employed. Firm opacity is also linked to the beta spread between high and low return intervals (Gilbert et al. 2014). Kamara et al. (2016) report that the beta on the small size portfolio varies over a horizon. Return intervals are associated with the heterogeneities of investors. Brennan and Zhang (2019) propose the extended CAPM that considers a stochastic horizon, since investors do not liquidate their portfolios continuously. Lewellen and Nagal (2006), Ang and Kristensen (2012), and Hollstein et al. (2019) address that betas vary over time to reflect economic conditions, and propose employing the conditional CAPM. The important difference between this study and the previous multi-investment horizon literature is that our ICAPM should be tested with the same relative risk-aversion parameter across all portfolios as in Bali (2008), and Bali and Engle (2010). This restriction is substantial, since it maintains cross-sectional consistency across portfolios and generates economically reasonable risk-aversion values. Most ICAPM studies ignore this point, while Merton (1973) shows that the common relative risk-aversion parameter represents the average risk-aversion for investors.

Investors' rebalance frequencies affect stock market returns. For example, investors tend to make their investment decisions at the end of the year, since there are Christmas, end-of-year bonuses, and tax consequences of capital gains and losses (Jagannathan and Wang, 2007). Abel et al. (2013) present that information and transaction costs lead to infrequent rebalancing, and Bogousslavsky (2016) reports that infrequent rebalancing produces autocorrelations of daily data. Return intervals are also associated with information delays. Stock market prices underreact because of gradual information diffusion processes (Hong and Stein, 1999). Hou and Moskowitz (2005) find that information of small firms tends to be delayed, and Hou (2007) reveals lead-lag effects from large firms to small firms in the same industries.

To investigate multiscale covariance risk between the market and test portfolios, we employ a wavelet approach which decomposes time series into high-frequency and low-frequency components. The wavelet approach is powerful in isolating a cyclical component at different timescales, and allows us to extract long-run and short-run components separately. It is reasonable to employ daily data for a short-run analysis and quarterly (or annual) data for a long-run analysis as in the previous literature. The approach, however, has a drawback in that data focus only on a specific interval and lose other information. The wavelet approach deals with the drawback, since it decomposes one time series into multiple scales; hence, we conduct long-run and short-run exploration simultaneously. The timescale is not limited to two states such as long-run and short-run, and hence it illustrates investors' decisions at multiple scales (In and Kim, 2006). Furthermore, this approach can handle a wider variety of non-stationary data (Ramsey and Lampard, 2002; Jammazi, et al., 2015), and this feature is advantageous for financial data, since they include jumps in short-run intervals and cyclical components in long-run intervals[1]. Introducing the timescale concept does matter in financial market data. For instance, Gençay et al. (2005) present that risk-return trade-off relations are stronger in longer horizons.[2] Rua and Nunes (2009) find that the degree of international stock market co-movements depends upon time frequencies. Moreover, Granger causality relationships between oil and U.S stock markets vary based upon time frequencies (Bekiros et al., 2016).

The second contribution of this study is to investigate whether size and value factors bear risk premia (Fama and French, 1992, 1993). If size and value are risk factors, then covariance with the market should have positive risk premia. Fama and French (1993) and Davis et al. (2000) conclude that average high returns on value stocks are

[1] The wavelet approach is also adopted in order to investigate market friction at high-frequency timescales (Conlon et al., 2018; Hasbrouck, 2018). Ortu et al. (2013) and Dew-Becker and Giglio (2016) explore consumption fluctuations by the wavelet methods.

[2] Rua and Nunes (2012) report that betas estimated by a wavelet approach vary over time. This study focuses upon a risk aversion parameter and a theoretical motivation that assumes a time-varying risk aversion is weak (Nagel, 2013). Hence, this study concentrates upon effects of time frequencies.

compensation for risk [3]. Some important studies, however, propose alternative explanations. For instance, Lakonishok et al. (1994) argue that a value premium comes from the overreactions of investors. A firm characteristic explanation is proposed by Daniel and Titman (1997) who show that the factor loading on the value factor is not associated with the expected return after controlling for firm characteristics. Bali and Engle (2010) also examine whether size and value are risk factors by adopting Merton's (1973) ICAPM, while they do not focus upon time horizons. Introducing time horizons is important since exposures on size and value risks depend upon investment horizons (Kamara et al. 2016).

To preview our results, we find that covariance with the market bears a positive risk premium in a shorter horizon, which means that there is a trade-off between risk and returns. At an approximately weekly frequency (2 days to 16 days), however, there is a negative risk-return relation, which might be related to a correction process of overreaction. Finally, there is a strong risk-return relation at an approximately monthly frequency (16 days to 32 days), which suggests that infrequent investment decisions based upon the end of the month is associated with our results. Moreover, size and value are regarded as risk factors only in shorter horizons.

[3] A recent study finds that the size premium is associated with a volatility regime (Cho, 2019).

The rest of the paper is organized as follows: Section 2 lays out the econometrics, Section 3 describes the data, Section 4 presents the empirical results, Section 5 conducts various robustness tests, and Section 6 concludes.

2. Estimation Methodology

This section describes our estimation methods. Subsection 2.1. introduces the multiscale analysis and we employ the wavelet approach. Subsection 2.2. provides the estimation methodology for intertemporal relations between risk and return. We use the Asymmetric Dynamic Conditional Correlation (DCC)-GARCH model of Engle (2002) and Cappiello et al. (2006) to estimate covariance risk with the market portfolio.

2.1. *Multiscale Analysis*

A wavelet transformation allows us to decompose a time series into sets of coefficients relating to a scale. Let $\psi_{\tau,s}(t)$ be wavelets where τ is the time proposition, s is the scale, and *t* is the time and they are described as waves which grow and decay in limited time periods. We follow Ramsey and Lampart (1998), and Rua and Nunes (2009), $\psi_{\tau,s}(t)$ is decomposed by a mother wavelet $\psi(t)$ and is denoted as:

$$\psi_{\tau,s}(t)= \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \quad (1)$$

where $1/\sqrt{s}$ ensures that wavelet transforms are comparable across scales and time series. Any function $f(t)$ can be described as a sequence of projections onto father and mother wavelets, $\Phi_{J,k}$ and $\psi_{j,k}$, indexed by both time domain $\{k\}$, $k = \{0,1,2,\cdots\}$ and by wavelet scale $\{s\}=2^j$, $j = \{1,2,3,\cdots\}$:

$$f(t)= \sum_k s_{J,k}\Phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \quad (2)$$

where $s_{J,k}$ and $d_{j,k}$ are coefficients of the expansion and are written as:

$$s_{J,k}= \int f(t)\, \Phi_{J,k}(t)dt, \quad d_{j,k}= \int f(t)\, \psi_{j,k}(t)dt, \quad j=1,\cdots, J \quad (3)$$

where the large *J* is the highest level of dilation used for the low-frequency, and the small *j* is the higher-frequency detail coefficients. The father wavelet $\Phi_{J,k}$ is given by:

$$\Phi_{J,k} = 2^{\frac{-J}{2}}\Phi\left(\frac{t-2^J k}{2^J}\right), \quad \int \Phi(t)dt = 1, \quad (4)$$

and the mother wavelet $\psi_{j,k}$ is given by:

$$\psi_{j,k} = 2^{\frac{-j}{2}}\Phi\left(\frac{t-2^j k}{2^j}\right), \quad \int \psi(t)dt = 0. \quad (5)$$

Following Ramsey (2002), this study adopts low pass and high pass filters to obtain wavelet coefficients. A Haar wavelet is used as the father and the mother wavelets, which is suitable to decompose asset return series (e.g. Ramsey, 2002; Gençay et al., 2005). We follow Gençay et al. (2005) and consider $j = 0, 1, \cdots, 6$, which starts from the shortest interval (2 days) to the longest interval (128 days). A stock return is replaced with $f(t)$ in Equation (2) and is decomposed into low and high-frequency series.

2.2. *The intertemporal relation between expected return and risk*

This subsection describes the ICAPM. Let μ_{t+1} be the $n \times 1$ vector of conditional mean stock returns r_{t+1} at time $t+1$, $r_{f,t+1}$ be the $n \times 1$ vector of risk free rate, and x_{t+1} be the $k \times 1$ vector of state variables. Merton (1973) presents that the conditional mean excess returns on stock portfolios are liner functions of its expected conditional covariance between r_{t+1} and the market return $r_{mkt,t+1}$, and its covariance with investment opportunity set x_{t+1}:

$$\mu_{t+1} - r_{f,t+1} = A \cdot Cov_t(r_{t+1}, r_{mkt,t+1}) + Cov_t(r_{t+1}, x_{t+1}) \cdot S \tag{6}$$

where estimated parameters, *A* is the scalar and *S* is the $k \times 1$ vector. *A* indicates the average relative risk-aversion of investors. This study follows Bali (2008), and Bali and Engle (2010), portfolios are estimated simultaneously by the system of Equation (6). Parameters, *A* and *S*, are constrained to have the same values across all portfolios. Note that Equation (6) of portfolio *i* is written as Equation (7):

$$r_{i,t+1} = C_i + A \cdot \sigma_{i,mkt,t+1} + S \cdot X_t + e_{i,t+1} \tag{7}$$

where $r_{i,t+1}$ is the excess return of portfolio i, the covariance term $\sigma_{i,mkt,t+1} =$

$Cov_t(r_{i,t+1}, r_{mkt,t+1})$, X_t is a vector of state variables, and $e_{i,t+1}$ is an error term. Equation (7) indicates that the expected return of portfolio i, $E[r_i]$ is determined by the market beta β_i, the expected market risk premium $E[r_{mkt}] = A \cdot \sigma^2_{mkt}$, and the state variable term as:

$$E[r_i]=C_i + A \cdot \sigma^2_{mkt} \frac{\sigma_{i,mkt}}{\sigma^2_{mkt}} + S \cdot X = C_i + (A \cdot \sigma^2_{mkt}) \cdot \beta_i + S \cdot X. \quad (8)$$

We follow Bali and Engle (2010) and employ the Dynamic Conditional Correlation (DCC)-GARCH model of Engle (2002) to estimate the covariance terms, $Cov_t(r_{t+1}, r_{mkt,t+1})$ and $Cov_t(r_{t+1}, x_{t+1})$. Furthermore, we allow asymmetric shocks as in Cappiello et al. (2006) and the detail is explained by the Appendix.

Following Gençay et al. (2005), level j wavelet covariance is written as $Cov_t(r_{j,t+1}, r_{j,mkt,t+1})$ and $Cov_t(r_{j,t+1}, x_{j,t+1})$. We estimate covariance for each portfolio and the market at the wavelet level j by the bivariate DCC-GARCH model. After obtaining all covariance variables, we estimate Equation (6) by the Seemingly Unrelated Regressions (SUR).

3. Data

This section explains data and we use the value-weighted index of the Center for Research in Security Prices (CRSP) as the market portfolio ranging from 2 January 1963 and 31

July 2019. Daily returns are calculated as the daily percentage change in the index. Following Bali and Engle (2010), we also employ value-weighted decile portfolios sorted by book-to-market ratios, firm size, and momentum which are widely used in the literature. One-month T-bill yield is adopted as the risk free rate. These data sets are downloaded from Kenneth French's website.

The first state variable is the VIX index that is implied volatility of synthetic at-the-money option contract on the S&P100 index (Ang et al., 2006; Guo and Whitelaw, 2006; Bali and Engle, 2010). VIX starts in 2 January 1986 and that is provided by the Chicago Board options Exchange (CBOE) website. VIX is strongly related to market stock and bond conditions (Adrian et al., 2019). The second state variable is the default spread that is the difference between corporate bond yield on BAA and 10-year T-bill yield (Welch and Goyal, 2008, Ang and Kristensen, 2012). The third state variable is the term spread between three month and 10-year T-bill yields (Ferson and Harvey, 1999; Maio and Santa-Clara, 2012). The fourth variable is the Federal Fund rate that is linked to the U.S. monetary policy, and is also associated with investment opportunity sets (Bali and Engle, 2010). The default spread, the term spread, and the Federal Fund rate are obtained from the Federal Reserve Bank at St. Louis. We use the default spread, the term spread, and the Federal Fund rate in a same model. The default spread covers the shortest period among the three variables and is available from 2 January 1986. Therefore, our estimation starts from 2 January 1986, when we control for these state variables.

4. Empirical Results

4.1. *Multiscale risk-return trade-off without intertemporal hedging demand*

We begin with discreate wavelet transform (DWT) on the excess market return. Figure 1 demonstrates the original data series and DWT on it.[4] We note that each decomposition is different from the original data series and contains different information.

Next, we move on to the empirical results for multiscale intertemporal relations between risk and expected returns. First, we estimate them without the risk aversion parameter restriction as a benchmark. Table 1 reports the estimated risk-aversion parameter, A in Equation (6) for the 10 size portfolios.[5] We present the results for the raw data which are not decomposed by the wavelet and those for scale 4 which are approximately one month. We observe that only five out of the ten portfolios are positive and statistically significant at least at the 5% level for the raw data, and three out of the ten are for scale 4. Moreover, the parameter on the portfolio 1 for the raw data is negative

[4] We do not report the decomposition of level 6 to save space.

[5] The other test portfolio results are available from the author on request.

and statistically significant at the 1% level, which is an opposite sign expected by the theory.

Having found the weak relationship between risk and returns, we introduce the common the risk-aversion parameter restriction. Table 2 reports the parameter estimates for the average risk-aversion for the 10 size, book-to-market, and momentum portfolios. We impose the restriction that the risk-aversion parameter A in Equation (6) takes the same value across 10 test portfolios, as mentioned in the previous section. We run the system of Equation (6), while we include intercepts and do not consider a state variable x_t in these estimations. Wald test results are also reported, which tests the joint hypothesis that all intercepts are equal to zero.

The estimated average risk-aversion coefficients are in the range of 0.85 to 3.05 and statistically significant at least at the 5% level for the raw data in Panels A, B and C in Table 2. The results of Wald test indicate that some intercepts are not equal to zero. According to Equation (8), variance of the market portfolio σ^2_{mkt} is about 0.00001, and the range of the expected market risk premium $A \cdot \sigma^2_{mkt}$ covers 0.86% to 7.52%, assuming 252 trading days in a year. The value for the size portfolios is relatively small compared to that of Bali and Engle (2010), and we will examine the robustness section.

Interestingly, the results for scales 2 and 3 in Panels A, B and C in Table 2 uncover that the parameters are negative and statistically significant at least at the 10% level, which suggests that there is no trade-off between risk and returns. We move on to the results for scale 4 and find that the risk-aversion parameters flip the sign again, being statistically significant at least at the 5% level. The range of the expected risk premium is from 8.64% to 11.10%. Furthermore, the results of Wald test indicate that the joint null hypothesis that all parameters are equal to zero is not rejected for all three test portfolios. These imply that there is no abnormal return that is not explained by the conditional covariance with the market portfolio. These correspond to the literature which reports that the CAPM is not rejected in longer time horizons (e.g. Handa et al., 1993; Gilbert et al., 2014; Brennan and Zhang 2019). Bali and Engle (2010) present the empirical evidence that there are abnormal returns for the momentum portfolios, while our scale 4 results for the momentum portfolios hold for the ICAPM since the results pass the following two conditions: (i) risk-aversion parameter is positive and (ii) intercepts are zero. This result highlights the importance of introducing the multiscale analysis.

4.2. *Controlling for unexpected news in market volatility*

The literature shows that market volatility is associated with future stock returns (Campbell, 1993; Ang et al., 2006; Guo and Whitelaw, 2006). Following Engle and Bali

(2010), we include a change in VIX at time t, ΔVIX_t, in the system of equations. Equation (7) for scale j is written as:

$$r_{i,j,t+1} = C_{i,j} + A_j \cdot \sigma_{i,j,mkt,t+1} + S_j \cdot \Delta VIX_{j,t} + e_{i,j,t+1} \quad (9)$$

where $C_{i,j}$, A_j and S_j indicate the estimated parameters, and $e_{i,j,t+1}$ is an error term.

Panels A, B and C in Table 3 present that the estimated risk-aversion parameters are statistically significant and maintain the similar magnitudes for the raw data results. The estimated parameters on the change in VIX are positive for the book-to-market and the momentum portfolios, while negative for the size portfolios. The change in VIX is more important for the size portfolio for scale 1, since the risk-aversion parameter is insignificant and the parameter on ΔVIX is positive. This suggests that an increase in market volatility leads to increases in the portfolio returns. The risk-aversion parameters flip the signs for scales 2 and 3, which is consistent with the findings of the Table 2. Finally, we observe the positive and statistically significant risk-aversion parameters for the book-to-market and the momentum portfolios for scale 4, which corresponds to almost one month.

In summary, we find that the risk-aversion parameter is positive at an approximately monthly frequency (scale 4) and this result is robust after controlling for market volatility.

4.3. *Risk premia on other factors*

Next, we investigate whether Fama and French's (1993) three factors (MKT, SML and HML) and Carhart's (1997) momentum factor (UMD) bear additional risk premia. We employ covariance terms with the market factor as in Bali and Engle (2010) and Equation (6) for scale *j* is written as:

$$r_{i,j,t+1}=C_{i,j}+A_j\cdot\sigma_{i,j,mkt,t+1}+S1_j\cdot\sigma_{i,j,SML,t+1}+S2_j\cdot\sigma_{i,j,HML,t+1}+S3_j\cdot\sigma_{i,j,UMD,t+1}+e_{i,j,t+1} \quad (10)$$

where $\sigma_{i,j,SML,t+1}$, $\sigma_{i,j,HML,t+1}$, and $\sigma_{i,j,UML,t+1}$ are conditional covariance between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors, respectively, at time $t+1$ and for scale *j*. $C_{i,j}$, A_j, $S1_j$, $S2_j$, and $S3_j$ indicate the estimated parameters, and $e_{i,j,t+1}$ is an error term.

Panels A, B and C in Table 4 present the empirical evidence that the conditional covariance terms with the market factor are positive and statistically significant for the raw data, as well as scales 1 and 4 after controlling for four factors, except for the result of the size portfolios for scale 4. When we turn to the covariance terms with four factors, SML and HML are positive and statistically significant at the 1% level in all three test portfolios for scale 1. Interestingly, these size and value terms are insignificant for scale 4, which implies that the market factor is more important in a longer horizon. This is associated with the findings of Gençay et al. (2005) who report that the CAPM is held in a long horizon. Moreover, Handa et al. (1989) uncover that the size effect disappears when they employ a longer return interval, and our results also support this finding. Our result is contrast to the findings of Kamara et al. (2016) who address that the HML factor is important for the intermediate horizon. This is attributed to different strategies for estimations, and they neither focus upon each portfolio nor impose the common slope upon the risk-aversion parameter.

4.4. *Further discussion*

Given the heterogeneous risk-return relations for different scales, we will consider a mechanism for generating these results. We observe the positive risk premium on the covariance with the market factor for the raw data and scale 1, while there are negative risk premia for scales 2 and 3 (from 4 days to 16 days). The strongest positive results are observed for scale 4 (from 16 days to 32 days), which corresponds to almost monthly return data. Monthly portfolio rebalances are widely used and might impact stock market return patterns. Ogden (1990) highlights that the payment system of the U.S. is related to the monthly effect; that is to say, U.S. stock returns rise from the end of the month (Ariel, 1987). The payment system causes investors to realize their cash receipts at the end of the month. Jagannathan and Wang (2007) address that tax consequences of capital gains and losses are related to infrequent investment decisions for investors. Furthermore, Jagannathan et al. (2008) support Jagannathan and Wang's (2007) finding by examining U.K. data which have different tax year ends, and reveal that the decisions of U.K. investors are based upon the tax year ends within the U.K.

The negative risk-return relations for scales 2 and 3, however, might be associated with the correction of investors' overreactions in a shorter term. Individual investors tend to buy stocks in the news (Barber and Odean, 2008). This buying pressure leads to increases in the attention- grabbing stocks; however, these temporary rises in prices are not based upon fundamental changes in the firms. Hence, they will be adjusted in the near future. This slow adjustment is consistent with gradual information diffusion processes (Hong and Stein, 1999; Hou and Moskowitz, 2005). Stocks which co-move with the market portfolio on the same day are regarded as risky stocks for investors, while stocks which co-move over a few weeks are not. This is a possible reason as to why the covariance term with the market does not bear risk premia for scales 2 and 3.

5. Robustness

The results in the previous section have shown that the conditional covariance terms with the market factor are positive for the short and long horizons, while they are negative for the intermediate horizons. This section presents additional evidences: (i) controlling for macroeconomic variables; (ii) employing a different wavelet filter; (iii) subsample analysis; (iv) using additional risk factors; (v) adopting other test portfolios.

5.1. *Controlling for unexpected news in macroeconomic variables*

Macroeconomic variables are widely used as measures of investment opportunity sets. We follow Bali and Engle (2010) and choose changes in the following three macroeconomic variables at time *t*: Federal Fund rate (ΔFED_t), term spread ($\Delta TERM_t$), and default spread (ΔDEF_t). We estimate the average risk-aversion for scale j with controlling for the changes in the macroeconomic variables as:

$$r_{i,j,t+1} = C_{i,j} + A_j \cdot \sigma_{i,j,mkt,t+1} + S1_j \cdot \Delta FED_{j,t} + S2_j \cdot \Delta TERM_{j,t} + S3_j \cdot \Delta DEF_{j,t} + e_{i,j,t+1} \quad (11)$$

where $C_{i,j}$, A_j, $S1_j$, $S2_j$ and $S3_j$, indicate the estimated parameters, and $e_{i,j,t+1}$ is an error term.

Table 5 shows the estimation results and the risk-aversion parameters for the raw data range from 0.92 to 3.29 corresponding to the annualized market risk premia from 2.26 % to 8.12%, and are statistically significant at least at the 5% level. The risk-aversion parameters for scales 2 and 3 are negative and statistically significant, except for the result in the momentum portfolios for scale 3. The signs on the risk-aversion parameter flip for scale 4, and only the parameter on the book-to-market portfolios is statistically significant at the 1% level, and the annualized market risk premium is 10.13%. The parameter on the momentum portfolios is insignificant, but the standard error of the risk-aversion parameter is relatively small in terms of the coefficient, and hence the parameter is marginally insignificant. We also find that ΔDEF_t is positively related to the expected returns for the raw data, which is consistent with the findings of Bali and Engle (2010). Furthermore, ΔDEF_t is positive and statistically significant for scales 1 and 2.

Overall, we see the consistent changes in the risk-aversion parameter over scales after controlling for changes in the macroeconomic variables.

5.2. *Daubechies least asymmetric wavelet filter*

We explore whether our results are sensitive to wavelet filters in this subsection. Following In and Kim (2006), and Fan and Gençay (2010), we employ the Daubechies least asymmetric wavelet filter of length 8 (Daubechies, 1992) and controlling for four factors as in Table 4. Table 6 reports that the risk-aversion parameters for scales 1 and 4 are statistically significant, which is consistent with the previous results. Moreover, when we focus upon the size portfolio, the parameter for scale 4 is statistically significant at the 5% level, which is not observed in the previous results. This suggests that using the Daubechies least asymmetric wavelet filter may improve our results.

5.3. *Multiscale risk-return trade-off: Subsample analysis*

Given the small value of the expected market risk premium for the size portfolios in Table 3, we conduct subsample estimations in this subsection. We split the sample data, and the first half sample period is from 2 January 1963 to 31 December 1990, then the second half sample period is from 2 January 1991 to 31 July 2019. Table A1 indicates that the estimated average risk-aversion parameter in the first half sample for the raw data is 2.55, which is higher than that of the second half sample. Moreover, the result of the second half sample is not statistically significant, which suggests that the risk-return trade-off relation in the size portfolios becomes weaker in the recent period. This is the main reason that the inconsistency between Bali and Engle's (2010) and our results. In addition, the average risk-aversion parameter in the second half sample for scale 4 is also insignificant.

5.4. *Five factors*

We also control for five factors proposed by Fama and French (2015). In addition to three factors (MKT, SML, and HML), Fama and French (2015) propose the profitability (RMW) and the investment (CMA) factors. Table A2 reports that the pattern of the average risk-aversion parameters is similar to that of the previous results, which means that the signs of those are positive in short and long horizons, but negative in intermediate horizons. Panel B of Table A2 shows that the parameters on the conditional covariance of the profitability factor are statistically significant from the raw data to scale 4. This suggests that the profitability factor is regarded as a risk factor in multiple timescales.

5.5. *Other test portfolios.*

In addition, we adopt other test portfolios. The first test portfolios are the value-weighted decile industry portfolios and the second test portfolios are the value-weighted six portfolios sorted by size and book-to-market. We include covariance of the size and the value factors, $\sigma_{i,j,SML,t+1}$ and $\sigma_{i,j,HML,t+1}$, as controlling variables.[6] Tables A3 and A4 present the empirical evidence and these changes do not affect our main findings.

[6] The covariance term of momentum factor, $\sigma_{i,j,UML,t+1}$, is excluded since it provides unreasonably large values.

6. Conclusion

We explore the intertemporal relations between covariance risk with the market and the expected returns. Merton (1980) suggests that all portfolios have the same slope that indicates investors' relative risk-aversion, and Bali (2008) and Bali and Engle (2010) find that the relative risk-aversion parameter takes an economically sensible value. We extend the relation to multiple timescales, since heterogeneous investors have different investment horizons. The literature presents empirical evidence that return horizons are related to expected returns and the magnitudes of betas (Handa et al., 1993; Gilbert et al., 2014; Brennan and Zhang 2019). This study employs a wavelet approach in order to decompose financial data into multiple timescales without losing information, which are successful in extracting short-run and long-run information (Gençay et al., 2005; Conlon et al., 2018; Hasbrouck, 2018).

We reveal that there are risk-return trade-offs at daily and longer (about one month) frequencies. The estimated relative risk-aversion has an economically sensible value and the annualized market risk premium ranges from 8.6% to 11.1%. In contrast, there exist negative relations between covariance risk and the expected returns at intermediate frequencies (4 to 16 days). This is associated with the correction of investors' overreactions in a shorter term. Investors prefer to buy attention-grabbing stocks (Barber and Odean, 2008), and hence high beta stocks are regarded as being risky in a short-run horizon. These overpriced stocks, however, are adjusted gradually, which means that they are not regarded as risky stocks for intermediate investors. Finally, the positive risk-return relation in a longer horizon is derived by infrequent investment decisions. Our longer horizon corresponds to about one month, and investors make investment decisions at the end of the month (Ogden, 1990).

Data Source

1)Kenneth French's website
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
2)Chicago Board options Exchange (CBOE) website
http://www.cboe.com/vix
3) Federal Reserve Bank at St. Louis
https://fred.stlouisfed.org

Acknowledgements

I would like to thank two anonymous Referees for helpful comments in preparing my manuscript for publication. I am grateful to Kenneth French for sharing the data. I have benefited from discussion with Xiao Jing Cai, Setsuya Kurahashi, Takahiro Obata and Junsuke Senoguchi. All errors belong to the author. This work was supported by KAKENHI(20K22092).

References

Abel, A.B., Eberly, J.C. and Panageas, S. (2013). Optimal inattention to the stock market with information costs and transactions costs. *Econometrica*, **81**, 1455-1481.

Adrian, T., Crump, R.K., and Vogt, E. (2019). Nonlinearity and flight to safety in the risk-return trade-off for stocks and bonds. *Journal of Finance*, **74**, 1931-1973.

Ang, A., Hodrick, R.J., Xing, Y., and Zhang, X., (2006). The cross-section of volatility and expected returns. *Journal of Finance*, **61**, 259-299.

Ang, A., and Kristensen, D. (2012). Testing conditional factor models. *Journal of*

Financial Economics, **106**, 132-156.
Ariel, R. A. (1987). A monthly effect in stock returns. *Journal of Financial Economics*, **18**, 161-174.
Bali T.G. (2008). The intertemporal relation between expected returns and risk. *Journal of Financial Economics,* **87**, 101-131.
Bali, T.G. and Engle, R.F. (2010). The intertemporal capital asset pricing model with dynamic conditional correlations. *Journal of Monetary Economics*, **57**. 377-390.
Barber, B. and Odean, T. (2008). All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors. *Review of Financial Studies,* **21**, 785-818.
Bekiros, S., Nguyen, D.K., Uddin, G., and Sjö, B. (2016). On the time scale behavior of equity-commodity links: Implications for portfolio management. *Journal of International Financial Markets, Institutions and Money*, **41**, 30-46.
Bogousslavsky, V. (2016). Infrequent rebalancing, return autocorrelation, and seasonality. *Journal of Finance,***71**, 2967-3006.
Brennan, M.J. and Zhang, Y. (2019). Capital asset pricing with a stochastic horizon. *Journal of Financial and Quantitative Analysis,* Forthcoming.
Campbell, J. Y. (1987). Stock returns and the term structure. *Journal of Financial Economics*, **18**, 373-399.
Campbell, J.Y. (1993). Intertemporal asset pricing without consumption data. *American Economic Review*, **83**, 487–512.
Cappiello, L., Engle, R.F. and Sheppard, K. (2006) Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*,**4**, 537-572
Carhart, M. (1997). On persistence in mutual fund performance. *Journal of Finance,* **52**, 57–82.
Cho, S., (2019). The size premium and macrovolatility risks: Evidence from U.S. and U.K. equity markets. *International Journal of Finance and Economics,* **24**, 1271-1286.
Conlon, T., Cotter, J., and Gençay, R. (2018) Long-run wavelet-based correlation for financial time series. *European Journal of Operational Research*, **271**, 676-696.
Daniel, K.and Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns, *Journal of Finance*, **52**, 1-33.
Daubechies, I. (1992). Ten Lectures on Wavelets. CBMS-NSF Regional Conference Series in Applied Mathematics, Philadelphia: SIAM.
Davis, J.L., Fama E.F., and French, K.R. (2000). Characteristics, covariances, and average returns: 1929 to 1997. *Journal of Finance*, **55**, 389-406.

Dew-Becker, I. and Giglio, S. (2016). Asset pricing in the frequency domain: theory and empirics. *Review of Financial Studies*, **29**, 2029-2068.

Engle, R.F. (2002). Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, **20**, 339-350.

Fama, E.F. and French, K.R. (1992). The cross-section of expected stock returns. *Journal of Finance*, **47**, 427–465.

Fama, E.F. and French, K.R. (1993). Common risk factors in the returns on stock and bonds. *Journal of Financial Economics*, **33**, 3–56.

Fama, E.F. and French, K.R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, **116**, 1–22.

Fan, Y. and Gençay, R. (2010). Unit root tests with wavelets. *Econometric Theory*, **26**, 1305-1331.

Ferson, W.E. and Harvey, C. (1999). Conditioning variables and the cross section of stock returns. *Journal of Finance*, **54**, 1325-1360.

French, K., Schwert, G., and Stambaugh, R., (1987). Expected stock returns and volatility. *Journal of Financial Economics*, **19**, 3-29.

Gençay, R., Selçuk, F., and Whitcher, B. (2005). Multiscale systematic risk. *Journal of International Money and Finance,* **24**, 55-70.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2005). There is risk-return trade-off after all. *Journal of Financial Economics*, **76**, 509-548.

Gilbert, T., Hrdlicka, C., Kalodimos, J., and Siegel, S. (2014). Daily data is bad for beta: Opacity and frequency-dependent betas. *Review of Asset Pricing Studies*, **4**, 78-117.

Glosten, L.R., Jagannathan, R., and Runkle, D.E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, **48**, 1779-1801.

Handa, P., Kothari, S. P., and Wasley, C. (1989). The relation between the return interval and betas: Implications for the size effect. *Journal of Financial Economics*,**23**,79-100.

Handa, P., Kothari, S. P., and Wasley, C. (1993). Sensitivity of multivariate tests of the capital asset-pricing model to the return measurement interval. *Journal of Finance*,**48**, 1543-1551

Hasbrouck, J. (2018). High-frequency quoting: Short-term and volatility in bids and offers. *Journal of Financial and Quantitative Analysis,* **53**, 613-641.

Hollstein, F., Prokopczuk, M., and Wese Simen, C. (2019). The conditional CAPM revisited: evidence from high-frequency betas. *Management Science,* Forthcoming.

Hong, H., and Stein, J.C. (1999). A unified theory of underreaction, momentum trading and overreaction in asset markets. *Journal of Finance*, **54**, 2143-2184.

Hou, K. (2007). Industry information diffusion and the lead-lag effect in stock returns. *Review of Financial Studies*, **20**, 1113–1138.

Hou, K., and Moskowitz, T.J. (2005). Market frictions, price delay, and the cross-section of expected returns. *Review of Financial Studies*, **18**, 981–1020.

In, F. and Kim, S. (2006). The hedge ratio and the empirical relationship between the stock and futures markets: a new approach using wavelet analysis. *Journal of Business*, **79**, 799-820.

Jagannathan, R. and Wang, Y. (2007). Lazy investors, discretionary consumption, and the cross-section of stock returns. *Journal of Finance*, **62**, 1623–1661.

Jagannathan, R., Marakani, S., Takehara, H., and Wang, Y. (2008). Calendar cycles, infrequent decisions and the cross section of stock returns. *Management Science,* **58**, 507-522.

Jammazi, R., Lahiani, A., and Nguyen, D.K. (2015). A wavelet-based nonlinear ARDL model for assessing the exchange rate pass-through to crude oil prices. *Journal of International Financial Markets, Institutions and Money*, **34**, 173-187.

Kamara, A., Korajczyk, R., Lou, X., and Sadka, R. (2016). Horizon pricing. *Journal of Financial and Quantitative Analysis,* **51**, 1769-1793.

Lakonishok, J., Shleifer, A., and Vishny, R.W. (1994), Contrarian investment, extrapolation, and risk, *Journal of Finance*, **49**, 1541-1578.

Lewellen, J., Nagel, S., (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics*, **82**, 289-314.

Merton, R. C. (1973). An intertemporal asset pricing model. *Econometrica,* **41**, 867–887.

Merton, R. C. (1980). On estimating the expected return on the market: an exploratory investigation. *Journal of Financial Economics,* **8**, 323–361.

Maio, P., Santa-Clara, P., (2012). Multifactor models and their consistency with the ICAPM, *Journal of Financial Economics*, **106**, 586-613.

Nagel, S. (2013). Empirical cross-sectional asset pricing, *Annual Review of Financial Economics,* **5**, 167-199.

Ogden, J.P. (1990). Turn-of-month evaluations of liquid profits and stock returns: A common explanation for the monthly and January effects. *Journal of Finance,* **45**, 1259–1272.

Ortu, F., Tamoni, A., and Tebaldi, C. (2013). Long-run risk and the persistence of consumption shocks. *Review of Financial Studies*, **26**, 2876-2915.

Ramsey, J. B. and Lampart, C. (1998). Decomposition of economic relationships by timescale using wavelets. *Macroeconomic Dynamics*, **2**, 49-71.

Ramsey, J. B. (2002). Wavelets in economics and finance: Past and future. *Studies in Nonlinear Dynamics & Econometrics*, **6**, 1-29.

Rua, A. and Nunes, L.C. (2009). International comovement of stock market returns: A wavelet analysis. *Journal of Empirical Finance,* **16**, 632–639.

Rua, A. and Nunes, L.C. (2012). A wavelet-based assessment of market risk: The emerging markets case. *The Quarterly Review of Economics and Finance,* **52**, 84-92.

Welch, I. and Goyal, A. (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies*, **21**, 1455-1508.

Table 1 Risk return trade-off without common slope restriction

	Size							
	Raw				Scale 4			
	C		$\sigma_{i,mkt,t+1}$			C	$\sigma_{i,mkt,t+1}$	
Small	0.040	***	-1.436	***	Small	-0.016	1.330	
	(0.008)		(0.417)			(0.045)	(2.859)	
P2	0.029	***	0.170		P2	-0.037	2.410	
	(0.010)		(0.377)			(0.054)	(3.169)	
P3	0.033	***	0.257		P3	-0.029	1.704	
	(0.010)		(0.386)			(0.053)	(2.937)	
P4	0.029	***	0.401		P4	-0.032	2.709	
	(0.010)		(0.380)			(0.051)	(2.816)	
P5	0.029	***	0.553		P5	-0.034	2.657	
	(0.010)		(0.373)			(0.051)	(2.770)	
P6	0.024	**	0.920	**	P6	-0.007	2.820	
	(0.009)		(0.371)			(0.048)	(2.629)	
P7	0.024	***	0.899	**	P7	-0.024	4.005	*
	(0.009)		(0.351)			(0.046)	(2.194)	
P8	0.021	**	1.134	***	P8	-0.048	6.588	***
	(0.009)		(0.350)			(0.048)	(2.523)	
P9	0.017		1.235	***	P9	-0.044	6.491	**
	(0.009)		(0.344)			(0.046)	(2.563)	
Large	0.005		1.934	***	Large	-0.049	5.553	***
	(0.009)		(0.385)			(0.041)	(2.000)	

Notes: This table shows the estimated average relative risk-aversion parameter *A* and constant term *C* using the system of Equation (6) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted size portfolios. We do not impose the common slope restriction. The table presents raw return results and the following wavelet scale results: Scale 4: 16–32 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 2 Risk return trade-off at different wavelet scales

	Panel A Size $\sigma_{i,mkt,t+1}$		Wald	
Raw	0.853	**	8.50	***
	(0.328)		[0.00]	
Scale 1	0.829	**	4.03	***
	(0.391)		[0.00]	
Scale 2	-2.288	***	2.10	**
	(0.778)		[0.02]	
Scale 3	-5.191	***	1.70	*
	(0.982)		[0.08]	
Scale 4	3.609	**	0.99	
	(1.686)		[0.45]	
Scale 5	3.841		0.94	
	(2.591)		[0.49]	
Scale 6	8.741		0.24	
	(17.948)		[0.98]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (6) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted size portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in square brackets. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 2 Continue

	Panel B Book-to-market $\sigma_{i,mkt,t+1}$		Wald	
Raw	1.253	***	6.16	***
	(0.328)		[0.00]	
Scale 1	2.759	***	5.30	***
	(0.392)		[0.00]	
Scale 2	-2.257	**	2.41	**
	(0.768)		[0.01]	
Scale 3	-3.909	***	1.55	
	(1.106)		[0.12]	
Scale 4	4.635	**	1.35	
	(1.874)		[0.21]	
Scale 5	-9.670	*	1.37	
	(3.918)		[0.20]	
Scale 6	-20.156		0.41	
	(5.989)		[0.93]	

Notes: This table shows the estimated average relative risk-aversion parameter *A* using the system of Equation (6) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted book-to-market portfolios. The *F*-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the *p*-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 2 Continue

	Panel C Momentum $\sigma_{i,mkt,t+1}$		Wald	
Raw	3.045	***	3.46	***
	(0.311)		[0.00]	
Scale 1	1.198	***	2.90	***
	(0.372)		[0.00]	
Scale 2	-1.295	*	1.40	
	(0.753)		[0.18]	
Scale 3	-3.422	***	1.22	
	(1.021)		[0.28]	
Scale 4	4.810	***	1.42	
	(1.796)		[0.17]	
Scale 5	2.508		0.14	
	(3.757)		[0.99]	
Scale 6	-8.477	*	0.67	
	(4.71)		[0.75]	

Notes: This table shows the estimated average relative risk-aversion parameter *A* using the system of Equation (6) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted momentum portfolios. The *F*-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the *p*-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 3 Risk return trade-off after controlling for market volatility

	Panel A Size					
	$\sigma_{i,mkt,t+1}$		ΔVIX_t		Wald	
Raw	0.902	**	-0.019	***	4.81	***
	(0.407)		(0.004)		[0.00]	
Scale 1	0.624		0.116	***	2.97	***
	(0.423)		(0.005)		[0.00]	
Scale 2	-2.273	**	-0.110	***	3.07	***
	(0.98)		(0.006)		[0.00]	
Scale 3	-3.004	***	-0.208	***	0.41	
	(1.032)		(0.011)		[0.93]	
Scale 4	1.174		-0.168	***	0.83	
	(1.128)		(0.012)		[0.59]	
Scale 5	6.293	**	-0.441	***	1.58	
	(2.886)		(0.035)		[0.12]	
Scale 6	14.548		-0.539	***	0.90	
	(19.663)		(0.005)		[0.52]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (9) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted size portfolios. We employ the past change in VIX, ΔVIX_t, as the state variable. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from 2 January 1986 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 3 Continue

	Panel B Book-to-market $\sigma_{i,mkt,t+1}$		ΔVIX_t		Wald	
Raw	1.473	***	0.010	**	3.08	**
	(0.394)		(0.005)		[0.01]	
Scale 1	2.492	***	0.195	***	5.21	***
	(0.430)		(0.006)		[0.00]	
Scale 2	-2.588	***	-0.115	***	3.32	***
	(0.897)		(0.009)		[0.00]	
Scale 3	-2.368	**	-0.253	***	0.13	
	(1.121)		(0.013)		[0.99]	
Scale 4	4.542	***	-0.213	***	1.50	
	(1.400)		(0.013)		[0.14]	
Scale 5	-12.906	***	-0.456	***	2.02	**
	(4.014)		(0.036)		[0.03]	
Scale 6	-29.038	***	-0.552	***	4.85	***
	(9.935)		(0.045)		[0.00]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (9) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted book-to-market portfolios. We employ the past change in VIX, ΔVIX_t, as the state variable. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from 2 January, 1986 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 3 Continue

	Panel C Momentum $\sigma_{i,mkt,t+1}$		ΔVIX_t		Wald	
Raw	3.371	***	0.034	***	1.51	
	(0.378)		(0.004)		[0.14]	
Scale 1	1.309	***	0.212	***	3.08	***
	(0.408)		(0.006)		[0.00]	
Scale 2	-0.993		-0.094	***	1.60	
	(0.912)		(0.009)		[0.11]	
Scale 3	-0.585		-0.215	***	0.23	
	(1.068)		(0.013)		[0.99]	
Scale 4	2.996	**	-0.205	***	0.90	
	(1.384)		(0.013)		[0.52]	
Scale 5	1.794		-0.463	***	0.02	
	(4.137)		(0.037)		[0.99]	
Scale 6	-9.195		-0.531	***	1.71	*
	(5.924)		(0.047)		[0.08]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (9) and standard errors are reported in parentheses. The test portfolios are the 10 value-weighted momentum portfolios. We employ the past change in VIX, ΔVIX_t, as the state variable. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from 2 January, 1986 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 4 Risk return trade-off after controlling for four factors

	Panel A Size									
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,UMD,t+1}$		Wald	
Raw	0.627	*	-2.373	***	1.957	*	-0.150		8.50	***
	(0.350)		(0.871)		(1.065)		(0.637)		[0.00]	
Scale 1	1.034	**	4.205	***	5.803	***	-1.044		5.44	***
	(0.453)		(0.966)		(1.525)		(0.961)		[0.00]	
Scale 2	-2.639	***	-13.522	***	-2.077		-0.670		2.36	**
	(0.889)		(1.942)		(2.167)		(1.408)		[0.01]	
Scale 3	-5.875	***	11.237	***	5.563	*	0.003		1.55	
	(1.121)		(3.297)		(3.330)		(1.504)		[0.12]	
Scale 4	2.286		-2.395		-8.187		-1.285		0.99	
	(1.909)		(6.023)		(5.326)		(2.303)		[0.45]	
Scale 5	-1.225		18.351		-6.300		0.684		1.33	
	(4.733)		(14.261)		(7.621)		(2.774)		[0.22]	
Scale 6	-4.889		54.070		-29.477	*	-2.141		0.01	
	(19.122)		(37.095)		(15.090)		(5.759)		[0.99]	

Notes: This table shows the estimated average relative risk-aversion parameter *A* using the system of Equation (10) and standard errors are reported in parentheses. We include the conditional covariance terms between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors (Fama and French, 1993; Carhart, 1997). The test portfolios are the 10 value-weighted size portfolios. The *F*-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the *p*-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 4 Continue

	Panel B Book-to-market									
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,UMD,t+1}$		Wald	
Raw	0.811	***	0.427		0.236		-1.363	**	7.47	***
	(0.405)		(1.156)		(0.828)		(0.632)		[0.00]	
Scale 1	2.912	***	4.579	***	6.368	***	1.994	**	4.75	***
	(0.522)		(1.208)		(1.215)		(0.964)		[0.00	
Scale 2	-1.712	*	-11.181	***	-2.906	*	4.292	***	0.50	
	(0.950)		(2.368)		(1.661)		(1.364)		[0.88]	
Scale 3	-3.236	***	3.512		1.190		2.640		1.31	
	(1.188)		(5.545)		(2.607)		(1.735)		[0.23]	
Scale 4	4.523	**	-9.448		0.998		-0.842		1.11	
	(2.029)		(7.519)		(4.910)		(2.480)		[0.35]	
Scale 5	-6.035		-12.329		1.351		5.026		1.52	
	(4.739)		(15.299)		(6.703)		(3.747)		[0.13]	
Scale 6	0.487		-19.273		-7.666		8.160	**	0.91	
	(10.631)		(18.494)		(10.045)		(3.429)		[0.51]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (10) and standard errors are reported in parentheses. We include the conditional covariance terms between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors (Fama and French, 1993; Carhart, 1997). The test portfolios are the 10 value-weighted book-to-market portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 4 Continue

	Panel C Momentum									
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,UMD,t+1}$		Wald	
Raw	2.606	***	1.038		0.684		-1.658	***	5.83	***
	(0.367)		(1.078)		(0.856)		(0.535)		[0.00]	
Scale 1	1.787	***	6.793	***	5.295	***	0.096		1.98	**
	(0.478)		(1.107)		(1.278)		(0.840)		[0.04]	
Scale 2	-1.107		-8.800	***	1.028		2.162	*	2.36	**
	(0.927)		(2.620)		(1.886)		(1.148)		[0.01]	
Scale 3	-1.148		-2.208		5.530	**	5.729	***	0.28	
	(1.163)		(4.969)		(2.591)		(1.502)		[0.98]	
Scale 4	5.569	***	-1.234		4.841		-3.153		1.63	
	(1.936)		(6.674)		(4.793)		(2.248)		[0.10]	
Scale 5	3.675		-0.069		2.683		1.872		0.54	
	(4.610)		(8.314)		(5.279)		(3.080)		[0.85]	
Scale 6	0.993		4.964		-19.972	*	3.695		1.92	**
	(7.894)		(14.481)		(9.398)		(3.517)		[0.05]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (10) and standard errors are reported in parentheses. We include the conditional covariance terms between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors (Fama and French, 1993; Carhart, 1997). The test portfolios are the 10 value-weighted momentum portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 5 Risk return trade-off after controlling for macroeconomic variables

Panel A Size	$\sigma_{i,mkt,t+1}$		ΔFED_t	$\Delta TERM_t$		ΔDEF_t		Wald	
Raw	0.919	**	0.017	0.367		-0.261		5.04	***
	(0.409)		(0.043)	(0.269)		(0.281)		[0.00]	
Scale 1	0.589		-0.028	0.346		1.839	***	3.10	***
	(0.447)		(0.052)	(0.357)		(0.361)		[0.00]	
Scale 2	-2.237	**	0.022	0.134		0.813		2.75	***
	(0.998)		(0.085)	(0.492)		(0.547)		[0.00]	
Scale 3	-4.918	***	0.035	1.005		-3.085	***	0.09	
	(1.091)		(0.159)	(0.800)		(0.871)		[0.99]	
Scale 4	0.673		-0.059	1.622		1.173		0.28	
	(1.079)		(0.237)	(1.162)		(1.305)		[0.98]	
Scale 5	5.771	*	-0.053	-1.178		-7.552	***	0.49	
	(2.970)		(0.274)	(1.611)		(1.458)		[0.89]	
Scale 6	1.278		0.033	-5.203	**	-8.839	***	0.20	
	(16.720)		(0.440)	(2.068)		(1.788)		[0.99]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (11) and standard errors are reported in parentheses. We employ changes in Federal Fund rate (ΔFED_t), term spread ($\Delta TERM_t$), and default spread (ΔDEF_t) as the state variables. The test portfolios are the 10 value-weighted size portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from 2 January, 1986 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 5 Continue

	Panel B Book-to-market $\sigma_{i,mkt,t+1}$		ΔFED_t	$\Delta TERM_t$		ΔDEF_t		Wald	
Raw	1.338	***	-0.034	0.396		1.425	***	3.63	***
	(0.396)		(0.056)	(0.351)		(0.366)		[0.00]	
Scale 1	2.518	***	-0.050	0.930	*	3.037	***	4.68	***
	(0.452)		(0.073)	(0.503)		(0.509)		[0.00]	
Scale 2	-2.480	***	-0.080	0.397		1.705	**	2.99	***
	(0.909)		(0.116)	(0.673)		(0.749)		[0.00]	
Scale 3	-4.552	***	0.069	0.599		-1.185		0.33	
	(1.175)		(0.197)	(0.993)		(1.080)		[0.96]	
Scale 4	4.100	***	-0.128	1.026		2.241		0.56	
	(1.431)		(0.286)	(1.405)		(1.577)		[0.83]	
Scale 5	-14.554	***	0.071	-0.543		-5.513	***	4.80	***
	(4.069)		(0.308)	(1.814)		(1.643)		[0.00]	
Scale 6	-33.198	***	-0.747	-3.147		-5.797	***	1.14	
	(10.363)		(0.492)	(2.300)		(2.000)		[0.33]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (11) and standard errors are reported in parentheses. We employ changes in Federal Fund rate (ΔFED_t), term spread ($\Delta TERM_t$), and default spread (ΔDEF_t) as the state variables. The test portfolios are the 10 value-weighted book-to-market portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts,$C_i$equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from 2 January, 1986 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 5 Continue

	Panel C Momentum $\sigma_{i,mkt,t+1}$		ΔFED_t	$\Delta TERM_t$		ΔDEF_t		Wald	
Raw	3.289	***	0.000	0.665	*	1.411	***	1.52	
	(0.379)		(0.006)	(0.343)		(0.358)		[0.14]	
Scale 1	0.895	**	-0.022	0.905	*	2.850	***	2.67	***
	(0.431)		(0.073)	(0.502)		(0.509)		[0.00]	
Scale 2	-0.828		-0.015	0.619		1.473	**	1.41	
	(0.921)		(0.112)	(0.651)		(0.724)		[0.18]	
Scale 3	-2.158	**	0.214	1.484		0.545		0.04	
	(1.110)		(0.186)	(0.937)		(1.018)		[1.00]	
Scale 4	2.252		-0.202	2.208		2.936	*	0.42	
	(1.425)		(0.275)	(1.351)		(1.517)		[0.93]	
Scale 5	0.834		0.252	-2.165		-5.740	***	0.24	
	(4.191)		(0.310)	(1.826)		(1.654)		[0.99]	
Scale 6	-13.788		-0.459	-2.779		-6.832	***	1.23	
	(6.373)		(0.466)	(2.181)		(1.895)		[0.27]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (11) and standard errors are reported in parentheses. We employ changes in Federal Fund rate (ΔFED_t), term spread ($\Delta TERM_t$), and default spread (ΔDEF_t) as the state variables. The test portfolios are the 10 value-weighted momentum portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from 2 January, 1986 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 6 Risk return trade-off with the Daubechies least asymmetric wavelet filter

	Panel A Size									
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,UMD,t+1}$		Wald	
Scale 1	1.417	***	3.326	***	6.056	***	6.056		4.12	***
	(0.499)		(0.960)		(1.601)		(1.009)		[0.00]	
Scale 2	-0.808		4.638	**	-5.911	***	-0.140		0.20	
	(0.595)		(2.111)		(1.866)		(1.043)		[0.99]	
Scale 3	-1.809		13.172	***	-2.688		-2.952		-1.44	
	(1.306)		(3.851)		(3.329)		(1.966)		[0.17]	
Scale 4	6.035	**	-14.863	***	-3.691		1.243		1.08	
	(2.702)		(5.757)		(5.179)		(2.826)		[0.37]	
Scale 5	-0.572		5.294		15.669		0.488		0.08	
	(3.504)		(9.008)		(11.023)		(3.363)		[0.99]	
Scale 6	-9.764		-18.081		3.014		1.319		1.33	
	(8.807)		(30.119)		(5.708)		(2.786)		[0.21]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (10) and standard errors are reported in parentheses. We use the Daubechies least asymmetric wavelet filter (Daubechies, 1992). We include the conditional covariance terms between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors (Fama and French, 1993; Carhart, 1997). The test portfolios are the 10 value-weighted size portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 6 Continue

	Panel B Book-to-market $\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,UMD,t+1}$		Wald	
Scale 1	3.027	***	3.049	**	7.134	***	7.134	***	5.60	***
	(0.587)		(1.180)		(1.327)		(1.067)		[0.00]	
Scale 2	-1.821	***	11.097	***	-2.861	*	-0.244		0.79	
	(0.690)		(3.131)		(1.469)		(1.029)		[0.63]	
Scale 3	-0.829		-6.487		-0.161		3.473	*	0.27	
	(1.342)		(5.032)		(2.322)		(1.980)		[0.98]	
Scale 4	12.912	***	-18.064	**	3.039		1.824		3.56	***
	(2.866)		(8.821)		(4.661)		(2.836)		[0.00]	
Scale 5	-7.896	*	-24.610	***	21.684	**	10.778	**	1.85	*
	(4.184)		(9.068)		(9.639)		(4.329)		[0.05]	
Scale 6	3.003		-56.235	**	4.615		0.726		0.50	
	(6.885)		(25.617)		(5.236)		(3.583)		[0.88]	

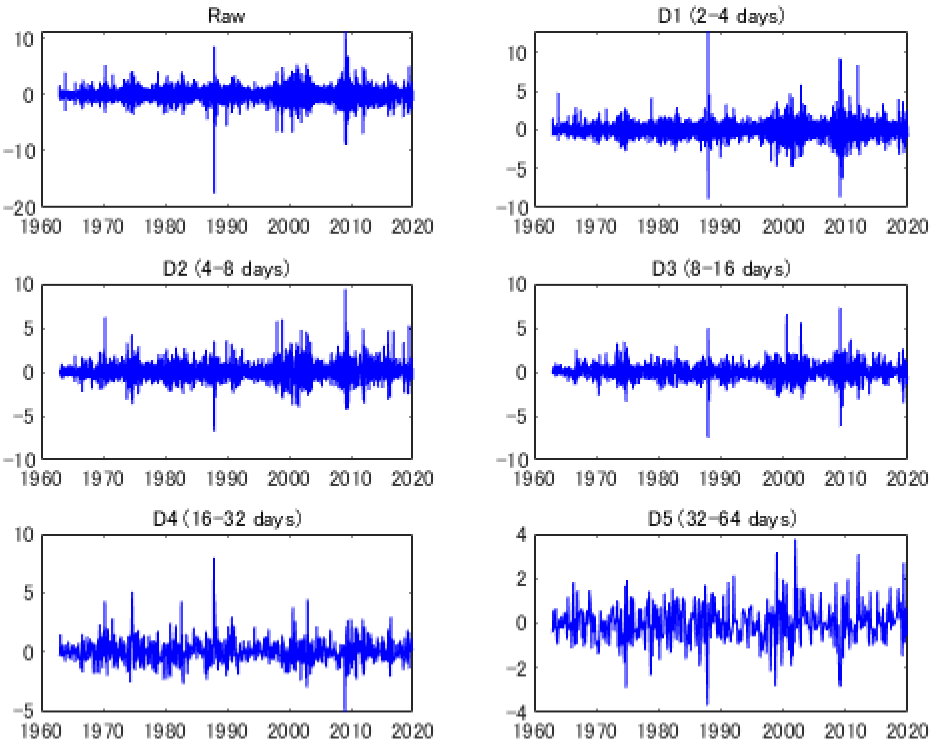
Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (10) and standard errors are reported in parentheses. We use the Daubechies least asymmetric wavelet filter (Daubechies, 1992). We include the conditional covariance terms between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors (Fama and French, 1993; Carhart, 1997). The test portfolios are the 10 value-weighted book-to-market portfolios.The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 6 Continue

	Panel C Momentum									
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,UMD,t+1}$		Wald	
Scale 1	2.233	***	6.510	***	5.504	***	5.504	***	2.85	***
	(0.547)		(1.153)		(1.423)		(0.921)		[0.00]	
Scale 2	-2.225	***	-1.404		-10.009	***	0.425		0.37	
	(0.624)		(2.666)		(1.516)		(0.857)		[0.95]	
Scale 3	0.731		7.699		-0.193		-0.259		0.45	
	(1.486)		(4.885)		(2.617)		(1.926)		[0.91]	
Scale 4	5.571	**	-7.885		2.495		-3.898		1.65	
	(2.538)		(7.484)		(4.863)		(2.595)		[0.10]	
Scale 5	1.110		-13.996		0.916		4.694		1.49	
	(3.814)		(10.964)		(11.333)		(3.751)		[0.15]	
Scale 6	0.379		9.087		18.408	***	0.836		1.51	
	(6.528)		(20.737)		(6.002)		(2.966)		[0.14]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (10) and standard errors are reported in parentheses. We use the Daubechies least asymmetric wavelet filter (Daubechies, 1992). We include the conditional covariance terms between the market portfolio and the size (SML), value (HML) and momentum (UMD) factors (Fama and French, 1993; Carhart, 1997). The test portfolios are the 10 value-weighted momentum portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Figure 1 Market return and DMT decomposition



Notes: This figure illustrates the market excess return (top left) and discreate wavelet transform (DWT) on it. Decompositions for levels 1 to 5 are presented.

Appendix

A1. Haar wavelet

This section describes low pass and high pass filters of the Haar wavelet. Let $l(k)$ represent a low pass filter and $h(k)$ represent a high pass filter. These filters are obtained from father and mother wavelets as in Ramsey (2002):

$$l(k) = \frac{1}{\sqrt{2}} \int \Phi(t)\, \Phi(2t - k) dt \tag{A1}$$

$$h(k) = \frac{1}{\sqrt{2}} \int \Psi(t)\, \Phi(2t - k) dt \tag{A2}$$

where Φ is the father wavelet and Ψ is the mother wavelet.

We follow Ramsey (2002) and Gençay et al. (2005) and the low and the high pass filters obtained by the Harr wavelet for N=2 are given by:

$$l(k) = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\} \tag{A3}$$

$$h(k) = \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}. \tag{A4}$$

A2. Daubechies wavelet

We also employ the Daubechies wavelet that is more general case of the Haar wavelet (Daubechies, 1992). Following Wavelet and scaling coefficients $W_{t,1}$and $V_{t,1}$ are given by:

$$W_{t,1} = \sum_{p=0}^{L-1} h_p y_{2t-p,} \qquad V_{t,1} = \sum_{p=0}^{L-1} g_p y_{2t-p,} \tag{A5}$$

where $\{h_p\}$ is the wavelet filter, $\{g_p\}$ is the scaling filter, and $t = L_1, L_1 + 1, \cdots, T/2$ with $L_1 = L/2$.

A3. Dynamic Conditional Correlation (DCC)-GARCH estimation

This section describes an estimation method of conditional correlations. This study employs the Asymmetric Dynamic Conditional Correlation (ADCC)-GARCH model proposed by Engle (2002) and Cappiello et al. (2006). This approach allows obtaining time-varying conditional correlations between market and test portfolios. Let μ_{t+1} be the $n \times 1$ vector of conditional mean stock returns r_{t+1} at time $t + 1$, $r_{f,t+1}$ be the $n \times 1$ vector of risk free rate, and x_{t+1} be the $k \times 1$ vector of state variables and the following mean return process is considered:

$$y_{t+1} \equiv \begin{pmatrix} r_{t+1} \\ r_{mkt,t+1} \\ x_{t+1} \end{pmatrix} = \alpha_0 + \alpha_1 y_t + \mu_t + \varepsilon_{t+1} \tag{A6}$$

where μ_t is given by Equation (6), ε_{t+1} is the residual term, estimated parameters, α_0 and α_0 are diagonal matrices. Variance of ε_{t+1} is written by a diagonal matrix of conditional standard deviations D_{t+1}, and a conditional correlation matrix ρ_{t+1}:

$$Var_t[\varepsilon_{t+1}] = D_{t+1}\rho_{t+1}D_{t+1} \tag{A7}$$

where D_{t+1}^2 is given by

$$D_{t+1}^2 = b_0 + b_1 y_t^2 + b_2 D_t^2 \tag{A8}$$

where parameters b_0, b_1, and b_2 are diagonal matrices. The conditional correlation matrix ρ_{t+1} is modelled as a function of lagged standardised residuals $u_t = D_t^{-1}\varepsilon_t$, and lagged conditional correlation ρ_t:

$$\rho_{t+1} = S(1 - \gamma_1 - \gamma_2) - \gamma_3 S^- + \gamma_1 u_t u_t' + \gamma_2 \rho_t + \gamma_3 u_t^- u_t^{-\prime} \tag{A9}$$

where u_t^- are the zero-threshold standardised residuals which are equals to u_t when less than zero else zero otherwise, S is the unconditional correlation matrix of ε_t, and S^- is the unconditional correlation matrix of u_t^-.

Table A1　Subsample results

	Panel A				Panel B			
	Size				Size			
	$\sigma_{i,mkt,t+1}$		Wald		$\sigma_{i,mkt,t+1}$		Wald	
Raw	2.547	***	0.31	***	0.197		5.64	***
	(0.561)		[0.97]		(0.488)		[0.00]	
Scale 1	-2.422	***	0.82		2.250	***	4.37	***
	(0.684)		[0.60]		(0.568)		[0.00]	
Scale 2	-1.218		0.65		-2.093	**	3.59	**
	(1.354)		[0.75]		(1.063)		[0.00]	
Scale 3	-1.508		0.17		-7.774	***	1.76	*
	(1.163)		[1.00]		(1.37)		[0.07]	
Scale 4	3.352	**	0.35		-4.535		0.21	
	(1.410)		[0.96]		(2.793)		[0.99]	
Scale 5	-6.29		0.01		1.531		0.16	
	(9.502)		[1.00]		(3.54)		[1.00]	
Scale 6	-14.649		0.70		-3.715		1.16	
	(19.416)		[0.71]		(5.609)		[0.32]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation (6) and standard errors are reported in parentheses. Panel A indicates the first half sample period that covers from 2 January 1963 to 31 December 1990 and Panel B does the second half sample period that covers from 2 January 1991 to 31 July 2019. The test portfolios are the 10 value-weighted size portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in square brackets. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A2 Risk return trade-off after controlling for five factors

Panel A

Size

	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,RMW,t+1}$		$\sigma_{i,CMA,t+1}$	Wald	
Raw	1.903	***	0.490		0.123		7.217	***	1.913	2.92	***
	(0.475)		(1.127)		(1.282)		(1.932)		(2.306)	(0.00)	
Scale 1	0.849		4.092	***	7.259	***	2.756		-5.790	4.27	***
	(0.663)		(1.305)		(1.947)		(3.224)		(3.434)	(0.00)	
Scale 2	0.783		-7.075	***	-5.558	**	18.047	***	9.585	0.26	
	(1.016)		(2.178)		(2.564)		(3.847)		(4.598)	(0.98)	
Scale 3	-4.703	***	13.351	***	-0.073		0.640		13.347	1.15	
	(1.356)		(3.872)		(3.408)		(5.380)		(7.290)	(0.32)	
Scale 4	3.239		-5.431		-8.638		-5.442		10.572	0.67	
	(2.495)		(6.833)		(5.687)		(8.193)		(10.296)	(0.74)	
Scale 5	-4.696		0.009		1.613		-27.989	***	-12.262	0.30	
	(5.324)		(14.153)		(7.750)		(7.916)		(10.567)	(0.97)	
Scale 6	-13.320		-87.597		-37.652	**	12.279		32.491	0.43	
	(42.767)		(55.836)		(15.248)		(11.597)		(21.208)	(0.92)	

Notes: This table shows the estimated average relative risk-aversion parameter *A* using the system of Equation and standard errors are reported in parentheses. We include the conditional covariance terms between the market portfolio and the size (SML), value (HML), profitability (RMW) and the investment (CMA) factors (Fama and French, 2015). The test portfolios are the 10 value-weighted size portfolios. The *F*-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the *p*-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A2　Continue

	Panel B Book-to-market											
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,RMW,t+1}$		$\sigma_{i,CMA,t+1}$		Wald	
Raw	3.7114	***	5.963	***	0.838		11.995	***	3.823	*	3.21	***
	(0.533)		(1.363)		(0.828)		(2.132)		(2.120)		(0.00)	
Scale 1	2.246	***	3.914	***	7.309	***	7.240	**	-9.639	***	3.71	***
	(0.674)		(1.446)		(1.248)		(3.049)		(3.216)		(0.00)	
Scale 2	-0.980		-7.535	***	-6.440	***	12.950	***	4.537		1.28	
	(1.105)		(2.688)		(1.768)		(4.811)		(3.985)		(0.24)	
Scale 3	-1.889		14.274	**	3.417		28.380	***	-10.218		1.00	
	(1.487)		(5.700)		(2.170)		(6.199)		(6.642)		(0.44)	
Scale 4	6.810	***	-1.816		2.624		15.743	*	7.316		1.49	
	(2.616)		(8.415)		(5.122)		(9.425)		(9.401)		(0.15)	
Scale 5	-4.756		-15.463		-11.826		-43.197	**	26.040		0.69	
	(6.222)		(15.542)		(8.085)		(19.222)		(18.288)		(0.72)	
Scale 6	0.705		-24.749		-32.738	***	-2.004		30.493		0.60	
	(12.215)		(20.199)		(9.548)		(12.824)		(19.926)		(0.79)	

Notes: This table shows the estimated average relative risk-aversion parameter *A* using the system of Equation and standard errors are reported in parentheses. We include the conditional covariance terms between the market portfolio and the size (SML), value (HML), profitability (RMW) and the investment (CMA) factors (Fama and French, 2015). The test portfolios are the 10 value-weighted book-to-market portfolios. The *F*-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the *p*-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A2 Continue

	Panel C Momentum											
	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		$\sigma_{i,RMW,t+1}$		$\sigma_{i,CMA,t+1}$		Wald	
Raw	3.538	***	3.113	**	2.619	***	4.467	**	-2.355		2.93	***
	(0.522)		(1.399)		(0.828)		(1.882)		(1.996)		(0.00)	
Scale 1	0.981		5.430	***	7.495	***	3.381		-9.592	***	2.10	**
	(0.662)		(1.395)		(1.295)		(2.872)		(3.135)		(0.03)	
Scale 2	0.060		-4.825		-2.839		7.713	**	4.505		0.68	
	(1.131)		(3.049)		(1.999)		(3.650)		(4.167)		(0.73)	
Scale 3	-2.279	*	3.076		-1.560		-0.724		7.905		0.87	
	(1.372)		(5.432)		(2.210)		(5.653)		(6.363)		(0.55)	
Scale 4	3.549		-8.320		7.898		-11.264		-5.845		0.78	
	(2.401)		(8.070)		(4.899)		(7.817)		(8.875)		(0.63)	
Scale 5	3.093		1.974		7.088		15.546		-12.897		0.31	
	(5.025)		(8.582)		(6.272)		(13.227)		(11.539)		(0.97)	
Scale 6	-2.027		2.178		-26.339	***	-0.492		38.759		1.11	
	(7.921)		(14.802)		(8.425)		(10.559)		(25.838)		(0.36)	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation and standard errors are reported in parentheses. We include the conditional covariance terms between the market portfolio and the size (SML), value (HML), profitability (RMW) and the investment (CMA) factors (Fama and French, 2015). The test portfolios are the 10 value-weighted momentum portfolios. The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A3 Risk return trade-off: Industry portfolios

	Industry $\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		Wald	
Raw	1.691	***	-0.569		1.647	**	3.12	***
	(0.360)		(1.148)		(0.677)		[0.00]	
Scale 1	3.487	***	8.964	***	3.572	***	4.12	***
	(0.475)		(1.290)		(1.007)		[0.00]	
Scale 2	-2.898	***	-8.866	***	-2.870	**	1.62	
	(0.837)		(2.352)		(1.462)		[0.10]	
Scale 3	-3.229	***	3.530		5.727	***	1.41	
	(1.240)		(5.178)		(2.205)		[0.18]	
Scale 4	5.500	**	5.623		14.751	***	0.62	
	(2.480)		(7.225)		(5.217)		[0.78]	
Scale 5	6.898		-22.042		0.648		0.79	
	(5.274)		(14.704)		(6.851)		[0.62]	
Scale 6	16.110		-42.905		-17.893		0.43	
	(14.919)		(28.229)		(12.407)		[0.92]	

Notes: This table shows the estimated average relative risk-aversion parameter *A* using the system of Equation and standard errors are reported in parentheses. The test portfolios are the 10 industry portfolios. We include the conditional covariance terms between the market portfolio and the size (SML) and value (HML) factors (Fama and French, 1993). The *F*-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the *p*-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A4 Risk return trade-off: 2×3 portfolios

Size and book-to-market

	$\sigma_{i,mkt,t+1}$		$\sigma_{i,SMB,t+1}$		$\sigma_{i,HML,t+1}$		Wald	
Raw	0.857	**	-3.653	***	2.086	***	9.95	***
	(0.362)		(0.948)		(0.694)		[0.00]	
Scale 1	2.772	***	5.701	***	4.304	***	8.46	***
	(0.470)		(0.963)		(1.037)		[0.00]	
Scale 2	-3.067	***	-11.686	***	-5.932	***	2.52	**
	(0.839)		(2.125)		(1.420)		[0.03]	
Scale 3	-3.530	***	10.408	***	-0.174		1.32	
	(1.243)		(3.730)		(1.931)		[0.25]	
Scale 4	5.594	***	-5.116		5.467		1.09	
	(1.846)		(6.793)		(5.319)		[0.36]	
Scale 5	1.517		-12.416		0.085		0.62	
	(4.424)		(4.424)		(6.222)		[0.69]	
Scale 6	-35.862	**	30.611		-21.951	*	0.53	
	(17.190)		(21.741)		(11.239)		[0.75]	

Notes: This table shows the estimated average relative risk-aversion parameter A using the system of Equation and standard errors are reported in parentheses. The test portfolios are the six portfolios sorted by size and book-to-market. We include the conditional covariance terms between the market portfolio and the size (SML) and value (HML) factors (Fama and French, 1993). The F-test statistics of Wald test are reported. The joint null hypothesis is that all intercepts, C_i equal zero and the p-values are reported in parentheses. The table presents raw return results and the following wavelet scale results: Scale 1: 2–4 day periods, Scale 2: 4–8 day periods, Scale 3: 8–16 day periods, Scale 4: 16–32 day periods, Scale 5: 32–64 day periods, and Scale 6: 64–128 day periods. The market portfolio is the value weighted NYSE/AMEX/NASDAQ market portfolio. The sample period covers from January 2, 1963 to July 31, 2019. Asterisk *,**, and *** indicate significance at the 10%, 5%, and 1% level, respectively.