

Application of Dead Beat Control method to the Water Level Control of Small-scale Hydroelectric Power Plant

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Small Scale Hydroelectric Power Generation is made using the water drop caused by the gradient difference between that of the river and that of the penstock. The water flow in this system apt to vary with time. Especially, at the time of the flood or that of the dry, the flow in the water stream varies largely. To realize the stable generation, it is necessary to maintain the water level in the head tank located in the mid way in the stream channel at a constant level. In this paper, variation of water level is forecasted using two-tank model for the penstock and the head tank and the water level control algorithm is proposed by deadbeat control. The effectiveness of the algorithm is evaluated beforehand using the simulation program of the water environment for the hydroelectric generation system.

1. INTRODUCTION

The Hydropower generation is expected as one of the surrogate method of energy supply for the petroleum.

As for the hydropower generation, large scale plant accompanies many geographical limitations. On the other hand, small scale plant has the lower limitation and there exist several thousand candidate places for electric plant around the whole country. This is the reason of the big expectation for small scale hydropower generation.

The majority of the small-scale hydropower generation is the type of the water guide generation using water stock called a head tank and the energy induced by the difference in the gradients of the river and that of the penstock. The important things to stabilize and to attain high efficiency in such small scale power generation are as follows.

- (1) The position of the head tank is to be placed at the nearest position to the water site and useless over flow from the head tank is must be prevented.
- (2) To maintain the water level variation within a certain duration.
- (3) To minimize the movement of guide valve for water level regulation.

However, it is not so easy to maintain the water level in the head tank because of its small capacity[1],[2],[3],[4]. Furthermore, due to the nonlinear characteristics of the process, conventional feedback control can not meet the requirement. To overcome the problem, high accuracy water level control method by the deadbeat control algorithm[5] is studied.

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2. WATER LEVEL CONTROL METHOD

Figure 1 shows the configuration of the control system for water level control in hydropower generation system. As shown in Figure 1, opening of the guide valve is calculated from the difference of water level in the head tank from its reference value. The flow through the turbine is regulated by guide valve and the water level in the head tank is controlled within a permissible range. In the following, modeling and design of control algorithm are described.

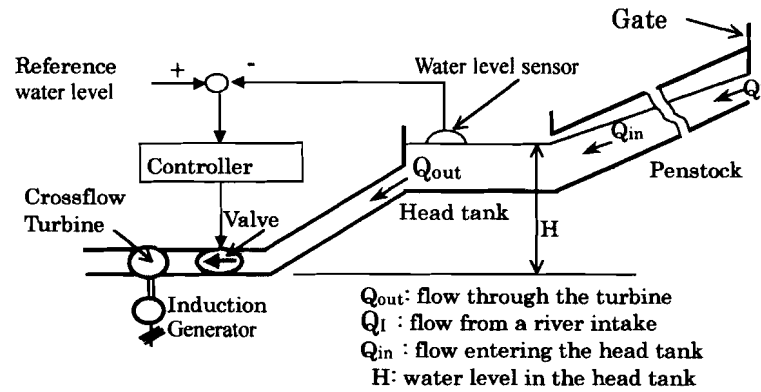


Figure 1 Schematic diagram of system considered

2.1 Construction of two-tank model

In the design of control system, target process is described by the state equation or transfer function. We described transfer function for the penstock and the head tank. The penstock of non pressure type can be treated as a water vessel. So, we consider the combined two-tank model shown in Figure 2.

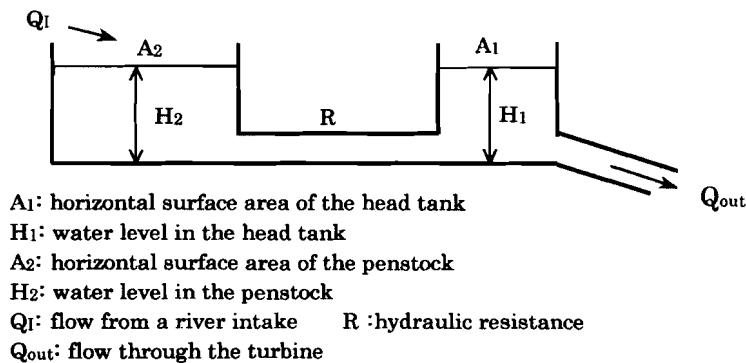


Figure 2 Two-tank model

The relations between state variables in Figure 2 are described in the following equations.

$$\frac{dH_1}{dt} = \frac{1}{A_1} \left\{ \frac{1}{R} (H_2 - H_1) - Q_{out} \right\} = \frac{1}{T_1} (H_2 - H_1) - \frac{Q_{out}}{A_1} \quad (1)$$

$$\frac{dH_2}{dt} = \frac{1}{A_2} \left\{ Q_I - \frac{1}{R} (H_2 - H_1) \right\} = \frac{1}{T_2} (H_2 - H_1) - \frac{Q_I}{A_2} \quad (2)$$

where, $T_1=A_1R$ and $T_2=A_2R$.

The step response of the water level $h(t)$ in the head tank is written by

$$h(t) = \left\{ \frac{1}{A_1 + A_2} t - \frac{R A_1 / A_2}{\left(1 + A_1 / A_2\right)^2} \left(1 - e^{-\left(\frac{1}{T_1} + \frac{1}{T_2}\right)t}\right) \right\} \Delta Q_{in} - \left\{ \frac{1}{A_1 + A_2} t + \frac{R}{\left(1 + A_1 / A_2\right)^2} \left(1 - e^{-\left(\frac{1}{T_1} + \frac{1}{T_2}\right)t}\right) \right\} \Delta Q_{out} \quad (3)$$

Generally speaking, as the penstock is very long, we can assume that $A_2 \gg A_1$.

In this case, equation (3) can be rewritten as follows,

$$h(t) = \left\{ \frac{1}{A_2} t - R \frac{A_1}{A_2} \left(1 - e^{-\frac{t}{T_1}}\right) \right\} \Delta Q_{in} - \left\{ \frac{1}{A_2} t + R \left(1 - e^{-\frac{t}{T_1}}\right) \right\} \Delta Q_{out} \quad (4)$$

Using the above equation, time response $h(t)$ against the step change of ΔQ_{in} and ΔQ_{out} are calculated as shown in Figures 3(a) and 3(b).

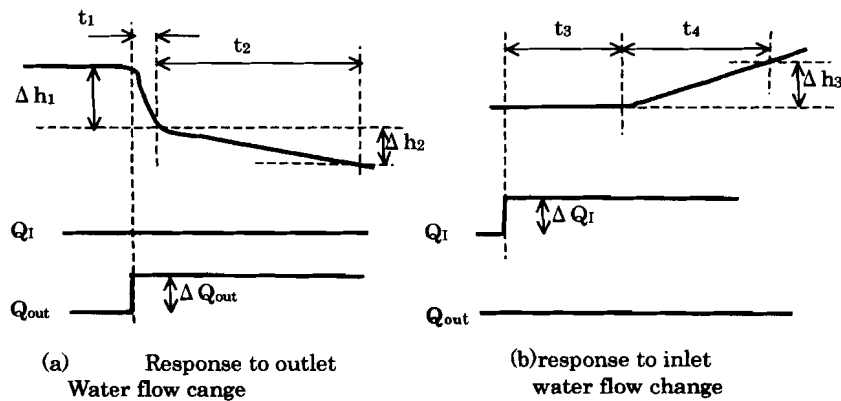


Figure 3 Response of two-tank model

We can estimate system parameters A_2 and R using the data in Figure 3. That is, from Figure 3(a), parameters A_2 and R are

$$A_2 = \Delta Q_{out} \times t_2 / \Delta h_2 \quad (5)$$

$$R = \Delta h_1 / \Delta Q_{out} \quad (6)$$

From Figure 3(b), parameter A_2 is

$$A_2 = \Delta Q_{in} \times t_4 / \Delta h_3 \quad (7)$$

As written in equations (5) to (7), we can determine the values of model parameters from the results of step response of water level of the head tank.

Generally speaking, the right hand side in equation (5) and the left hand side in equation (7) is not equal. So, rewriting A_2 in equation (7) to A'_2 . Then we have the following equation (8).

$$A'_2 = \frac{\Delta Q_1 \times t_4}{\Delta h_3} \tag{8}$$

Using the definition of equation (8), equation (4) can be written as follows:

$$h(t) = \left\{ \frac{1}{A_2}t - R \frac{A_1}{A_2} (1 - e^{-\frac{t}{T_1}}) \right\} \Delta Q_1 - \left\{ \frac{1}{A_2}t + R (1 - e^{-\frac{t}{T_1}}) \right\} \Delta Q_{out} \tag{9}$$

As for the first term of the right hand side in equation (9), we can approximate with respect to time. These are, as follows;

In the case when time t is small $h(t) \approx 0$ (10)

In the case when time t is large $h(t) \approx (1/A'_2)t$ (11)

Considering this, we can draw the block diagram from equation (9) as shown in Figure 4.

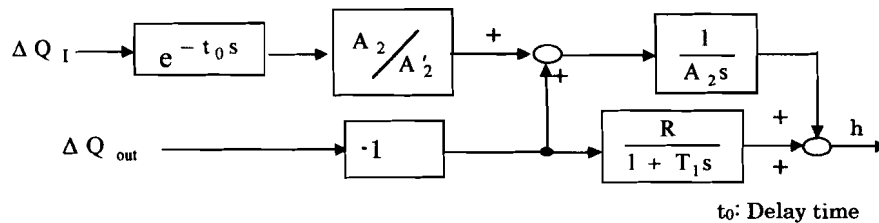


Figure 4 Block diagram of the two-tank model

2.2 Water level control by deadbeat control algorithm

To simplify the control algorithm, sampling time T_0 is chosen such as $T_0 \gg T_1$. Adding the controller to the two-tank system, we have the control block as shown in Figure 5.

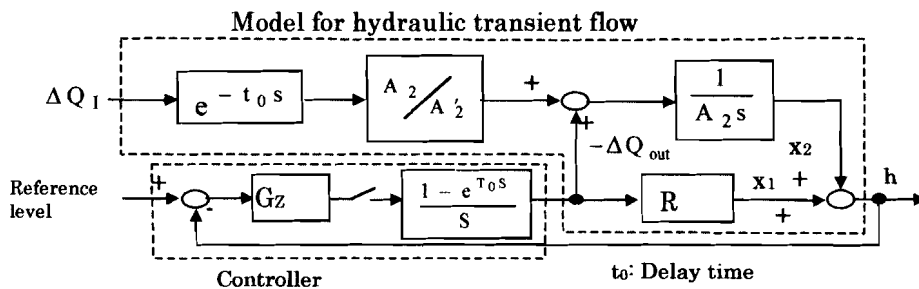


Figure 5 Control block diagram

At any time during the control action, it is required that the water level stays in the permissible range between $+\delta h$ and $-\delta h$. To attain the deadbeat control under the

permissible state of the water level, the following condition is required to hold.

Where, the sign + represents the condition that the water level reaches to its upper limit, $+\delta h$, and the sign - corresponds to the state where the water level reaches to its lower limit, $-\delta h$. Figure 6 shows the example where the water level reaches to its upper limit.

If $\Delta Q_{out} = \Delta Q_1 = 0$ and $x_2 = \pm \delta h$ at time T_{1a} , the output of controller becomes

$$\Delta Q_{out} = \mp \delta h \cdot m \quad (12)$$

and

$$x_1 = \mp R \cdot \delta h \cdot m \quad (13)$$

$$x_2 = \pm \delta h \quad (14)$$

where m is the gain of the controller. Next at time T_{2a} when the time elapses sufficiently after T_{1a} , the value of x_1 and x_2 becomes the following.

$$x_1 = \mp R \cdot \delta h \cdot m \quad (15)$$

$$x_2 = \pm \delta h \mp \frac{T}{A_2} \delta h \cdot m \quad (16)$$

Where T is the time from T_{1a} to T_{2a} .

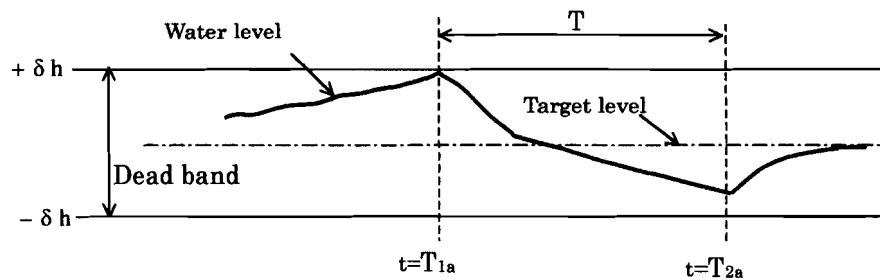


Figure 6 Water level variation

At this state, the necessary conditions that the variation of water level exist between $+\delta h$ and $-\delta h$ are as follows.

$$x_1 + x_2 \geq -\delta h \quad (17)$$

and

$$x_1 + x_2 \leq +\delta h \quad (18)$$

Substituting equations (15) and (16) to these conditions the following relation holds.

$$R \cdot m + \frac{1}{A_2} \cdot m \cdot T \leq 2 \quad (19)$$

On the while, if we put x_2 equals to zero at time T_{2a} , we have the following relation.

$$\frac{1}{A_2} \cdot m \cdot T = 1 \quad (20)$$

Solving equation (19) and (20) we have the condition for control gain and time T as

follows.

$$m \leq \frac{1}{R} \tag{21}$$

$$T = \frac{A_2}{m} \tag{22}$$

Putting $m = 1/R$ the output of the controller can be calculated from equation (12) such as

$$\Delta Q_{out} = \frac{\mp \delta h}{R} \tag{23}$$

On the other hand, to make dx_2/dt of equation (24) to be zero, it is necessary that $\Delta Q_{out} = \Delta Q_1$.

$$\dot{x}_2 = \frac{dh}{dt} = (\Delta Q_1 - \Delta Q_{out}) \frac{1}{A_2} \tag{24}$$

Using equation (8), the value of ΔQ_1 can be written as equation (25).

$$\Delta Q_1 = \Delta h \left(\frac{A_2'}{T_0} \right) \tag{25}$$

Therefore, necessary flow modification at time T_{2a} is given by

$$\Delta Q_{out} = \frac{\mp \delta h}{R} + \Delta h \frac{A_2'}{T_0} \tag{26}$$

The example of control action is shown in Figure 7.

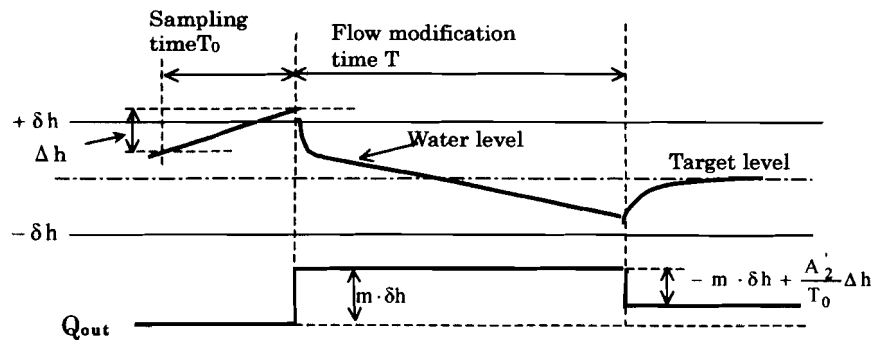


Figure 7 Water level and Q_{out} distribution in water level control

3. SIMULATION MODEL AND DETERMINATION OF CONTROL PARAMETER

3.1 Modeling of the system

The mathematical model for the penstock is the following distributed parameter system whose variables are the flow rate of the penstock and the water level.

System Dynamics:

$$\frac{1}{gA} \frac{\partial Q}{\partial t} - \frac{2QB}{gA^2} \frac{\partial B}{\partial t} + \frac{2Qq}{gA^2} - \frac{Q^2 B}{gA^3} \left(i + \frac{\partial H}{\partial x} \right) + \frac{\partial H}{\partial x} + \frac{|Q|Q}{K^2} = 0 \tag{27}$$

Equation for continuity:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{28}$$

where, $Q, H, A, B, K, q, x, t, g, i$ are inlet flow volume, water level, horizontal surface area of the penstock, width of the penstock, conductivity of the water surface, lateral flow volume of the penstock, distance (down stream direction as positive sign), time, gravity and gradient of the penstock respectively. The simulation is carried out based on the configuration shown in Figure 8 which is the combination of equations (27), (28) and the head tank model. The steps of the calculation are as follows. The initial conditions are the water level of head tank H and inlet flow volume of the upper stream river Q_i .

Solving equation (27),(28) by finite difference method[6], inlet flow volume to the head tank Q_{in} is calculated. Using this Q_{in} and out let flow volume Q_{out} , the water level in the head tank is calculated. This water level H is replaced with the initial water level and the procedures described above are iterated.

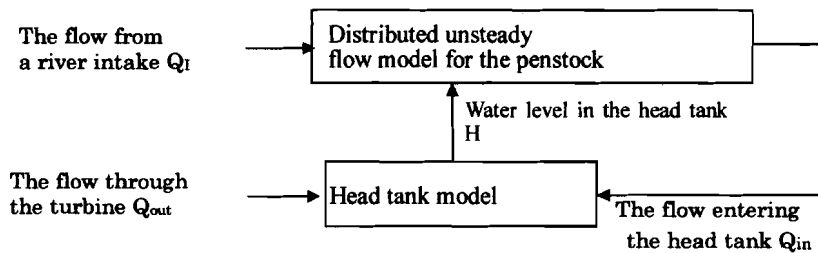


Figure 8 Simulation model

3.2 Determination of parameters of controller

Based on the simulated response characteristics, control parameters are determined. As for the parameters in water environment are fixed at the actual values. Making outlet disturbance of $\Delta Q_{out} 0.14m^3/sec$, at the inlet flow volume of $1.4 m^3/sec$, the response performance is calculated as shown in Figure 9. From this result the value of A_2 is assumed to $1500 m^2$ and the value of R is assumed to 0.13 . While from Figure 10, the simulated result of water level variation against the variation in inlet flow volume of upper stream river the value of A_2 is to $3300 m^2$. To simplify the calculation R is set to 0.1 . Then the control gain m and control time T are determined as 10.0 and 150 sec respectively.

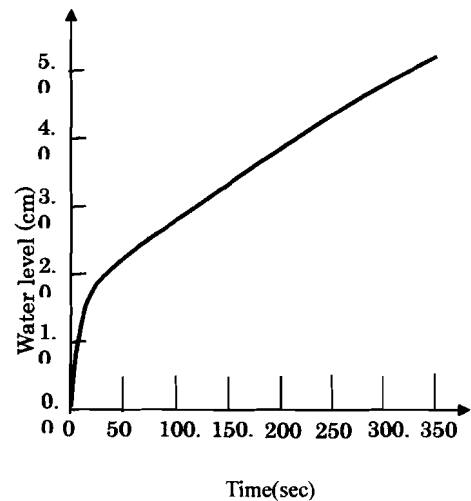


Figure 9
Water level response at $\Delta Q_{out} = 0.14m^3/sec$

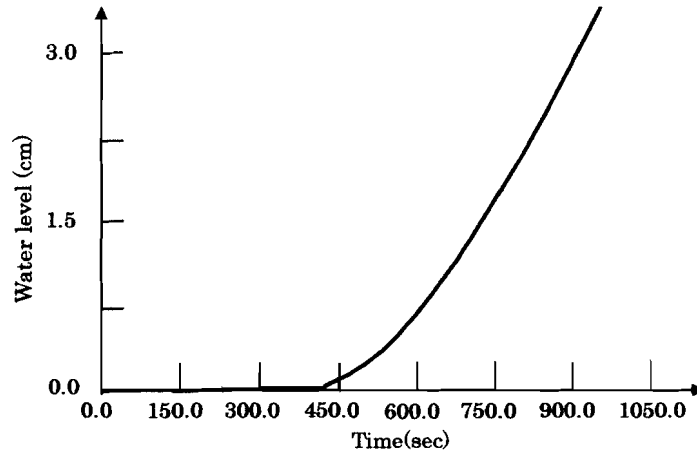


Figure 10 Water level response at $\Delta Q_1 = 0.24 \text{ m}^3/\text{sec}$

As stated before to simplify the control algorithm the sampling time T_0 is set more than ten times of the time constant of the system. However, excessive length of the sampling time leads to long time delay of measurement and induces over flow of the water level. Empirically, the following condition is necessary to hold for stable control.

$$\frac{\Delta Q_{out}}{A_2} T_0 \leq 0.005(5\text{mm})$$

From this we have the upper limit value of the sampling time T_0 as 82sec assuming the value of ΔQ_{out} be $0.2 \text{ m}^3/\text{sec}$. Therefore the reasonable value of sampling time must be between 40 sec and 80 sec.

4. APPLICATION TO ACTUAL SYSTEM

In the following, applied result of this control method to actual water environment is described. The physical data of the actual system are listed in Table 1.

Table 1 Parameters of the system

Length of the penstock	1800m
Width of the penstock	2.1m
Maximum flow from river intake	$1.1 \text{ m}^3/\text{sec}$
Horizontal surface area of the head tank	66 m^2

As for the control parameter, based on the simulation results, sampling interval and control time are set at 50 sec and 200sec respectively. Reference value of the control is 5cm below the water crossing point of the bank and dead band is set at $\pm 2\text{cm}$. Water level is

sampled every one second and stored in the table after filtered. The table is used to calculate variation of water level Δh and stores its newest 130 data.

4.1 Water level control

Here, water level control based on the algorithm described in chapter 2 is tried. Figure 11 shows control procedure of the water level in the head tank.

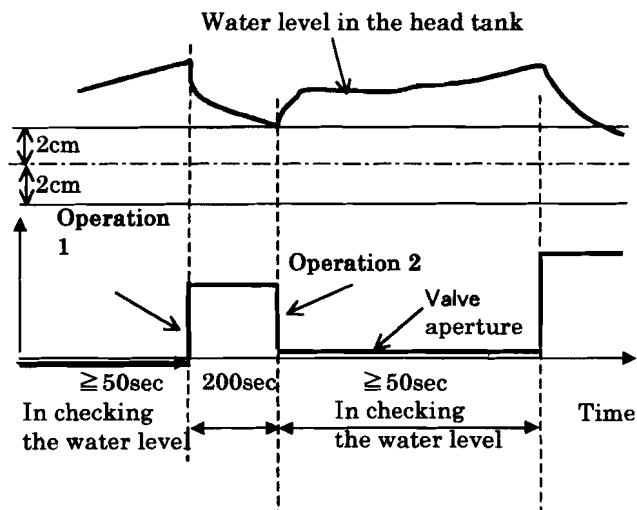


Figure 11 Automatic operation at water level control

In checking the water level, if the water level exceeds the dead band, the derivation of water level is calculated. And opening the guide valve for a certain interval starts after the end of operation 1 (equation (23)). After 200sec of the completion, control flow volume is calculated for the operation 2 (equation (26)) of guide valve opening. Waiting 50sec from this operation, water check procedure is beginning again.

Figure 12 shows the flow chart of water level control.

4.2 Experimental Results

Figures 13 and 14 show the experimental results. Figure 13 shows variation of water level and that of the guide valve opening in the case where water control is started from over flowed condition. By opening the guide valve, outlet flow volume is increased and over flowed water level is removed to the target level. On the other hand, Figure 14 shows the water level when the control is done by changing outlet flow and the inlet flow to the penstock. Water level is also successfully converge to its target level.

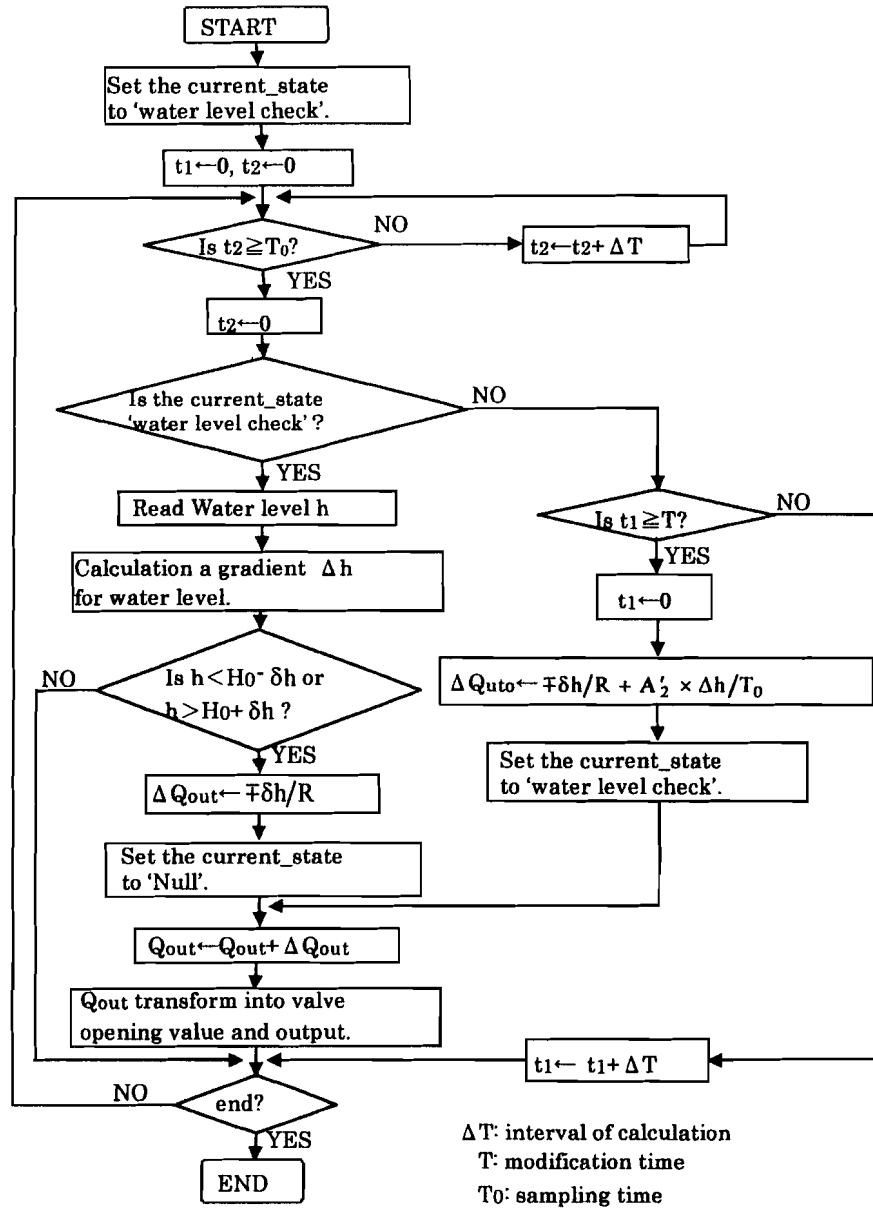


Figure 12 Flow chart of water level control

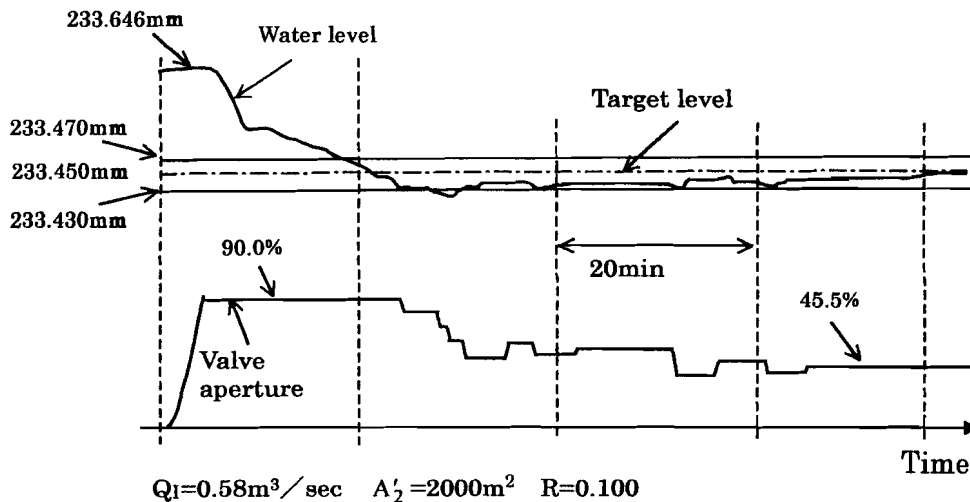


Figure 13 Example data of water level control

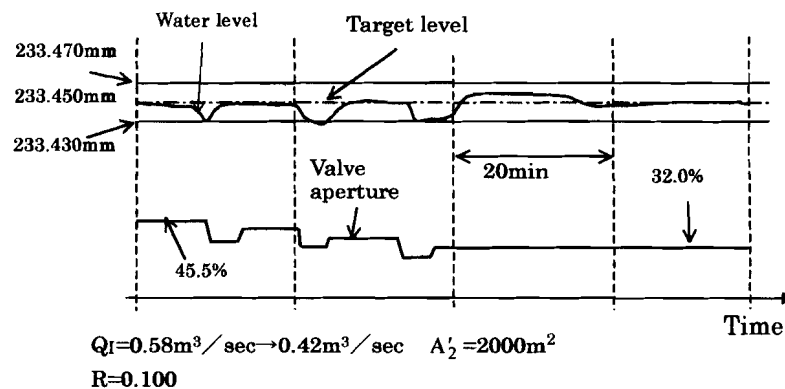


Figure 14 Example data of water level control

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5. CONCLUSION

In this paper, the control algorithm for water level control in the head tank of small scale hydropower generation. This system has the strong nonlinear characteristics and suffers disturbance from inlet flows fluctuation because of the climate change.

By making preliminary estimation of the system performance and control scheme of deadbeat control, high accuracy water level control in the head tank is confirmed.

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Appendix: Accuracy of the two-tank model

Figure A shows the comparison of the detailed calculation by equations (29) and (30) with the calculation by two-tank model. In the comparison, variation in water level of head tank is calculated changing ΔQ_{out} from 0 to 0.14 m³/sec. As valid from the figure two models are in good accordance. This means that the simplified two-tank model is enough to be used for the design of control algorithm.

As for the ΔQ_1 , same comparison is made and similar result was obtained.

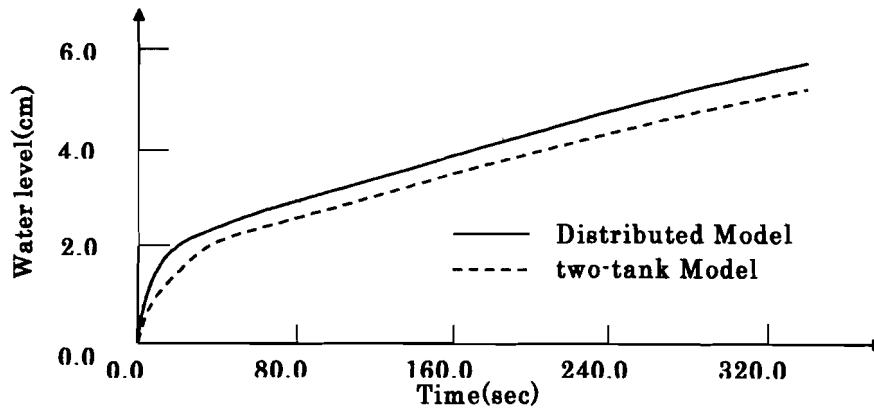


Figure A Water level response