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Price-Mediated Exchange Equilibrium:
Its Two Limiting Cases

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1. INTRODUCTION

This article is a part of the research which mainly purports to describe the exchange process mediated by prices. In the work of Maruyama [9], under the specification of the transaction rule in the price-mediated exchange economy, we gave some equilibrium concept (namely the price-mediated exchange equilibrium). This definition takes its rise in the modification of Morishima's characterization of Walrasian exchange equilibrium, and assumes a natural constraint imposed on each agent's transaction mediated by prices and is endowed with the essential property of voluntary exchange. This equilibrium concept is a general one, because of the fact that it includes a Walrasian exchange equilibrium as its unique efficient state and the several non-Walrasian equilibria(e.g. monopolistic equilibrium and fixprice equilibrium) as its special states. We will attempt to pose a unified approach to the explanation of several representative configurations of equilibria by imposing special restrictions on the general structure of price-mediated exchange economy. At first, in this article we will examine two limiting cases which extract the efficient state of the price-mediated exchange economy (i. e. the Walrasian exchange equilibrium), hence, by doing so, we will capture the essential meanings of Walrasian exchange equilibrium.

The first limiting case we will consider here is the economy which consists of a sufficiently large numder of agents and each agent's volume of transaction is relatively so small that we can treat it as a negligible one. Then we can show the limit theorem : the equilibrium concept of a price-mediated exchange economy coincides with the Walrasian exchange equilibrium when the economy is in our limiting case. Note that this limit theorem is *prima facie* giving the same line of reasoning as the one of usual cooperative market exchange game model, but they are distinct from each other, because the former explicitly presupposes the structure of exchange mediated by prices. The limit theorem of the cooperative market exchange model says that the core, which is difinitely Pareto efficient, shrinks to Walrasian exchange equilibria as the number of agent approaches infinity, however, our limit theorem says that the set of price-mediated exchange equilibria, which are in general shown to be Pareto inefficient, shrinks to the efficient state of it. This result may be interpreted as follows: the general inefficient property of our equilibria originates from the essential nature of voluntary exchange mediated by prices, if the economy is large, hence, there are many similar agents and each agent's volume of transaction is negligibly small, then the effect of each agent's selfish behaviour sinks under the ocean and the efficient state can only be extracted as the price-mediated exchange equilibrium.

The second limiting case we will examine in this article is the terminal state of some assumed dynamic exchange process when time goes to infinity. We will suppose that any transaction is actually carried out only at the price-mediated exchange equilibria. After the transaction, however, if the state is not a price-mediated exchange equilibrium when viewed itself as an initial state, then we will suppose that the assumed recontract mediated by prices is reiterated, which generates some new price-mediated exchange equilibrium, the allocation is altered, and this dynamic exchange process continues ad infinitum. Then we will show that this sequence of allocations generated by the recurrence of price-mediated exchange game will converge to the Walrasian exchange equilibrium. This result implies that the recurrence of recontract may exhaust mutually advantageous contracts and so extracts the efficient price-mediated exchange equilibrium as its limiting case.

In the final section we will partially examine the movement of prices which is associated with the sequence of price-mediated exchange equilibria. It will be shown that the movement of prices directed by actual market participants *a posteriori* obeys the *Law of Supply and Demand.* The present state of the research area concerning to the price-adjustment from the market-wide perspective lacks a systematic explanation, hence, our result may serve as one possible clue to it.

2. *Preliminary Remarks*

Let $\mathscr{E} = \{(X_i, U_i, \omega_i)_{i \in I}, J\}$ denote an exchange economy with a finite set of agents $I = \{i \mid i = 1, 2, \ldots, n\}$ and a set of commodities $J = \{j \mid j = 1, 2, \ldots, m\}$. $X_i \subset R^m$ denotes agent i's feasible consumption set,

U_i denotes his utility function, and ω_i denotes his initial endowment of commodities. Concerning to the characteristics of each agent we assume the following: for any agent $i \in I$

Assumption 1 : feasible consumption set $X_i =$ non-negative orthant of m-dimensional Euclidean space R^m_+,

Assumption 2 : initial endowment vector ω_i is positive,

Assumption 3 : utility function U_i is strictly quasi-concave, monotone increasing and continuously differentiable, and

Assumption 4 : $\{x_i \in X_i \mid U_i(x_i) \geqq U_i(\omega_i)\} \cap \partial R^m_+ = \phi$

, where ∂R^m_+ denotes the boundary of R^m_+.

Now we will define the structure of exchange and the solution concept of the price-mediated exchange economy.

Definition 1 : We will say that a coalition C can *p-block* the allocation $(x_i)_{i \in I}$ from the initial allocation $(\omega_i)_{i \in I}$ is there exists a pair of price vector p and the allocation $(p, (\bar{x}_i)_{i \in C})$, such that

[i] $\sum_{i \in C} \bar{x}_i = \sum_{i \in C} \omega_i$,

[ii] $(\forall i \in C) : (\sum_j p_j \bar{x}_{ij} = \sum_j p_j \omega_{ij})$,

[iii] $(\forall i \in C) : (U_i(\bar{x}_i) \geqq U_i(x_i))$ and $(\exists i \in C) : (U_i(\bar{x}_i) > U_i(x_i))$, and

[iv] $(\forall i \in C) : (\nexists t \in [0, 1]) : (U_i(x_i(t)) > U_i(\bar{x}_i))$,

where x_i denotes agent i's consumption vector, x_{ij} and ω_{ij} denote his consumption and initial endowment of commodity j, respectively, and $x_i(t) = \omega_i + t(\bar{x}_i - \omega_i)$.

This definition differs from the usual one of cooperative market game in two points, condition [ii] and [iv]: in the price-mediated exchange economy any agent is naturally supposed to trade subject to his budget constraint and in the voluntary exchange economy it is natural to suppose that nobody can be forced to trade beyond he wishes. In the following, when we consider two commodity exchange economy, we often use one special transaction rule(proportional rationing scheme).

Definition 2 : We say that a coalition C allocates its initial allocation $(\omega_i)_{i\in C}$ among members according to a *proportional rationing scheme* at the price vector p; if the allocation $(\bar{x}_i)_{i\in C}$ is given as follows:

$$\bar{x}_i = \tilde{z}_i(p) + \omega_i \quad \text{for any agent} \quad i \in S(p)$$

$$\bar{x}_i = \delta\tilde{z}_i(p) + \omega_i \quad \text{for any agent} \quad i \in L(p),$$

where $\tilde{z}_i(p)$ denotes agent i's Walrasian net trade vector at p, that is, $\tilde{z}_i(p) = (\tilde{z}_{i1}(p), \tilde{z}_{i2}(p))$, $S(p)$ denotes the set of agents who belong to the short side of markets at p, which is defined as $S(p) = \{i \in C \mid \tilde{z}_{i1}\sum_{i\in C}\tilde{z}_{i1} \leqq 0\}$, $L(p)$ denotes the set of agents who belong to the long side of markets at p, which is defined as $L(p) = \{i \in C \mid \tilde{z}_{i1}\sum_{i\in C}\tilde{z}_{i1} > 0\}$, and δ is a rationing coefficient of both markets, which is defined as $\delta = -\sum_{i\in S}\tilde{z}_{i1}/\sum_{i\in L}\tilde{z}_{i1}$.

It can be easily seen that the allocation $(\bar{x}_i)_{i\in C}$ satisfies conditions [i], [ii] and [iv] in *Definition 1*, if $(\bar{x}_i)_{i\in C}$ is given by the special transaction rule specified above. Hence we will define the special concept of *p-blocking* in the following manner.

Definition 3 : We will say that under a special transaction rule a coalition C can *p-block* the allocation $(x_i)_{i\in I}$ from the initial allocation $(\omega_i)_{i\in I}$ is there exists a pair of price vector and the allocation $(p, (\bar{x}_i)_{i\in C})$ such that

[i] $(\bar{x}_i)_{i\in C}$ is given by the transaction rule in *Definition 2*, and
[ii] $(\forall\, i\in C):(U_i(\bar{x}_i)\geqq U_i(x_i))$ and $(\exists\, i\in C):(U_i(\bar{x}_i)> U_i(x_i))$.

With this concept of *p-blocking* we define the following concept.

Definition 4 : We say the allocation $(x_i)_{i\in I}$ is *p-Pareto optimal*, when it is feasible, that is, $\sum_{i\in I}x_i=\sum_{i\in I}\omega_i$ and $x_i\in X_i$ for any agent i and it is not *p-blocked* by any coalition which consists of the whole of agents. Furthermore if the allocation is feasible and it is not *p-blocked* by any coalition we say that this allocation belongs to the *p-core*.

Now we will restate the equilibrium concept in the price-mediated exchange economy, which was employed in Maruyama [9]. Our concept of equilibrium is directly related to the characterization of Walrasian exchange equilibrium given by Morishima [10]: the allocation $(x_i)_{i\in I}$ is a Walrasian exchange equilibrium one relative to the initial allocation $(\omega_i)_{i\in I}$ if and only if it is Pareto optimal and the conditions

[a] $(\exists\, p\geqq 0):(\forall\, i\in I):(\sum_j p_j x_{ij}=\sum_j p_j\omega_{ij})$, and

[b] $(\forall\, i\in I):(\nexists\, t\in[0,1]):(U_i(x_i(t))> U_i(x_i))$

are satisfied, where $x_i(t)=\omega_i+t(x_i-\omega_i)$. The condition [a] is a fundamental constraint which is imposed on each agent's transaction in the price-mediated exchange economy, and the condition [b] presents the essential property of voluntary exchange. But, the concept of Pareto optimality is defined independently from the fact that exchange is mediated by prices. We are now explicitly considering the exchange mediated by prices, hence we will employ the concept of *p-Pareto optimality* in place of the Pareto optimality in the usual sense.

Definition 5 : We say the pair of price vector and the allocation $(p,(x_i)_{i\in I})$ a *price-mediated exchange equilibrium* (abbreviated as *p. e. e.*) relative to the initial allocation $(\omega_i)_{i\in I}$ when it is *p-Pareto optimal* and the above two conditions [a] and [b] are satisfied. Furthermore we say $(p,(x_i)_{i\in I})$ a *strong price-mediated. exchange equilibrium* (abbreviated as *strong p. e. e.*) relative to $(\omega_i)_{i\in I}$ when it is *p-core* and the above two conditions [a] and [b] are satisfied.

Here will give some remarks on the *p. e. e.*

Remark 1 : From the definition of *p. e. e.* we can directly derive the statement that the pair of price vector and the allocation $(p,(x_i)_{i\in I})$ is a Walrasian exchange equilibrium if and only if it is a *p. e. e.* and the allocation $(x_i)_{i\in I}$ is Pareto optimal in the usual sense. As a corollary of this result we can state that any *p. e. e.* allocation except for Walrasian exchange equilibrium one (namely any *non-trivial p. e. e.* allocation) is not Pareto optimal in the usual sense.

Remark 2: If the initial allcation is positive and is not Pareto optimal, then it is not a *p. e. e.* allocation at all.

This remark can be easily justified as follows. First we note the fact that if the initial allocation is positive, then the existence of Walrasian exchange equilibrium is assured under our *Assumptions*, and Walrasian exchange equilibrium is a *p. e. e.* and its allocation is Pareto optimal. Then, it is clear that if the initial allocation is not Pareto optimal, it is not a *p-Pareto optimal* allocation, hence, is not a *p. e. e.* allocation.

3. *On the Two Limiting Cases*

We have already given the characterization of *p. e. e.* and explored the close relation between *p. e. e.*, Walrasian exchange equilibrium and non-Walrasian equilibrium in [9]. Hereafter we will focus our attention on *Remark 1*: Walrasian exchange equilibrium is the unique efficient *p. e. e.* That is, in this section we will examine two limiting cases which extract the efficient state of the price-mediated exchange economy(i.e. the Walrasian exchange equilibrium). Hence, by doing so, we will capture the essential meaning of Walrasian exchange equilibrium. A representative approach of Debreu and Scarf [3] characterized a Walrasian exchange equilibrium as a limiting case of the solution of cooperative market exchange game(i. e. the core), where economy consists of a sufficiently large number of agents. That is, they showed the limit theorem: as the number of agents approaches infinity, the core allocation coincides with the one of Walrasian exchange equilibrium. There are various interpretations about this limit theorem. In my view, the solution concepts of the cooperative market exchange game (i. e. the core) is defined independently from the fact that exchange is mediated by prices. However, Walrasian exchange equilibrium is nothing but a equilibrium concept of the price-mediated exchange economy. In other words, Walrasian exchange equilibrium, in its definition, prespposes a special structure of exchange: price-mediated exchange. The concept of the core in the usual cooperative market exchange game and the Walrasian exchange equilibrium presuppose virtually different structures of exchange, respectively. In the following we will explore the limit theorem in the price-mediated exchange economy. Here we will focus our attention on the *strong p. e. e.* which is given in *Definition 5.* By strengthening the equilibrium concept in this way can we directly extract the efficient state of the price-mediated exchange economy? Consider a two commodity-two agent exchange economy. Then in this economy we can easily show that *p. e. e.* is equivalent to *strong p. e. e.* and so that *non-trivial strong p. e. e.* is not Pareto optimal in the usual sense. Hence, in general, we can say that *strong p. e. e.* does not necessarily coincide with the Walrasian exchange equilibrium.

Here we will examine the property of *strong p. e. e.* in the limiting case where the economy consists of a sufficiently large number of agents. We can derive the following result in the two commodity exchange economy, where any coalition uses the special transaction rule specified in *Definition 2.*

Proposition 1: In a sufficiently large *r-fold replica economy*, the *strong p. e. e.* allocation coincides with the Walrasian exchange equilibrium one.

Proof: We will show this by reductio ad absurdum. Suppose the *r-fold replica economy* $\mathscr{E}^r$: there are n types of agents, with r agents of each type and every agent of the same type has precisely the same preference and the same endowment. Let $(\bar{p}, \bar{x})$ be the pair of *strong p. e. e.* price vector and allocation of the sufficiently large *r-fold replica economy.* Suppose that it is not the pair of Walrasian exchange equilibrium. Then from *Remark 1* the allocation $\bar{x}$ is not Pareto optimal. Now for any chosen type i of agents who belong to the short side of markets at $\bar{p}$ we can take some price vector $\hat{p}$ such that $\hat{p}$ is sufficiently close to $\bar{p}$ and $U_i(\tilde{z}_i(\hat{p})+\omega_i) > U_i(\tilde{z}_i(\bar{p})+\omega_i)$ for any agent of type i. If we choose any type k of agents who belong to the long side of markets at $\bar{p}$, then from the fact that $U_k(\tilde{z}_k(\bar{p})+\omega_k) > U_k(\bar{x}_k)$ and the continuity of $\tilde{z}_k(p)$ and his utility function it is clear that $U_k(\tilde{z}_k(\hat{p})+\omega_k) > U_k(\bar{x}_k)$ for any agent of type k. Here we must note that an allocation of *strong p. e. e.* assigns the same consumption to all agents of the same type in our specified exchange economy.

Now we suppose the sequence of integer r_i such that it approaches infinity. We define $r_k(r_i)$ as the minimum integer r_k which satisfies the relation $r_i|\tilde{z}_{i1}(\hat{p})| \leqq r_k|\tilde{z}_{k1}(\hat{p})|$. Suppose the coalition which consists of r_i agents of type i and $r_k(r_i)$ agent of type k. If this coalition makes a transaction specified in *Definition 2* at the price $\hat{p}$, the resulting consumption vector of each agent is:

$\hat{x}_i = \tilde{z}_i(\hat{p}) + \omega_i$ for any agent of type i

$\hat{x}_k = \delta(r_i)\tilde{z}_k(\hat{p}) + \omega_k$ for any agent of type k,

where $\delta(r_i)$ is a rationing coefficient of both markets, which can be given as $\delta(r_i) = r_i|\tilde{z}_{i1}(\hat{p})| / r_k|\tilde{z}_{k1}(\hat{p})|$ by inspecting the construction of the coalition. From the definition of $r_k(r_i)$ it can be justified that $\lim_{r_i \to \infty}\delta(r_i) = 1$. Then it is clear that this coalition *p-blocks* the allocation $\bar{x}$ when the economy is sufficiently large *r-fold replica economy*. This contradicts the supposition that $(\bar{p}, \bar{x})$ is a pair of *strong p. e. e.* This completes the proof. ‖

This result implies that, when we presuppose a price-mediated exchange economy, and formulate it in a cooperative game manner, then the limiting case of equilibrium coincides with Walrasian exchange equilibrium, where there is a sufficiently large number of agents. And note that our approach here is different from the limit theorem of the usual cooperative market exchange game model in the point : presupposition of the structure of exchange (price-mediated exchange). Recently, many authors focus their attention on the structure of price-mediated exchange and formulate it in a non-cooperative game manner. (See Schmeidler [13], Shapley [14], Shapley and Shubik [15], Roberts and Postlewaite [12], and works in [11]). Roberts and Postlewaite also explicitly assume that exchange is guided by prices, then they show that under some assumptions the incentive to manipulate prices will disappear in a large economy. Maruyama [8] formulates the price-mediated exchange economy in a non-cooperative game manner and shows that equilibrium in this economy coincides with Walrasian exchange equilibrium when the economy consists of a large number of agents. In combining this result with the above result we may be able to support the usual interpretation of Walrasian exchange equilibrium in the price-mediated exchange economy.

Next we will examine the alternative limiting case of *p. e. e.* That is, we will examine the limiting case of some dynamic process of exchange when time goes to infinity. Now we will examine the limit of sequence generated by the following hypothetical exchange process. At first we suppose that any exchange is actually carried out only at the price-mediated exchange equilibrium. That is, by noting the property that any *p. e. e.* allocation is *p-Pareto optimal*, this supposition implicitly assumes that recontracting process mediated by prices (information-exchange process where price plays the role of essential means of communication) exhausts any mutually advantageous trading contract as possible and only after that any exchange is actually carried out. In this article we will not precisely define this recontracting process mediated by prices, rather we simply assume some recontracting process such that if there is a room of recontract, then it is done and ultimately it generates some *p. e. e.* as its stable point (the precise definition of recontract mediated by prices is given in Maruyama [7]).

For any given initial allocation, some *p. e. e.* allocation is generated by this assumed recontracting process and then the transition of initial allocation to this *p. e. e.* allocation is undertaken. Here we must note the fact that under our *Assumption 4* any *p. e. e.* allocation must belong to the interior of non-negative orthant. Hence, if this *p. e. e.* allocation is not Pareto optimal, then by noting *Remark 2*, it is not a *p. e. e.* allocation when viewed itself as a initial allocation, therefore, the assumed recontract is reiterated at this new initial allocation. This recurrence of recontract mediated by prices generates the sequence of *p. e. e.* allocation $\{\omega(t)\}_{t=1}^{\infty}$, which satisfies the relation:

$$\omega(t+1) \in P^*(\omega(t)) \subset P_u(\omega(t)),$$

where $\omega(t)$ denotes the allocation at time t, $P^*(\omega(t))$ denotes the set of *p. e. e.* allocation relative to $\omega(t)$, and the set $P_u(\omega(t)) = \{\omega = (\omega_i)_{i\in I} \mid U_i(\omega_i) \geqq U_i(\omega_i(t))$ and $\omega_i \in X_i$ for any agent i, and $\Sigma_{i\in I}\omega_i = \Sigma_{i\in I}\omega_i(t)\}$.

In the following we will examine the limit of sequence of *p. e. e.* allocation. We note the Pareto improving property of the sequence. Then we can show the following result.

Proposition 2: The sequence of *p. e. e.* allocation $\{\omega(t)\}_{t=1}^{\infty}$ converges to a Pareto optimal allocation.

Proof: The sequence $\{\omega(t)\}_{t=1}^{\infty}$ has a convergent subsequence, because the set $P_u(\omega)$ is compact. So we choose the convergent subsequence $\{\omega(t_\nu)\}_{\nu\in Q}$ such that $\lim_{\nu\in Q}\omega(t_\nu)=\omega(\infty)$, where Q is an infinite set of positive integers. Let $U(\omega)=(U_i(\omega_i))_{i\in I}$, where U_i is consumer i's utility function. Then from the definition of the *p. e. e.* and the dynamic process it follows that $(\forall t):(U(\omega(t+1))\geqq U(\omega(t)))$. That is, the sequence $\{U(\omega(t))\}_{t=1}^{\infty}$ which corresponds to the sequence $\{\omega(t)\}_{t=1}^{\infty}$ is monotone increasing on the compact set, hence this sequence converges. Let $\lim_{t\to\infty}U(\omega(t))=\bar{U}$. From the continuity of the utility fuction we have $\lim_{\nu\in Q}U(\omega(t_\nu))=U(\lim_{\nu\in Q}\omega(t_\nu))=U(\omega(\infty))$. Here we have $U(\omega(\infty))=\bar{U}$ because of the fact that the convergent sequence $\{U(\omega(t))\}_{t=1}^{\infty}$ must have unique limit. Now we choose another subsequence $\{\omega(t_\nu+1)\}_{\nu\in Q}$. Because this is a sequence in the compact set, we have a subsequence $\{\omega(t_\nu+1)\}_{\nu\in Q_1}$ such that $\lim_{\nu\in Q_1}\omega(t_\nu+1)=\omega(\infty+1)$, where $Q_1\subset Q$. By the similar argument we have $U(\omega(\infty+1))=\bar{U}$. Hence it follows that $U(\omega(\infty))=U(\omega(\infty+1))$. By noting the fact that the infinite subsequence $\{\omega(t_\nu)\}_{\nu\in Q_1}$ of the convergent sequence $\{\omega(t_\nu)\}_{\nu\in Q}$ must be convergent, we have $\{\omega(t_\nu)\}_{\nu\in Q_1}$, $\lim_{\nu\in Q_1}\omega(t_\nu)=\omega(\infty)$, $\{\omega(t_\nu+1)\}_{\nu\in Q_1}$, $\lim_{\nu\in Q_1}\omega(t_\nu+1)=\omega(\infty+1)$, and $(\forall\nu):(\omega(t_\nu+1)\in P_u(\omega(t_\nu)))$. Then from the upper semi-continuity of $P_u(\omega)$, we have $\omega(\infty+1)\in P_u(\omega(\infty))$. Now suppose that $\omega(\infty)$ is not Pareto optimal. Then from *Remark 2* $\omega(\infty)$ is not a *p. e. e.* allocation. From the definition of the dynamic process we have $\omega(\infty)\neq\omega(\infty+1)$. This result implies that $U(\omega(\infty+1))\geq U(\omega(\infty))$, which contradicts the above result: $U(\omega(\infty+1))=U(\omega(\infty))$. Hence $\omega(\infty)$

must be a Pareto optimal allocation. This result implies that limit points of the sequence $\{\omega(t)\}_{t=1}^{\infty}$ are Pareto optimal.

Furthermore we can show that the sequence $\{\omega(t)\}_{t=1}^{\infty}$ converges to a Pareto optimal allocation. This can be done by showing the uniqueness of the limit point. We will show this by reductio ad absurdum. Let $\{\omega(t_\nu)\}_{\nu\in\bar{Q}}$ and $\{\omega(t_\nu)\}_{\nu\in\bar{\bar{Q}}}$ be subsequences of $\{\omega(t)\}_{t=1}^{\infty}$ converging, respectively, to $\bar{\omega}$ and $\bar{\bar{\omega}}$. Then from the above argument we have $U(\bar{\omega}) = U(\bar{\bar{\omega}})$. Now suppose that $\bar{\omega} \neq \bar{\bar{\omega}}$. Then by noting the strict quasi-concavity of U_i it follows that this supposition cotradicts the already shown result that both of limit points $\bar{\omega}$ and $\bar{\bar{\omega}}$ are Pareto optimal. Hence we verified that the limit point was unique and so that the sequence of allocations $\{\omega(t)\}_{t=1}^{\infty}$ converged to a Pareto optimal allocation. This completes the proof. ||

Considering the sequence of *strong p. e. e.* allocation, we may have a same result. The *strong p. e. e.* is a *p-core*, hence, the exchange process defined above can be interpreted as the recurrence of price-mediated exchange game. This line of analysis of the exchange process may accord with the approach suggested by Laroque [6], and is related to recent works by Schoumaker[16] and Tulkens and Zamir[17] in the literature of planning procedure (designing the mechanism of resouce allocation processes).

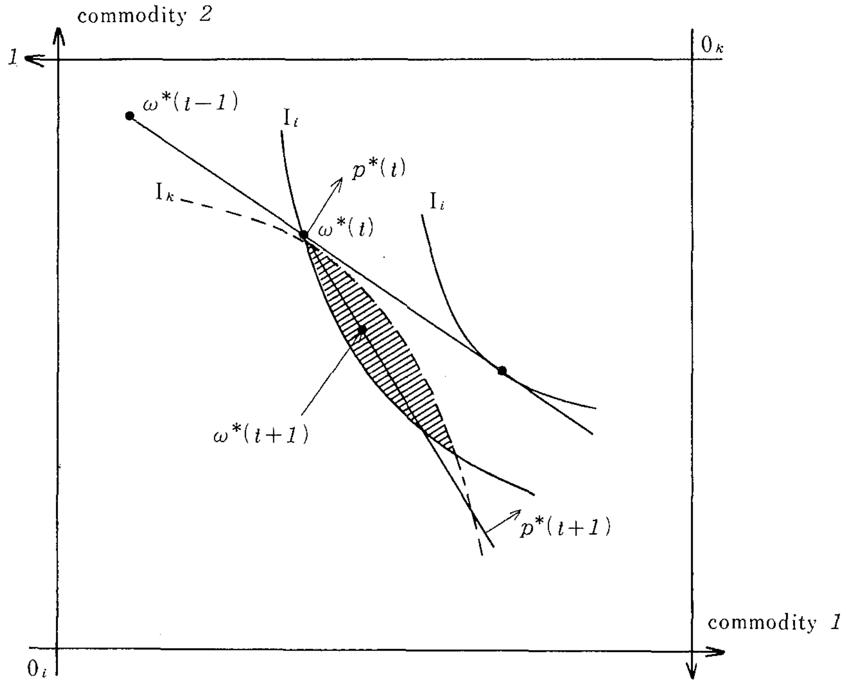
4. *On the Movement of Prices*

From the recurrence of recontract mediated by prices is derived the sequence of price-mediated exchange equilibria. In this section we will examine the property of the associated sequence of *p. e. e.*

price vectors, that is, the price vector at which any transaction is carried out. Can we say anything about the movement of prices? As a clue to this examination we begin with a simplified example. Consider a pure exchange economy with two agents indexed i and k, and two commodities indexed *1* and *2*. The recurrence of recontract mediated by prices can be illustrated as follows by using a usual Edgeworth's box-diagram. We denote 0_i as the agent i's origin and 0_k as the agent k's origin. The horizontal axis measures the quantity of commodity *1* and the vertical axis measures the quantity of commodity *2*.

We denote $(p^*(t), \omega^*(t))$ as a pair of *p. e. e.* price vector and allocation at time t. Suppose that *p. e. e.* allocation at time $t-1$ is illustrated as a point $\omega^*(t-1)$ in our diagram, then, by noting the *Definition* of price-mediated exchange equilibrium, *p. e. e.* allocation at time t must be such a point $\omega^*(t)$ that is shown in our diagram. In this case $\omega^*(t)$ is not Pareto optimal, where I_i and I_k are agent i's and agent k's indifference curves, respectively. Hence, from *Remark 1*, $\omega^*(t)$ is not a *p. e. e.* when viewed itself as a initial allocation. Consequently, after the transaction at time t some assumed recontract is re-opened, and next *p. e. e.* allocation, i.e., *p. e. e.* allocation at time $t+1$ will be generated. Here, *p. e. e.* allocation $\omega^*(t+1)$ must be Pareto superior to the initial allocation $\omega^*(t)$, hence, *p. e. e.* allocation at time $t+1$ must be such a point $\omega^*(t+1)$ which belongs to the mutually advantageous shaded area. Therefore *p. e. e.* price vector at time $t+1$ must be illustrated as $p^*(t+1)$ in our diagram. This is a sketch of the sequence of *p. e. e.* generated by the recurrence of recontract in a simplified exchange economy.

Here we pay attention to the movement of *p. e. e.* price vectors. Note that there is a excess demand of commodity *1* and a excess supply of commodity *2* at the price vector $p^*(t)$. Then this example shows that the movement of *p. e. e.* price vector from $p^*(t)$ to $p^*(t+1)$ obeys the *Law of Supply and Demand.* This may be an interesting feature of our exchange process mdiated by prices. In the following we will formulate this intuition by restricting our analysis to the case: two commodity exchange economy, where any coalitional transaction is specified by the one given in *Definition 2.* We will show the following proposition.



Proposition 3: The sequence of *p. e. e.* price vectors $\{p^*(t)\}_{t=1}^{\infty}$ in the above specified economy obeys the *Law of Supply and Demand.*

Proof: Let $(p^*(t), (z_i^*(t))_{i\in I})$ denote the pair of *p. e. e.* price vector and the list of net trade vectors at time t. When we denote $(\omega_i(t))_{i\in I}$ as the initial allocation at time t and $x^*(t)=(x^*(t))_{i\in I}$ as the *p. e. e.* allocation at time t, $z_i^*(t)=x_i^*-\omega_i(t)$ for any agent i. We say the movement of *p. e. e.* price vector obeys *the Law of Supply and Demand,* if the following condition holds: (L) $(p_j^*(t+1)-p_j^*(t))\sum_{i\in I}\tilde{z}_{ij}(p^*(t))>0$ for any commodity j, where $\tilde{z}_{ij}(p^*(t))$ denotes agent i's Walrasian net trade of commodity j at the price vector $p^*(t)$. We are considering now two commodity exchange economy, hence if we normalize the price vector as $p^*=(p_1{}^*, 1)$, then the above condition (L) coincides with the condition

$$p^*(t+1)\sum_{i\in I}\tilde{z}_i(p^*(t))>0.$$

We will show that this condition will be satisfied.

Here we choose any agent $i\in S(p^*(t))$, that is, the agent who belongs to the short side of markets at the price vector $p^*(t)$. See *Definition 2* and note the Pareto improving property of the sequence of *p. e. e.* allocation, then $p^*(t)z_i^*(t+1)\geqq 0$ for any agent $i\in S(p^*(t))$. Hence we have $\sum_{i\in S}p^*(t)z_i^*(t+1)\geqq 0$. Now we have $\sum_{i\in L}p^*(t)z_i^*(t+1)\leqq 0$ because of the fact that $\sum_{i\in I}z_i^*(t+1)=0$, where L denotes the set of agents who belong to the long side of markets at $p^*(t)$. Now we can show that for any agent i who belongs to the long side of markets at $p^*(t)$

$$p^*(t+1)\tilde{z}_i(p^*(t))>0.$$

We will show this by reductio ad absurdum. At first it can be easily seen that the sign of $p^*(t+1)\tilde{z}_i(p^*(t))$ is same and is not zero for any agent $i \in L(p^*(t))$. We must note this fact. Hence we suppose that $p^*(t+1)\tilde{z}_i(p^*(t)) < 0$ for any agent $i \in L(p^*(t))$. This implies that $p^*(t+1)\tilde{z}_i(p^*(t)) < p^*(t+1)\tilde{z}_i(p^*(t+1)) = 0$. Then, from the weak axiom of revealed preference we have $p^*(t)\tilde{z}_i(p^*(t+1)) > 0$ for any agent $i \in L(p^*(t))$. In our specified exchange economy this implies that $p^*(t)z_i^*(t+1) > 0$ for any agent $i \in L(p^*(t))$. Hence we have $\sum_{i \in L} p^*(t) z_i^*(t+1) > 0$. This is a contradiction.

In our specified exchange economy note that the following condition holds: $\sum_{i \in I} \tilde{z}_i(p^*(t)) = (1-\delta)\sum_{i \in L} \tilde{z}_i(p^*(t))$, where δ is a same rationing coefficient in the both markets, i.e., $\delta = -\sum_{i \in S} \tilde{z}_{i1}(p^*(t)) / \sum_{i \in L} \tilde{z}_{i1}(p^*(t))$ and $0 < \delta < 1$. Accordingly we have $p^*(t+1)\sum_{i \in I} \tilde{z}_i(p^*(t)) > 0$. This completes the proof. ||

The theory of price adjustment has been dominated by the idea that prices rise in the presence of excess demand and fall in the presence of excess supply, namely, the *Law of Supply and Demand* which crystallized the empirical fact. However, Arrow [1] and Koopmans [5] have already warned us against this treatment of the problem and posed several scandals surrounding the usual explanation. That is, the crucial property of the usual interpretation of the *Law of Supply and Demand* is that variations in prices are generated by the working of the impersonal force of the market and therefore, separated from the activities of the individual market participants. Comparing the fact that demand and supply functions are formulated in accordance with the individuals' rational maximizing behaviours, the mechanics of price change reflects no one's rational maximizing behaviour. In other words, whose behaviour is thereby expressed? And how is that behaviour motivated? Each individual market participant is supposed to take prices as given and determines his choices as to purchases and sales accordingly; there is no one left over whose job it is to make a decision on price. This fact speaks eloquently of ad hoc characteristics of the usual explanation of the price adjustment. In reply to this critique Barro [2] and Iwai [4] gave choice-theoretic reasonings for the *Law of Supply and Demand* by considering the optimal price adjustment rule of the monopolistic firm. The result of this section may be viewed as a clue to explaining the price adjustment from the market-wide perspective.

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References

[1] Arrow, K. J., "Toward a Theory of Price Adjustment," in *The Allocation of Economic Resources*, M. Abramovitz et al. (eds.), Stanford University Press (1959), pp. 41-51.
[2] Barro, R., "A Theory of Monopolistic Price Adjustment," *Review of Economic Studies* (1972), pp. 17-26.
[3] Debreu, G. and H. Scarf, "A Limit Theorem on the Core of an Economy," *International Economic Review* (1963), pp. 235-246.
[4] Iwai, K., "The firm in Uncertain Markets and Its Price, Wage and Employment Adjustment," *Review of Economic Studies* (1974), pp. 257-276.
[5] Koopmans, T. C., *Three Essays on the State of Economic Science*, McGraw-Hill (1957).
[6] Laroque, G., "On the Dynamics of Disequilibrium: A Simple Remark," *Review of Economic Studies* (1978), pp. 273-278.
[7] Maruyama, M., "On the Competitive Trading Process," (1979), mimeo.
[8] Maruyama, M., "Noncooperative Price-Mediated Exchange," *Okayama Economic Review* (1980), pp. 239-251.

[9] Maruyama, M., "A Characterization of the Price-Mediated Exchange Equilibrium," *Okayama Economic Review* (1980), pp. 477-495.

[10] Morishima, M., *Walras' Economics*, Cambridge University Press (1977).

[11] "Nonçooperative Approaches to the Theory of Perfect Competition," *Journal of Economic Theory* (1980), pp. 121-376.

[12] Roberts, D. J. and A. Postlewaite, "The Incentives for Price-Taking Behaviour in Large Exchange Economies," *Econometrica* (1976), pp. 115-127.

[13] Schmeidler, D., "Walrasian Analysis Via Strategic Outcome Functions," *Econometrica* (1980), pp. 1585-1593.

[14] Shapley, L. S., "Noncooperative General Exchange," in *Theory and Measurement of Economic Externalities*, S. A. Y. Lin (ed.), Academic Press (1976).

[15] Shapley, L. S. and M. Shubik, "Trade Using One Commodity as a Means of Payment," *Journal of Political Economy* (1977), pp. 937-968.

[16] Schoumaker, F., "Incentives in Planning with Private Goods," *Review of Economic Studies* (1979), pp. 315-318.

[17] Tulkens, H. and S. Zamir, "Surplus-sharing Local Games in Dynamic Exchange Processes," *Review of Economic Studies* (1979), pp. 305-313.