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Harrodian Instability Principle in Full Disequilibrium

Takao Fujimoto

1. Introduction

Whenever we speak on Harrod's instability principle in a classroom, using some existing models, we cannot be fully satisfied because those models presuppose, explicitly or implicitly, some equilibrium conditions even out of the warranted growth path, and behavioural assumptions of entrepreneurs are not clearly explained. Let us take up a typical exposition for an undergraduate course.

"……Suppose the actual rate of growth G_a in net output Y is greater than the warranted rate of growth G_w, then by definition

$$G_a \equiv s / C_a > s / C_r = G_w,$$

where s is the saving coefficient, and C_a and C_r are the actual and the desired capital-output ratio, respectively. Thus, $C_a < C_r$, giving entrepreneurs more incentive to invest. And the actual growth rate G_a will shift still further away from the warranted rate. ……"

Students naturally wonder whether the market for commodities is in equilibrium or not, and also whether the saving coefficient is the actual or the planned one. If planned saving is different from planned investment, what will happen? Do capitalists respond to such a disequilibrium state? Which variables do entrepreneurs plan or schedule?

How do they adjust their plan for the next period after observing disequilibrium of the current period?

There have been proposed several models to answer these questions. (For these models, see the references at the end of this article.) They are, however, not quite satisfactory either because a planned quantity of a certain variable is indeed realized, or because expected rates of changes of some variables are revised irrespective of their actual rates.

In the following section, we present a simple new model in 'full disequilibrium', which is, we hope, suitable for classroom teaching. Then in section 3, some analyses on the model are carried out with further refinements suggested thenceforth.

2. A Simple Model

We use the following notation.

Y_e : expected net output.

Y : actual net output.

K : capital stock at the beginning of period t.

$\overline{C}$: desired capital-output ratio (assumed to be constant).

g_e : planned rate of accumulation of K at the beginning of period t.

g : actual rate of accumulation of K.

I_e : planned investment at the beginning of period t.

I : actual investment.

D_e : planned demand for goods and services in period t.

$\overline{s}$: desired saving ratio (assumed to be constant).

s : actual saving ratio.

u : degree of utilization of capital stock.

Variables represent their respective magnitudes of period t, while $x(t)$ or $x(t+1)$ for variable x explicitly shows the size of x in period t or $t+1$.

At the beginning of period t, 'capitalists' as a whole wish to realize the rate of accumulation g_e, given K. Thus,

$$I_e = g_e K. \tag{1}$$

The planned or scheduled supply can be written as

$$Y_e = K / \overline{C}. \tag{2}$$

Capitalists will then negotiate with one another and also with workers about payments for materials and wages. Whence scheduled demand becomes

$$D_e = (1-\overline{s})\, Y_e + I_e. \tag{3}$$

Naturally this planned demand D_e consists of the part by workers and that by capitalists. We then assume

$$Y = m_1(Y_e, D_e). \tag{4}$$

Here $m_1(x, y)$ is a function which assigns an intermediate value between x and y, and is non-decreasing in each variable. If a function has this property, we say this function belongs to a class of *mid* functions. For example, $min(x, y)$, $max(x, y)$ or $(\beta x + (1 - \beta)y)$ with $\beta \in (0, 1)$ can be mentioned. That is, when $Y_e > D_e$, some efforts for sales promotion will be made, and the actual output will be greater than D_e but smaller than Y_e. Given $Y_e < D_e$, managers' will try to increase their output with more overtime work than expected to secure market shares, and again the actual output will be settled down at somewhere between Y_e and D_e. Thus, the degree of capital utilization is calculated from

$$Y = uY_e. \tag{5}$$

At the beginning of period t with K and g_e given, five variables I_e, Y_e,

D_e, Y, and u can be determined using eqs. (1)—(5). Quite simple. It is now necessary to explain how the actual investment and saving are realized. We assume

$$I = m_2(I_e, \bar{s}Y_e), \tag{6}$$

where m_2 is a function having the same property as m_1 above, i. e., a *mid* function. It is not difficult to see

$$I_e > \bar{s}Y_e \quad \text{iff} \quad D_e > Y_e.$$

Since $sY = I$, the actual saving ratio is known as

$$s = I / Y.$$

Now having computed every variable in period t, the capital stock in period $t + 1$ is

$$K(t + 1) = K(t) + I(t) = K + m_2(I_e, \bar{s}Y_e). \tag{a}$$

On the other hand, we assume that entrepreneurs adjust their planned rate of accumulation g_e after the experience in period t according to

$$g_e(t + 1) = g_e(t) + f(u, D_e / Y_e) = g_e + f(u, D_e, / Y_e). \tag{b}$$

The function $f(u, v)$ is such that $f(1, 1) = 0$ and f is non-decreasing in each variable. This function is meaningful only on the set

$$\{(u, v) \mid u, v \geq 1 \text{ or } 1 \geq u, v \geq 0\}$$

because $u \geq 1$ iff $D_e \geq Y_e$. Since these variables are in a sense coherent either one of the arguments in f is apparently redundant. And yet both of them should be included simply because some variants of the model may not have that coherency and the model should remain general as much as possible. Besides to be more realistic, $f(1 + \alpha, 1) = 0$ with α taking a small positive or negative value depending on the magnitude D_e / Y_e, thus again losing the precise coherency. This excuse itself, however, seems redundant in such a classroom macro-model. Whatever our excuse is, the requirement $f(1, 1) = 0$ means that 'managers' as a whole will not change their planned (average) rate of accumulation when the degree of capial utilization is unity, i. e., at the normal level, and the market for goods and services is inequilibrium in the sense that planned net output (supply) equals scheduled demand, and so they are actually realized, hopefully without any typhoons or cool summers. The higher is the degree of capital utilization and the greater is the ratio between scheduled demand and supply, the larger step will be taken in the adjustment of planned growth rate in capital stock.

The above adjustment eqs. (a) and (b) give the values $K(t + 1)$ and $g_e(t + 1)$, and the same process as described by eqs. (1) - (5) will be repeated in the next period $t + 1$.

3 . Instability Principle

We proceed to the existence problem of the warranted growth path. Suppose

$$g_e = g_w \equiv \bar{s} / \bar{C}.$$

then it follows from eqs. (1) - (4)

$$Y_e = K / \bar{C} = (\bar{s}K / \bar{C}) + (1 - \bar{s})\, K / \bar{C} = g_e K + (1 - s)\, Y_e$$
$$= D_e.$$

Therefore, $Y = Y_e = D_e$, implying $u = 1$ by (5). This means that planned rate g_e $(= g_w)$ will be maintained in the next period and 'for ever'.

Now let us take up the instability principle. When $g_e > g_w$, we have

$$Y_e = g_w K + (1 - \bar{s})\, Y_e < g_e K + (1 - \bar{s})\, Y_e = D_e.$$

We get also $u > 1$. Thus, g_e will be revised further upward. Similarly when $g_e < g_w$, g_e will be made still smaller. This property is *in no sense*

Harrod's instability. The relevant variable is the actual accumulation rate g, which can be expressed as

$$g = I / K = m_2(I_e, \bar{s}Y_e) / K = m_3(I_e / K, \bar{s}Y_e / K)$$
$$= m_3(g_e, g_w), \tag{7}$$

where m_3 is another *mid* function. (Almost needless to say, when m_2 is homogeneous of degree one such as *min* or *max*, m_2 can be indentical with m_3) Eq. (7) tells us that when g_e deviates from g_w, the actual rate g also departs from g_w, but in a milder way.

It should be noted that instability does not depend upon the formulation in which the planned rate of accumulation is adjusted. This can be seen from the fact that the adjustment equation (b) can be rewritten as

$$I_e(t + 1) = I_e(t) + f(u, D_e / Y_e) K = I_e + h(u, D_e / Y_e, K).$$

4 . Some Variants

To produce a variant, a key equation is (6), which shows how the actual investment is determined. As has been done in most of the previous contributions on this topic, one may assume $s = \bar{s}$. Then, we have

$$I = sY = \bar{s}Y = \bar{s}m_1(Y_e, D_e) = m_4(\bar{s}Y_e, \bar{s}D_e).$$

From this,

$$g = I / K = m_5(\bar{s}Y_e / K, \bar{s}D_e / K)$$
$$= m_5(g_w, g_e + (1 - \bar{s})(g_w - g_e)), \tag{8}$$

because

$$\bar{s}D_e = \bar{s}I_e + \bar{s}(1 - \bar{s})Y_e = \bar{s}g_eK + (1 - \bar{s})g_wK.$$

In these equations, each m_i belongs to a class of *mid* functions. In this case, the actual rate of accumulation of capital deviates from the warranted rate in the same direction as the planned rate from the warranted one, but in a considerably mild way. The area where the actual rate g wanders is depicted as the shaded region in Fig.1. Let us give a numerical example. When $\overline{C} = 4$, $\overline{s} = 0.2$, we have $g_w = 0.05$. Now suppose $g_e = 0.06$, then g is between 0.05 and 0.052.

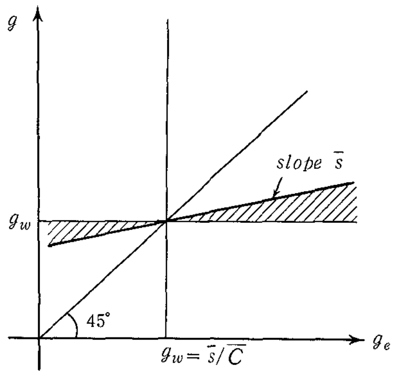

Fig. 1

Another restrictive assumption on the actual investment I is that the planned one is always realized. That is, $I = I_e$. In this variant, we easily obtain

$$g = g_e.$$

The actual rate is indeed as shaky as the planned rate, and we might agree to use the phrase 'knife-edge instability'.

The importance of the adjustment equation (b) should not be missed. To be more general and 'realistic', eq.(b) is to be

$$g_e(t + 1) = F(g_e, g, u, D_e, Y_e). \qquad \text{(b *)}$$

This general version says that in revising the planned rate, the actual (observed) rate should be taken into consideration. Because of eqs. (7) or (8), the movement of the planned rate g_e as well as the actual rate g is now somewhat tamed down, or even made stable depending on various functions and constants specified. It does not consume much time (though occupying a couple of pages) to derive sufficient conditions for stability of both g_e and g by using a certain norm of the Jacobian matrix for our system of nonlinear difference equations. Stable or unstable, we have to bear in mind the crucial role played by 'time and measure'. (See Atkinson (1969).)

5 . Remarks

Following an advice in Okishio (1984), we adopt a system of difference equations to describe the adjustment equations. In the literature, those models employing differential equations often blur the division line between the planned variable and the actual one, thus making implicit assumptions still more unnoticeable.

In this short essay, we do not comment on the works in the reference list below. This will be published in another note in the near future. We merely mention some points. Many authors assume that $I = I_e$, while some assume a *min* function in place of a *mid* function. To complete a model, it is quite desirable to give, in a more explicit way, the ceiling and the floor where divergent orbits are kicked back. Technical changes are to be brought in if one wishes to investigate the long-run nature of instability. Those changes may be made endogenous, thus either 'softening the blow' or embittering the misfortune. Surely in the long-run analysis, it is also necessary to treat two constants $\overline{C}$ and $\overline{s}$ as variables.

One more excuse. both words 'Harrod's' and 'Harrodian' are used in a loose (or even *sloppy*) way. The reader is quite free to interchange them

when it is regarded as suitable, blaming the author.

Originally the first draft of this article was written some years ago as a possible joint paper with Professor T. Shinozaki. Some mishaps on me, however, prevented us from working it out. Thanks are due to him for providing me with most relevant materials.

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