

《論 説》

The Effect of Public Education on the Long-Run Income Distribution

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1. Introduction

In developed countries, compulsory education is taken for granted as an essential public service. Education contributes to economic growth through forming individual human capital, promoting technological progress and so on. Furthermore, it serves for individuals to acquire fundamental learning skills. Then, compulsory education is regarded as one of the basic policy measures to realize economic growth and equal income distribution, and to achieve a stable and democratic society.

In economics literature on education, government interference in individual educational choice is justified by such economic factors as externalities [Weisbrod (1964), Pauly (1970)], the income redistribution effect [Hamada (1975), Bruno (1976), Ulph (1977)], and the achievement of economic efficiency [Welch (1970)]. In this paper, we focus on the intergenerational externalities, using an overlapping generation model. We assume here that parents make decisions on their educational expenditures for children on the basis of the so-called joy of giving motivation, and that parents' decisions have intergenerational externalities. That is, though education for children has influences upon

their human capital formation and future income, parents make decisions based on their own preferences for children's education rather than on their children's future welfare. It has been pointed out that parents' decisions based on these motivations yield insufficient amount of education [Eckstein and Zilcha (1994)]. Then government educational policies may be required to improve the situation.

In our model, we assume compulsory education which is provided for all individuals commonly as a government educational policy, and also assume that the individuals can purchase private education in addition to public education if they wish [Hare and Ulph (1979)].¹ Since in our model decisions on children's education are made by parents, parents' incomes and preferences for education have a critical influence upon children's human capital formation as well as public education. This characterizes the accumulation process of human capital through generations in each family. Recently, Eckstein and Zilcha (1994) analyzed the role of compulsory education, and showed that it enhances economic growth and makes the distribution of earnings more equal. We will discuss how introduction of compulsory public education affects human capital accumulation and income distribution among heterogeneous individuals. We also consider threshold effects in the accumulation process of human capital [Azariadis and Drazen (1990)], and examine how public education changes the accumulation process of human capital in the long run.

The remainder of the paper is organized as follows. Section 2 presents a basic model of individual decisions on private education and section 3 introduces the government to provide public education. Section 4 analyzes the long-run effects of public education on the level of individual human capital and the income distribution. Finally concluding comments follows

in section 5.

2. Decisions on private education

2.1 A basic model

We consider an overlapping generation economy in which an individual lives for two periods, receiving education in the first period (the period of childhood) and working in the second period (the period of parenthood). An individual, who has a child at the beginning of parent period, works for money, and makes a decision on education given to the child during that period. We assume that there is no population growth in the economy.

An individual who works in the t -period is called as t -generation and is assumed to have the following utility function :

$$u(c_t, a_{t+1}, E_{t+1}; \theta), \quad (1)$$

where c_t is the consumption of t -generation which includes the consumption of his / her child, and a_{t+1} and E_{t+1} are the bequest and the educational expenditures for $t + 1$ generation, respectively. We assume that the utility function has positive marginal utility with respect to each argument and is quasi-concave. This utility function implies that an individual has the joy of giving motivation for education and bequest to his / her offsprings.² Individuals have different utility functions identified by parameter θ . This parameter θ denotes an infinite sequence of individuals : more precisely each of them expresses a family (a parent and a child). In our model, the parameter stands for the degree of preference for education, and that a larger θ means a higher preference

for education, i. e., $u_{E\theta} > 0$. The preference parameter θ is distributed with a density function $f(\theta)$ which satisfies $f(\theta) \geq 0$ for $0 < \underline{\theta} \leq \theta \leq \bar{\theta} < \infty$ and $\int f(\theta)d\theta = 1$.³

Individuals spend income on education and bequest for their children as well as on consumption. A budget constraint of an individual of t – generation is expressed as

$$w_t h_t(\theta) + (1 + r_t)a_t(\theta) = c_t + E_{t+1} + a_{t+1}, \quad (2)$$

where w_t is the wage rate, r_t the interest rate in period t , $h_t(\theta)$ the level of human capital, and $a_t(\theta)$ the bequest left by a parent ($t - 1$ generation). We assume that an individual's labor supply is inelastic and proportional to his / her human capital.⁴ Assuming the proportional coefficient to be unity, the amount of labor supply is expressed by $h_t(\theta)$ in an efficiency unit. The left hand side of eq. (2) means that an individual earns the wage income, $w_t h_t(\theta)$, and receives the bequest with an interest on that, $(1 + r_t)a_t(\theta)$. Representing the income of an individual θ by $z_t(\theta)$, i.e., $z_t(\theta) \equiv w_t h_t(\theta) + (1 + r_t)a_t(\theta)$, eq. (2) is rewritten as

$$z_t(\theta) = c_t + E_{t+1} + a_{t+1}. \quad (3)$$

An individual is assumed to choose c_t , a_{t+1} and E_{t+1} so as to maximize the utility (1) subject to the budget constraint (3) with a_{t+1} given. The first order conditions are obtained for each θ as follows :

$$u_{c_t} - \lambda_t = 0, \quad (4)$$

$$u_{a_{t+1}} - \lambda_t = 0, \quad (5)$$

$$u_{E_{t+1}} - \lambda_t = 0, \quad (6)$$

$$z_t(\theta) = c_t + E_{t+1} + a_{t+1}, \quad (7)$$

where λ_t is the Lagrangean multiplier, and $u_{c_t} \equiv \partial u / \partial c_t$, $u_{a_{t+1}} \equiv \partial u / \partial a_{t+1}$, $u_{E_{t+1}} \equiv \partial u / \partial E_{t+1}$. Solving the above eqs. (4)–(7), the following demand functions can be obtained :

$$c_t(\theta) = c(z_t(w_t, r_t, h_t(\theta), a_t(\theta)); \theta), \quad (8)$$

$$a_{t+1}(\theta) = a(z_t(w_t, r_t, h_t(\theta), a_t(\theta)); \theta), \quad (9)$$

$$E_{t+1}(\theta) = E(z_t(w_t, r_t, h_t(\theta), a_t(\theta)); \theta). \quad (10)$$

We assume that all the goods and services are normal goods.

The total human capital of the economy has direct effects, together with the physical capital stock, upon production possibilities. The production function of the economy is defined as

$$Y_t = F(K_t, H_t), \quad (11)$$

where Y_t , K_t and H_t are total output, physical and human capitals in period t , respectively. The production function is assumed to have positive marginal products, to be quasi-concave and to be homogeneous of degree one with respect to both inputs. We assume the supply of capital consists of the only assets, which have been left for the following generation as bequest. Then the capital market clearance requires that the capital should be equal to the sum of individual assets. Denoting the total population by N , K_t and H_t are expressed as $K_t = N \int a_t(\theta) f(\theta) d\theta = N \bar{a}_t$ and $H_t = N \int h_t(\theta) f(\theta) d\theta = N \bar{h}_t$, where a_t and h_t are the average amount of individual physical capital and that of human capital, respectively. Using these symbols, the production function is rewritten in a per capita form :

$$\bar{y}_t = F(\bar{a}_t, \bar{h}_t), \quad (12)$$

where \bar{y}_t is per capita output. Assuming that the market is competitive,

each input is paid its marginal products : $r_t = F_{a_t}$, and $w_t = F_{h_t}$.

In equilibrium, the following market clearance condition must hold for output :

$$\bar{y}_t + \bar{a}_t = \bar{c}_t + \bar{E}_{t+1} + \bar{a}_{t+1}, \quad (13)$$

where $\bar{c}_t \equiv \int c_t(\theta)f(\theta)d\theta$ and $\bar{E}_{t+1} \equiv \int E_{t+1}(\theta)f(\theta)d\theta$ are the average amount of consumption and that of education in period t , respectively.

An individual human capital is accumulated in the educational process during his / her first period. We assume that the attained stock of human capital depends on the parent's level of human capital as well as the amount of education, which is given in childhood. Then the function of human capital formation is expressed as

$$h_{t+1}(\theta) = A(h_t(\theta))\psi(E_{t+1}(\theta), h_t(\theta)), \quad (14)$$

where a given function ψ is assumed to have positive derivatives with respect to both parameters and to be concave with respect to $h_t(\theta)$.⁵ The factor A represents the efficiency in human capital accumulation. We assume that the efficiency in human capital accumulation increases with a parent's human capital and that there are threshold effects in accumulation process as Azariadis and Drazen (1990) did.⁶ This means that education works more efficiently on human capital formation if a parent's human capital is not less than a certain level. To capture this nature of accumulation process, we define the scale factor function as :

$$A(h_t(\theta)) = \underline{A} \quad \text{for } h_t(\theta) < h^*, \quad (15)$$

$$A(h_t(\theta)) = \bar{A} \quad \text{for } h_t(\theta) \geq h^*, \quad (16)$$

where \underline{A} and \bar{A} are given positive constant parameters, and $\underline{A} < \bar{A}$. Eqs.

(15) and (16) mean that the scale factor function jumps at h^* .⁷ That is, h^* is a critical point which may cause radical differences in dynamic accumulation processes of human capital.⁸ We assume in addition that individuals have the identical function of human capital formation, and then that the critical point is also the same for all individuals. We call this point the threshold point.

2.2 Dynamic process

The dynamic accumulation processes of human and physical capitals are described by the following two difference equations :

$$h_{t+1}(\theta) = A(h_t(\theta))\psi(E_{t+1}(z_t(\theta), \theta), h_t(\theta)), \quad (17)$$

$$a_{t+1}(\theta) = a(z_t(\theta), \theta). \quad (18)$$

Since our system has threshold effects in eq. (17) , we have a unique or two equilibrium states corresponding to the scale factors \underline{A} and \bar{A} as will be explained in section 2. 3. Also, we assume that these equilibria are locally stable in a neighborhood of each equilibrium (see Appendix).

Restricting our concern to steady states, we can obtain equilibrium states for each individual by solving the dynamic system (17) and (18). Eq. (13) is automatically satisfied because of the budget constraint of each individual. It is not assured whether the individual's human capital is increasing or decreasing with θ in the long run. However, when the positive relation between educational preference and the demand for education is assumed, we can verify that under certain conditions, in a steady state the level of human capital and that of income both increase but the amount of bequest decreases with θ .⁹ In the discussion below, we will restrict our attentions to this case where $dh(\theta)/d\theta > 0$, $dz(\theta)/d\theta > 0$

and $da(\theta)/d\theta < 0$ in a neighborhood of equilibria for the same scale factor.

2.3 Threshold and steady states

We examine the threshold effect in the human capital formation function. Fixing the level of bequest at a certain equilibrium level a^* , we can derive a *potential* equilibrium point for any θ corresponding to each scale factor \underline{A} and \bar{A} regardless of a threshold point (see Fig. 1). Then we define the lower *potential* equilibrium point $\underline{h}(\theta)$ of the human capital formation function with scale factor \underline{A} , and the higher one $\bar{h}(\theta)$ with \bar{A} . According to where the threshold point h^* is, we can classify individuals into three cases as follows :

Case I . $h^* < \underline{h}(\theta) < \bar{h}(\theta)$

In this case, the steady state is unique. Every human capital accumulation path converges to the steady state $\bar{h}(\theta)$, regardless of the initial stock of individual's human capital. This case is shown in Fig. 2.

Case II . $\underline{h}(\theta) \leq h^* \leq \bar{h}(\theta)$

In this case, two steady states can be in equilibrium. It is the initial stock of human capital that decides which steady state will be realized. If the initial stock is below h^* , the accumulation path converges to the lower steady state $\underline{h}(\theta)$. On the other hand, if the initial stock is equal to or above h^* , the path converges to the higher steady state $\bar{h}(\theta)$. See Fig. 3.

Case III . $\underline{h}(\theta) < \bar{h}(\theta) < h^*$

In this case, the steady state is unique. Every human capital accumulation path converges to the steady state $\underline{h}(\theta)$, regardless of the

initial stock of individual's human capital. See Fig. 4.

Since the human capital formation function for the same scale factor shifts upward with educational preference θ by assumption, Case I is likely to be applied to individuals with a higher preference for education, Case II to individuals with a middle preference and Case III to individuals with a lower preference. We call the group of individuals in Case I the

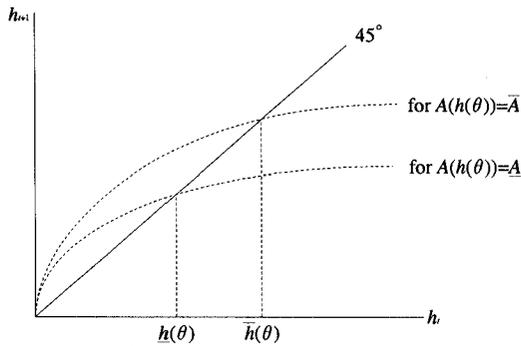


Fig. 1

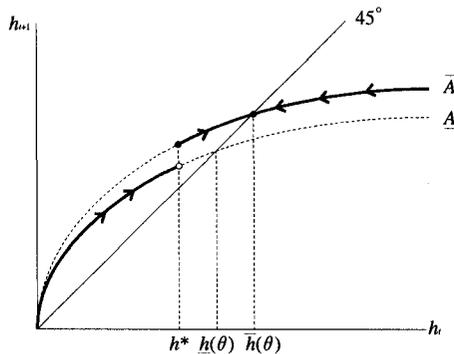


Fig. 2

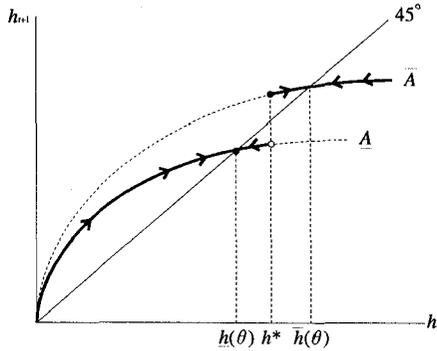


Fig. 3

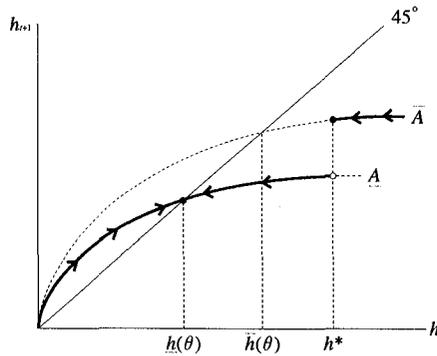


Fig. 4

higher preference group, those in Case II the middle preference group, and similarly those in Case III the lower preference group.

In both higher and lower preference groups, in the long run, the individuals with the higher preference attain the higher level of human capital than the individuals with the lower preference, regardless of the initial level of human capital. In the middle preference group, however, it is possible that the order of individual preference for education is not consistent with the order of the level of attained human capital. In other

words, even if the individual has a relatively higher preference for education, the attained human capital is possibly lower than that of individual with a relatively lower preference. It is because, in the case of the middle preference group, the initial level of human capital is critical in deciding which equilibrium is attained in the accumulation path.

3. The public provision of education

As stated in the above, a parent chooses the amount of education for his / her child in accordance with his / her own preference for education. In this case, the allocation of education is not socially optimum since the parent disregards the effects of education on future income and utility of his/her child. Eckstein and Zilcha (1994) showed that the decision of parents yields socially the under-investment in education and the government supply of education can improve the efficiency.

We introduce a government educational policy into the economy to improve the efficiency, in which the government imposes the comprehensive proportional income tax and spends the revenue on the provision of public education. Public education is assumed to be compulsory and provided for all individuals free of charge.

We examine the consumer's behavior in the presence of public education. Individuals may purchase education privately in addition to public education if they wish. For individuals, public education is regarded as a transfer in kind. Then we define $z_t^e(\theta)$ as the individual's after-tax income including a transfer in kind ; $z_t^e(\theta) \equiv (1 - \alpha_t)[w_t h_t(\theta) + (1 + r_t)a_t(\theta)] + g_{t+1}$, where α_t is an income tax rate, g_{t+1} public education per family supplied for the $t + 1$ generation.¹⁰

$E_{t+1}(\theta)$ in the previous section is reinterpreted here as the total expenditures on education which are the sum of public and private education. Then private education desired by individuals is expressed as $(E_{t+1} - g_{t+1})$. An Individual's demand functions are :

$$c_t(\theta) = c(z_t^e(w_t, r_t, h_t, a_t, g_{t+1}, \alpha_t); \theta), \quad (19)$$

$$a_{t+1}(\theta) = a(z_t^e(w_t, r_t, h_t, a_t, g_{t+1}, \alpha_t); \theta), \quad (20)$$

$$E_{t+1}(\theta) = E(z_t^e(w_t, r_t, h_t, a_t, g_{t+1}, \alpha_t); \theta). \quad (21)$$

For individuals who do not spend on private education, the total education equals public education, i. e., $E_{t+1}(\theta) = g_{t+1}$.

The government budget constraint is expressed as

$$\alpha_t \int z_i(\theta) f(\theta) d\theta = g_{t+1}. \quad (22)$$

The dynamic accumulation processes of human and physical capitals are derived as in the previous section assuming local stability.

Although we can derive the optimal condition for the level of public education in this economy as in Furumatsu (1997), our interest here is the long run comparative dynamics effects of public education on the economy consisting of heterogeneous individuals. We will examine in the next section how public education affects the individual level of human capital, and whether the income distribution is improved or not.

4. The effects of public education

4.1 The distribution effects

First, we examine the effects of public education on the steady state income distribution. The introduction of public education shifts human

capital formation function, changing the level of human capital in the steady state. In the steady state, each individual's human capital and asset should meet the following equations for a given g :

$$h(\theta) = A(h(\theta))\psi(E(z^e(\theta), \theta), h(\theta)) \quad \text{for all } \theta, \quad (23)$$

$$a(\theta) = a(z^e(\theta), \theta) \quad \text{for all } \theta, \quad (24)$$

where $h(\theta)$ and $a(\theta)$ are the values of human capital stock and asset for each individual in the steady state.

Totally differentiating these two equations with respect to g and taking account of the government budget constraint, we have

$$\begin{aligned} \begin{bmatrix} 1 - A(\psi_1 E_1 (1 - \alpha)w - \psi_2) & -A\psi_1 E_1 (1 - \alpha)(1 + r) \\ -a_1 (1 - \alpha)w & 1 - a_1 (1 - \alpha)(1 + r) \end{bmatrix} \begin{bmatrix} \frac{dh}{dg} \\ \frac{da}{dg} \end{bmatrix} \\ = \begin{bmatrix} A\psi_1 E_1 (1 - B(\theta)) \\ a_1 (1 - B(\theta)) \end{bmatrix}, \end{aligned} \quad (25)$$

where $B(\theta) = z^e(\theta) \frac{d\alpha}{dg} - \left(h(\theta) \frac{dw}{dg} + a(\theta) \frac{dr}{dg} \right)$. Solving eq. (25), we have

$$\frac{dh(\theta)}{dg} = \frac{A\psi_1 E_1 (1 - B(\theta))}{D}, \quad (26)$$

$$\frac{da(\theta)}{dg} = \frac{a_1 (1 - A\psi_2)(1 - B(\theta))}{D}, \quad (27)$$

where D denotes the determinant of the coefficient matrix on the left-hand side of eq.(25), i.e.,

$$D = (1 - A\psi_2)(1 - a_1(1 - \alpha)(1 + r)) - A\psi_1E_1(1 - \alpha)w.$$

The local stability conditions mean that the sign of D is positive.¹¹

The term $d\alpha/dg$ in $B(\theta)$ is deduced from the government budget equation $\alpha z = g$, where $z \equiv \int z(\theta)f(\theta)d\theta$:

$$\frac{d\alpha}{dg} = \frac{-\alpha z_g + 1}{z + \alpha z_\alpha},$$

which shows the change in the tax rate required to finance the increased public education. Since we consider the effects of introducing public education, we evaluate the derivatives at $\alpha = g = 0$. In this case, $d\alpha/dg = 1/z$, and then the function $B(\theta)$ can be expressed as follows:

$$B(\theta) = \frac{z^e(\theta)}{z} - \left(h(\theta) \frac{dw}{dg} + a(\theta) \frac{dr}{dg} \right). \quad (28)$$

In the following, we will examine how the effects of public education differ among individuals according to their preference parameter for education. The sign of eq. (26) is not assured, since the first term of the numerator is positive but the second term can take either sign. This means that introduction of public education does not always raise human capital for each individual. To see the sign of eq. (26), we will notice the level of $B(\theta)$. This term stands for the effects of public education on individual income through the labor–capital markets and the government budget. For the individuals whose $B(\theta)$ is smaller than unity, eq. (26) takes a positive sign, then introduction of public education accelerates the accumulation of human capital. On the other hand, for the individuals whose $B(\theta)$ is larger than unity, it is possible for introduction of public

education to depress human capital accumulation. This case must be for the individuals with the higher preference for education. It is because the first term of $B(\theta)$ is increasing and the second term is likely to be non-decreasing with θ .¹² Therefore, we can say that the individual with the higher preference for education has the larger possibility to depress human capital accumulation. Then we can conclude that, otherwise being equal, the distribution of human capital is possible to be more equalized by introduction of public education in the long run. These conjectures are also applicable to the sign of $da(\theta)/dg$.

Letting the disposable income be $z^d \equiv (1 - \alpha)(wh + (1 + r)a)$, we have

$$\frac{dz^d(\theta)}{dg} = \frac{(1 - \alpha)[(wA\psi_1E_1 + (1 + r)a_1(1 - A\psi_2))(1 - B(\theta))]}{D} - \frac{z(\theta)}{z}. \quad (29)$$

The interpretation of this equation is similar to the one for eqs. (26) and (27). Then we can also conclude that the income distribution is possibly more equalized by introduction of public education in the long run.

4.2 The efficiency effects

Second, we consider another effect of public education on accumulation of human capital. That is, we examine the possibility that, due to the threshold effects, introduction of public education changes radically the steady state to which the accumulation path of human capital converges.

As is suggested in section 4. 1, introduction of public education will shift the human capital function downward for the individuals whose $B(\theta)$ is larger than unity, that is, for ones whose preference parameter is higher than that of the individual with the average level of income. For

the individuals whose $B(\theta)$ is smaller than unity, the reverse will be the case. However, it is not assured to which group (*the higher, middle or lower preference group*) the individual with the average level of income belongs, since it depends on the initial distribution of human capital and the threshold point which are given exogenously. Therefore, it is possible for public education to shift the human capital function either upward or downward for individuals of all three groups, except an individual with the highest preference in the higher preference group and the one with the lowest preference in the lower preference group.

However, as will be made clear below, for individuals in the higher preference group, the existence of the threshold point does not have any substantial effect on the accumulation process of human capital when public education shifts the human capital function upward. Similarly, for the lower preference group, the downward shift of the human capital function due to introduction of public education does not have any substantial effect through the threshold effects. Therefore, in the following discussions, we will not refer to these cases, assuming that the average income individual belongs to the middle preference group.

The higher preference group

As noted in section 2.3, individuals in this group have equilibrium $\bar{h}(\theta)$. When public education shifts the human capital formation function downward, there may exist some individuals in the lowest end of this group who enter the middle preference group. For these individuals, if the initial level of human capital is lower than the threshold point h^* , the accumulation path converges to the lower equilibrium $\underline{h}^s(\theta)$ instead of $\bar{h}(\theta)$ in Fig. 5. This means that public education causes substantial

changes to lower the level of human capital of these individuals in the long run.

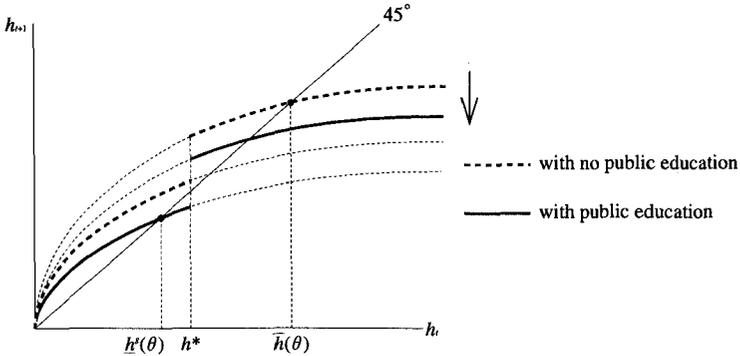


Fig. 5

The lower preference group

When public education shifts the human capital formation function upward, there may exist some individuals in the highest end of this group who enter the middle preference group. In the absence of public education, an individual in this group has equilibrium $\underline{h}(\theta)$. If the initial level of their human capital is higher than the threshold point h^* , then their human capital converges to the higher equilibrium level $\bar{h}^s(\theta)$ in Fig. 6 after introduction of public education. Public education brings about radical impacts for these individuals on human capital accumulation in the long run.

The middle preference group

Individuals in this group have two equilibria. Different from the above two groups, however, in the case of the middle preference group, the

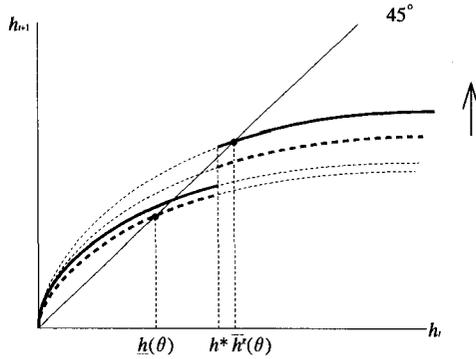


Fig. 6

existence of the threshold point has substantial impacts on human capital accumulation whatever effects of public education are on the human capital function. If public education shifts the human capital formation function upward, some individuals in the highest end of this group may become a member of the higher preference group after introduction of public education. Then they have equilibrium $\bar{h}^g(\theta)$ in Fig. 7-a. This suggests that, for the individuals whose previous convergence state is the lower equilibrium $\underline{h}(\theta)$, public education changes it to the higher one $\bar{h}^g(\theta)$.

If public education shifts the human capital formation function downward, individuals who belong to the lowest end of this group may become a member of the lower preference group. In this case, the convergence state becomes lower to $\underline{h}^g(\theta)$ in Fig. 7-b, even if his previous convergence state is $\bar{h}(\theta)$.

Introduction of public education has various impacts on the level of human capital in the long run for certain individuals in each group. These

effects are brought about by the threshold effects in the human capital formation function. Whether these effects increase the rate of growth of the economy or not depends on the distribution of the preference parameter.

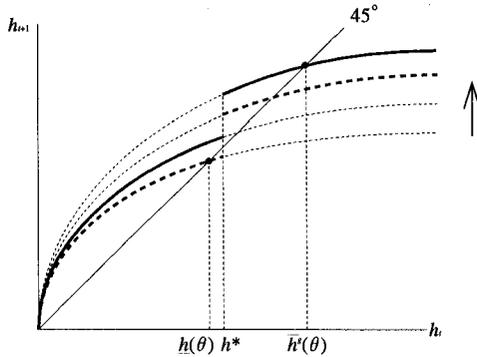


Fig. 7-a

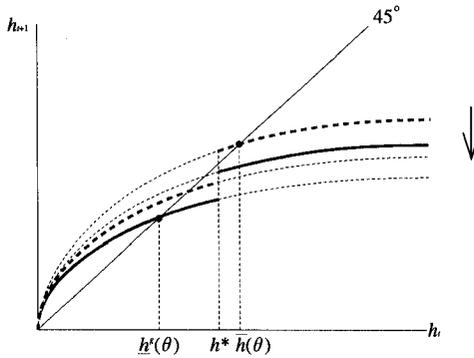


Fig. 7-b

5. Concluding comments

In our model, we assume that individuals have different preferences

for education which determine the level of human capital for children. If educational provision relies entirely on the individual decision, education would be insufficient to attain the efficient stock of human capital for a society as a whole. Using an overlapping generation model, we have analyzed how the introduction of public education exerts the different effects on individual human capital accumulation and income depending on the different educational preferences.

We have concluded particularly that the introduction of public education can possibly make the distributions of human capital and income more equal. This conclusion corresponds to a result in Eckstein and Zilcha (1994) which analyzed the effects of compulsory education using a model of heterogeneous individuals with different tastes regarding the choice of leisure and human capital.

We assume that the stock of a parent's human capital has the threshold effect on the accumulation of a child's human capital. The threshold effect may yield two equilibria. We showed that the introduction of public education will make a substantial difference in the convergence process, and that it is possible to change the level of human capital and the efficiency of the economy radically in the long run

Appendix

Let us examine the stability conditions of the model. To simplify the expressions, we omit the parameter when it does not make any confusion. To derive the local stability conditions, we differentiate eqs. (17) and (18) in a neighborhood of an equilibrium :

$$\begin{bmatrix} dh_{t+1} \\ da_{t+1} \end{bmatrix} = \begin{bmatrix} A(\psi_1 E_1 w + \psi_2) & A\psi_1 E_1(1+r) \\ a_1 w & a_1(1+r) \end{bmatrix} \begin{bmatrix} dh_t \\ da_t \end{bmatrix}, \quad (\text{A-1})$$

where the number of subscript denotes the partial differentiation with respect to corresponding variables. The characteristic polynomial of the matrix is

$$f(\lambda) = \lambda^2 - [A(\psi_1 E_1 w + \psi_2) + a_1(1+r)]\lambda + A\psi_2 a_1(1+r). \quad (\text{A-2})$$

Conditions for this system to satisfy locally stable solutions are met if $f(1) > 0$ and the value of λ to minimize the function (A-2) is between zero and one, since the characteristic polynomial has real roots and $f(0) > 0$. Then, we have

$$(1 - A\psi_2)(1 - a_1(1+r)) - A\psi_1 E_1 w > 0, \quad (\text{A-3})$$

$$A(\psi_1 E_1 w + \psi_2) + a_1(1+r) < 2. \quad (\text{A-4})$$

From eq. (A-3), we find that the first term is positive, that is, the signs of $(1 - A\psi_2)$ and $(1 - a_1(1+r))$ must be same, since $A\psi_1 E_1 w > 0$. Furthermore, eq. (A-4) is rewritten as follows :

$$A\psi_1 E_1 w < (1 - A\psi_2) + (1 - a_1(1+r)).$$

Since the left-hand side is positive, the right-hand side must be positive. The signs of both terms on the right-hand side are same, so that we obtain the following conditions :

$$A\psi_2 < 1, \quad a_1(1+r) < 1 \quad (\text{A-5})$$

In a similar way, the local stability conditions of the model in the presence of government intervention are obtained as follows :

$$(1 - A\psi_2)(1 - a_1(1-\alpha)(1+r)) - A\psi_1 E_1(1-\alpha)w > 0, \quad (\text{A-6})$$

$$A(\psi_1 E_1(1-\alpha)w + \psi_2) + a_1(1-\alpha)(1+r) < 2, \quad (\text{A-7})$$

and

$$A\psi_2 < 1, \quad a_1(1-\alpha)(1+r) < 1. \quad (\text{A-8})$$

Footnotes

- 1) In Hare and Ulph (1979), the government determines the allocation of public education among individuals according to their abilities taking into account their expenditures on private education. On the other hand, in Glomm and Ravikumar (1992) and Zhang (1996), public education provides every child with education of the same quality.
- 2) The joy of giving motivation has been discussed in the literature of bequest models. See Abel and Warshawsky (1988), and Kohlberg (1976).
- 3) The lower and the upper bounds of the integrals are omitted to simplify the expression.

- 4) In human capital theory, the labor productivity of a worker is often assumed to be equal to one's ability times the amount of human capital invested. Since we assume that individual 'ability' to use his / her human capital is the same for all individuals, labor in an efficiency unit is proportional to human capital. See Atkinson (1973), Sheshinski (1972), and Hamada (1975).
- 5) We assume that ψ is concave with respect to h , that is $\psi_{11}(E_1w)^2 + \psi_1E_{11}w^2 + 2\psi_{21}E_1w + \psi_{22} < 0$, and that ψ satisfies Inada conditions. We use subscripts to denote differentiation with respect to corresponding variables.
- 6) Azariadis and Drazen (1990) defined threshold externalities of human capital in the production function.
- 7) Futagami and Mino (1995) defined threshold externalities by the form of eqs. (15) and (16) in the case of public capital.
- 8) Rosenzweig and Wolpin (1994) showed empirically that there are increasing returns to the intergenerational production of human capital.
- 9) This requires $(1 - A\psi_1E_1w)a_2 < 0$, where $a_2 = \partial a / \partial \theta$, as well as the assumption of normal goods.
- 10) The price of public education and that of education purchased privately are both assumed to be unity. Individuals cannot, however, buy or sell public education in the market.
- 11) Local stability conditions when $\alpha = 0$ are given in the Appendix.
- 12) The homogeneity of the production function means $hdw/dg + adr/dg = 0$. Then, if $dw/dg \leq 0$, which means that introduction of public education increases total human capital, the second term in eq. (28) is non-decreasing with θ .

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The Effect of Public Education on the Long-Run Income Distribution

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Abstract

We consider public education provided obligatorily and equally for all individuals. It is usually said that compulsory public education ensures an equal opportunity of education for all individuals and contributes to human capital formation. We will discuss how the introduction of public education affects human capital accumulation and income distribution among heterogeneous individuals in an overlapping generation model. Particularly, we discuss those effects on the long-run equilibrium of individual human capital, considering the threshold effects of human capital stock.