

Popularization in the Higher Education and Optimal Educational Policies

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1. Introduction

The higher education has become popular with economic development in any advanced country. In Japan, the ratio of students who go on to high schools amounts to 95.8% and to universities and colleges 49.1% in 1999, while they were 57.7% and 10.3% in 1965, respectively. ⁽¹⁾

Education has both aspects of investment and consumption (Arai (1995), Blaug (1968)). Then, the high ratios should be explained by the high rate of return to education and / or the rising income of households. Fig.1 describes the time pattern of the relation between an enrolment rate for universities (and colleges) and an increasing rate of income (a worker's income) from 1950s to 1990s in Japan. ⁽²⁾ We can see from the figure that before 1975, there was a positive relation between them. However, after 1975, though the increasing rate of income was falling rapidly, the enrolment rate kept the level of 38% and then rose. That is, once the ratio of students who go on to universities rises, it won't fall easily, even though

(1) The data source is the report made public by the Ministry of Education.

(2) We use the worker's income per capita as the income. The data source is the Annual Report of the Family Income and Expenditure Survey (Statistics Bureau, Management and Coordination Agency).

the rate of income growth decreases. This fact implies that the rising enrolment rate cannot be explained only by the income factors. In this paper, we attempt to construct a model which explains a formation of demand for education, considering the individual preference for the higher education (university). We assume that the individual preference for education depends on the average level of education in a society and changes discontinuously at some level of the social average.⁽³⁾ That is, we consider a possibility of existence of a critical social average level of education where popularization of education can be accelerated very strongly.⁽⁴⁾

We examine a model in which the individual preference is affected by popularization of education, and show the optimal policies for the higher education in dynamic setting.⁽⁵⁾ The remainder of the paper is organized as follows. Section 2 presents a model and section 3 formulates the optimization problem and examines the optimal policies. Finally concluding comments follow in section 5.

2. Model

2.1 An individual behavior

We consider an economy consisting of homogeneous individuals, who form human capital through education. An individual human capital is

(3) Eckstein and Zilcha (1994) analyzed the optimal educational policies, considering the difference in the parent's preference for children's education.

(4) Trow (1972) called wide diffusion of education *popularization of education*.

(5) Furumatsu and Shirai (2000) analyzed the optimal subsidies to university with reference to education and research activities in dynamic setting.

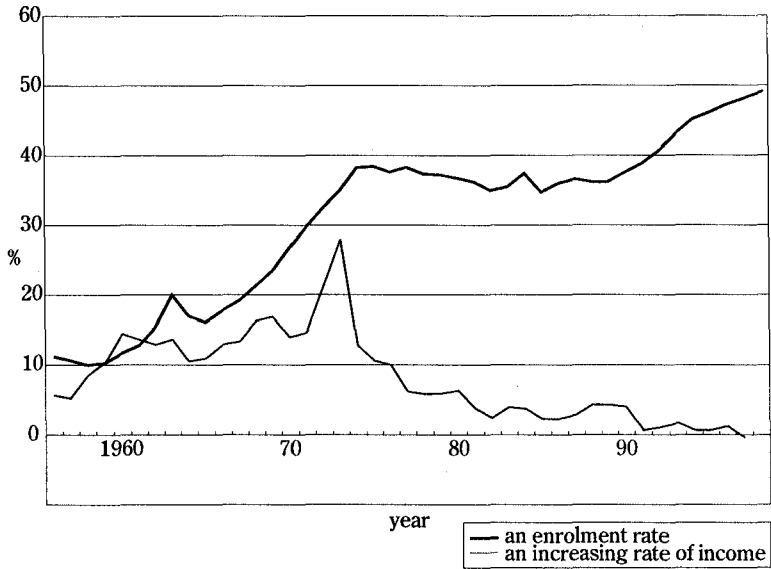


Fig. 1

formed as follows :

$$h = kH, \tag{1}$$

where h is the level of human capital, k the rate of embodying knowledge, and H the amount of knowledge in a society. The rate of embodying is assumed to depend on the level of education which individuals receive :

$$k = k(e), \tag{2}$$

where e is the level of education. We assume that $k(0) = 0$, $k_e > 0$, $k_{ee} < 0$, where subscripts mean derivatives with respect to the corresponding variables.

We assume that change in the amount of knowledge in a society depends on the average level of human capital at the time. Assuming that

knowledge depreciates at the rate δ ($0 < \delta < 1$), we define the accumulation function of knowledge in a society as follows :

$$\dot{H} = G(\bar{h}) - \delta H, \quad (3)$$

where \dot{H} is the change in the stock of knowledge, \bar{h} the average level of human capital in a society, and $G' > 0$.⁽⁶⁾

We assume that a representative individual has a separable utility function consisting of consumption and education :

$$U = u(c) + \alpha v(e), \quad (4)$$

where c is the amount of consumption. We assume that $u' > 0$, $u'' < 0$, $v' > 0$, $v'' < 0$. The factor α presents the weight on utility of education. We suppose that the weight changes with the amount of knowledge as follows :

$$\alpha(H) = \underline{\alpha} \quad \text{for } H < H^*, \quad (5)$$

$$\alpha(H) = \bar{\alpha} \quad \text{for } H \geq H^*, \quad (6)$$

where $\underline{\alpha}$ and $\bar{\alpha}$ are given positive constant parameters, and $\underline{\alpha} < \bar{\alpha}$. This means that the weight factor jumps at H^* . That is, H^* is a critical point which causes discontinuous change in an individual preference for education.⁽⁷⁾

Assuming that the individual labor supply is proportional to

(6) Since we consider the representative individual, the average level of human capital in the society is equal to the level of individual human capital.

(7) Futagami and Mino (1995) defined a point causing threshold externalities in the case of public capital by the same form. See Azariadis and Drazen (1990) about threshold externalities.

individual human capital and that the proportional coefficient is unity, the amount of labor supply is expressed by h in efficiency units. The individual budget constraint is

$$wh - T = c + pe, \tag{7}$$

where w is the wage rate, T the lump sum tax, and p the price of education which the individual faces. The individual maximizing problem is expressed as

$$\begin{aligned} \max U &= u(c) + \alpha(H)v(e) \\ \text{s.t. } wh - T &= c + pe \\ h &= k(e)H. \end{aligned}$$

The first order conditions are

$$u' - \lambda = 0, \tag{8}$$

$$\alpha(H)v' + \lambda(wk'H - p) = 0, \tag{9}$$

$$wk(e)H - T = c + pe, \tag{10}$$

where λ is a Lagrangean multiplier. Solving the above equations, we obtain the following demand functions :

$$c = c(w, T, p, H), \tag{11}$$

$$e = e(w, T, p, H). \tag{12}$$

Totally differentiating (8)–(10), we have

$$\begin{pmatrix} u'' & 0 & -1 \\ 0 & \alpha v'' + \lambda wk''H & wk'H - p \\ -1 & wk'H - p & 0 \end{pmatrix} \begin{pmatrix} dc \\ de \\ d\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ -\lambda(k'Hdw + wk'dH - dp) \\ -kHdw - wk dH + dT + edp \end{pmatrix}.$$

Solving (13), we obtain ⁽⁸⁾ (13)

$$\frac{\partial c}{\partial T} = \frac{\alpha v'' + \lambda w k'' H}{D} < 0, \quad (14)$$

$$\frac{\partial c}{\partial p} = \frac{-\lambda(wk'H - p) + e(\alpha v'' + \lambda w k'' H)}{D}, \quad (15)$$

$$\frac{\partial c}{\partial H} = \frac{\lambda w k'(wk'H - p) - w k(\alpha v'' + \lambda w k'' H)}{D}, \quad (16)$$

$$\frac{\partial e}{\partial T} = \frac{-u''(wk'H - p)}{D} < 0, \quad (17)$$

$$\frac{\partial e}{\partial p} = \frac{-u''e(wk'H - p) - \lambda}{D} < 0, \quad (18)$$

$$\frac{\partial e}{\partial H} = \frac{u''w k(wk'H - p) - \lambda w k'}{D}, \quad (19)$$

where D denotes the determinant of the coefficient matrix in the left hand side of (13), i.e.,

$$D = -u''(wk'H - p)^2 - (\alpha v'' + \lambda w k'' H) > 0.$$

2.2 Government behavior

The government imposes the income tax and spends the revenue on the subsidy for education. Assuming that the cost of education is unity, the educational subsidy is defined as $(1 - p)e$ ($0 < p < 1$). Through the choice of $1 - p$, the government can control the price of education which individuals face. The government budget constraint is expressed as

$$T = (1 - p)e. \quad (20)$$

(8) The weight factor $\alpha(H)$ is given positive constant parameter.

3. Optimal Policies

We assume that the government has an intertemporal utilitarian social welfare function defined by the utilities of the present and future individuals. The government chooses T and p so as to maximize the social welfare subject to the government budget, that is :

$$\begin{aligned} & \max \int_0^{\infty} (u(c) + \alpha(H)v(e))e^{-\rho t} dt \\ & s.t. \quad T = (1 - p)e \\ & \quad \dot{H} = G(\bar{h}) - \delta H, \end{aligned}$$

where the constant $\rho > 0$ is a discount parameter. We define the current value Hamiltonian H_h as

$$H_h \equiv u(c) + \alpha(H)v(e) + \gamma(T - (1 - p)e) + \eta(G(\bar{h}) - \delta H),$$

where γ is a Lagrangean multiplier of the government budget and η the costate variable. We obtain the first order conditions as follows :

$$u' \frac{\partial c}{\partial T} + \alpha v' \frac{\partial e}{\partial T} + \gamma \left(1 - (1 - p) \frac{\partial e}{\partial T} \right) + \eta G' k' H \frac{\partial e}{\partial T} = 0, \tag{21}$$

$$u' \frac{\partial c}{\partial p} + \alpha v' \frac{\partial e}{\partial p} + \gamma \left(- (1 - p) \frac{\partial e}{\partial p} + e \right) + \eta G' k' H \frac{\partial e}{\partial p} = 0, \tag{22}$$

$$\dot{\eta} = - \left(u' \frac{\partial c}{\partial H} + \alpha v' \frac{\partial e}{\partial H} - \gamma (1 - p) \frac{\partial e}{\partial H} + \eta \left(G' \left(k' H \frac{\partial e}{\partial H} + k \right) - \delta \right) \right) + \eta \rho. \tag{23}$$

Using (7), we can rewrite (21) and (22) as follows, respectively :

$$- u' + \gamma - \frac{\partial e}{\partial T} (\gamma(1 - p) - \eta G' k' H) = 0, \tag{24}$$

$$-u'e + \gamma e - \frac{\partial e}{\partial p}(\gamma(1-p) - \eta G'k'H) = 0. \quad (25)$$

From (24) and (25), we have

$$\left(\frac{\partial e}{\partial p} - e \frac{\partial e}{\partial T}\right)(\gamma(1-p) - \eta G'k'H) = 0. \quad (26)$$

Using (17) and (18), (26) is rewritten as follows :

$$-\frac{\lambda}{D}(\gamma(1-p) - \eta G'k'H) = 0. \quad (27)$$

Since λ is not zero,

$$\gamma(1-p) - \eta G'k'H = 0, \quad (28)$$

or

$$1-p = \frac{\eta}{\gamma} G'k'H. \quad (29)$$

The optimal level of educational subsidy is determined from (29).

Substituting (28) to (24), we have

$$u' = \gamma. \quad (30)$$

Using (7), (28) and (30), we rewrite (23) as follows :

$$\dot{\eta} = -\gamma \omega k - \eta(G'k - \delta) + \eta \rho. \quad (31)$$

We can investigate the optimal policies, using (3) and (31).

To simplify analyses, we specify the human capital formation function, the knowledge accumulation function and the individual utility function as $h = aeH$, $\dot{H} = g\bar{h}^{-\beta} - \delta H$, $U = c + \alpha(H)v(e)$, respectively. The constant values a , g and β are parametrically specified and positive, and $0 < \beta < 1$. The individual maximizing problem is revised as follows :

$$\begin{aligned} \max U &= c + \alpha(H)v(e) \\ \text{s.t. } wh - T &= c + pe \\ h &= aeH \end{aligned}$$

The first order conditions are

$$1 - \lambda = 0, \tag{32}$$

$$\alpha v' + \lambda(waH - p) = 0, \tag{33}$$

$$waeH - T = c + pe. \tag{34}$$

The demand functions are

$$c = c(w, T, p, H), \tag{35}$$

$$e = e(w, T, p, H). \tag{36}$$

From (32)–(36), we have

$$\frac{\partial c}{\partial T} = \frac{\alpha v''}{D} < 0, \tag{37}$$

$$\frac{\partial c}{\partial p} = -\frac{1}{D}(waH - p - \alpha v''e), \tag{38}$$

$$\frac{\partial c}{\partial H} = -\frac{1}{D}(-wa(waH - p) + \alpha v''wae), \tag{39}$$

$$\frac{\partial e}{\partial T} = 0, \tag{40}$$

$$\frac{\partial e}{\partial p} = -\frac{1}{D} < 0, \tag{41}$$

$$\frac{\partial e}{\partial H} = \frac{wa}{D} > 0, \tag{42}$$

where $D = -\alpha v'' > 0$.

The government problem is to choose T and p so as to maximize the social welfare subject to the budget as follows :

$$\begin{aligned} & \max \int_0^{\infty} (c + \alpha(H)v(e))e^{-\rho t} dt \\ & \text{s.t. } T = (1-p)e \\ & \quad \dot{H} = g\bar{h}^{-\beta} - \delta H. \end{aligned}$$

The first order conditions are

$$\frac{\partial c}{\partial T} + \alpha v' \frac{\partial e}{\partial T} + \gamma \left(1 - (1-p) \frac{\partial e}{\partial T} \right) + \eta \beta g a^{\beta} H^{\beta} e^{\beta-1} \frac{\partial e}{\partial T} = 0, \quad (43)$$

$$\frac{\partial c}{\partial p} + \alpha v' \frac{\partial e}{\partial p} + \gamma \left(-(1-p) \frac{\partial e}{\partial T} + e \right) + \eta \beta g a^{\beta} H^{\beta} e^{\beta-1} \frac{\partial e}{\partial p} = 0, \quad (44)$$

$$\dot{\eta} = - \left(\frac{\partial c}{\partial H} + \alpha v' \frac{\partial e}{\partial H} - \gamma(1-p) \frac{\partial e}{\partial H} + \eta \left(\beta g a^{\beta} H^{\beta} e^{\beta-1} \frac{\partial e}{\partial H} + \beta g a^{\beta} e^{\beta} H^{\beta-1} - \delta \right) \right) + \eta \rho. \quad (45)$$

Eqs. (43) and (44) are rewritten as follows, respectively :

$$(waH - p + \alpha v' - \gamma(1-p) + \eta \beta g a^{\beta} e^{\beta-1} H^{\beta}) \frac{\partial e}{\partial T} + \gamma - 1 = 0, \quad (46)$$

$$(waH - p + \alpha v' - \gamma(1-p) + \eta \beta g a^{\beta} e^{\beta-1} H^{\beta}) \frac{\partial e}{\partial p} + \gamma e - e = 0. \quad (47)$$

From (40), (41), (46) and (47), we can obtain

$$\gamma = 1, \quad (48)$$

$$1-p = \eta \beta g a^{\beta} e^{\beta-1} H^{\beta}. \quad (49)$$

Moreover, (45) is rewritten as follows :

$$\dot{\eta} = - wae - \eta \beta g (ae)^{\beta} H^{\beta-1} + \eta \delta + \eta \rho. \quad (50)$$

We describe two curves of $\dot{H} = 0$ and $\dot{\eta} = 0$ on the (H, η) plane. First, the curve $\dot{H} = 0$ is vertical at $\left(\left\{ \delta / (g a^{\beta} e^{\beta}) \right\}^{1/(\beta-1)}, 0 \right)$. We denote

$\Lambda = g (aeH)^\beta - \delta H$. Partially differentiating with respect to H on $\dot{H} = 0$, we have

$$\frac{\partial \Lambda}{\partial H} = \beta g a^\beta e^\beta H^{\beta-1} \left(\frac{H}{e} \frac{\partial e}{\partial H} - \frac{1-\beta}{\beta} \right). \tag{51}$$

If $(H/e)(\partial e/\partial H) < (1-\beta)/\beta$, then the sign of (51) is negative. We assume that $(H/e)(\partial e/\partial H) < (1-\beta)/\beta$. Second, for the curve $\dot{\eta} = 0$, totally differentiating $-wae - \eta\beta g (ae)^\beta H^{\beta-1} + \eta\delta + \eta = 0$, we have

$$\frac{d\eta}{dH} = - \frac{wa \frac{\partial e}{\partial H} + \eta\beta^2 g a^\beta e^\beta H^{\beta-2} \left(\frac{H}{e} \frac{\partial e}{\partial H} - \frac{1-\beta}{\beta} \right)}{\beta g a^\beta e^\beta H^{\beta-1} - (\delta + \rho)}. \tag{52}$$

In the positive quadrant in (H, η) plane, the denominator of the right hand side in (52) is negative by (50) along the curve $\dot{\eta} = 0$. However, the numerator can take either sign. Therefore, the sign of (52) is indeterminate. In either case, this curve is discontinuous at H^* .⁽⁹⁾ We denote $\Omega = -wae - \eta\beta g (ae)^\beta H^{\beta-1} + \eta\delta + \eta\rho$. Partially differentiating with respect to η , we have

$$\frac{\partial \Omega}{\partial \eta} = -\beta g a^\beta e^\beta H^{\beta-1} + (\delta + \rho). \tag{53}$$

Since we concentrate on the case of $\eta > 0$, the sign of (53) is positive.

In Fig.2 and Fig.3, as an example, we describe a case of $H^* < \left\{ \delta / (g a^\beta e^\beta) \right\}^{1/(\beta-1)}$. Fig.2 represents the case where the sign of (52) is positive. That is, the curve $\dot{\eta} = 0$ is increasing with H and jumps at H^* . This means that the educational subsidy increases discontinuously. In this case, as shown in Fig.2, there is an optimal convergent path to the

(9) The weight parameter $\alpha(H)$ changes from $\underline{\alpha}$ to $\bar{\alpha}$ at H^* . It causes sudden increase of demand for education. Then, to satisfy $-wae - \eta\beta g (ae)^\beta H^{\beta-1} + \eta\delta + \eta = 0$, the curve $\dot{\eta} = 0$ should shift up at H^*

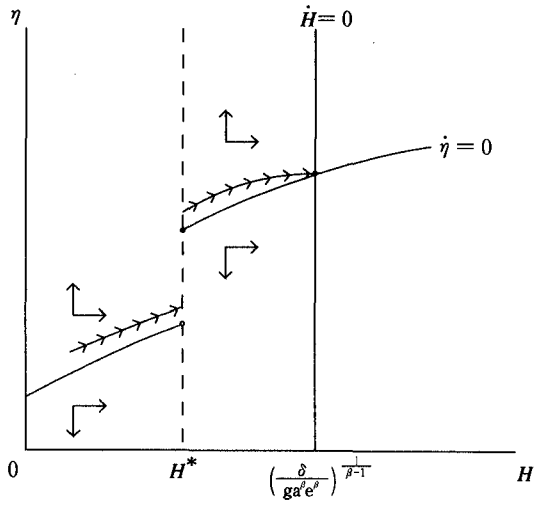


Fig. 2

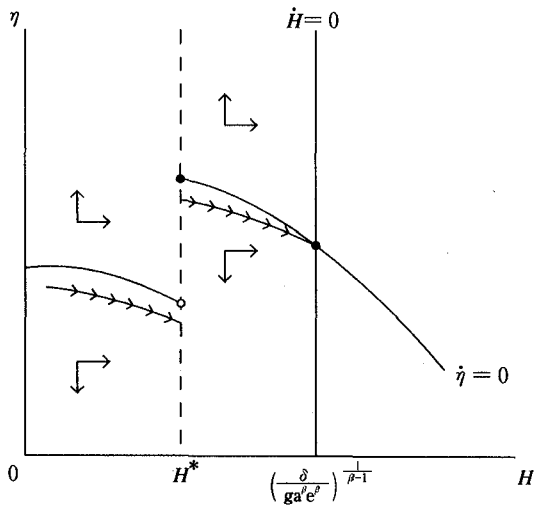


Fig. 3

steady state. Fig.3 represents the case where the sign of (52) is negative. That is, the curve $\dot{\eta} = 0$ is decreasing with H and jumps at H^* . Also in this case, there is an optimal path like Fig.3.

4. Concluding Comments

We have analyzed the optimal educational policies in dynamic setting. In our setting, the government subsidizes the higher education, because individual human capital formed through education contributes to creation of knowledge in a society. Though knowledge also contributes to formation of individual human capital, individuals do not take account of these effects when they decide on their level of education.

Furthermore, in our model, the individual decision for the level of education is affected by the preference for education. Under this setting, we have analyzed the phenomenon of popularization of education, which means the sudden increases in demand for the higher education by change of preference. We have found the optimal convergent path to the steady state, which jumps at the time when the individual preference changes, that is, at the beginning of popularization. We conclude that the government should substantially raise the subsidy for education at the time. The intuitive reason is that the price effect of education is reduced when the preference parameter for education shifts up, and the government needs to give the larger subsidy to change the level of education of individuals.

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In this paper, we construct a model which explains a change in demand for education, considering an individual preference for the higher education (university). We suppose that the individual preference for education depends on the average level of education in a society and changes drastically at some level. We consider a possibility that diffusion of education brings about discontinuous increase in demand for education, and examine the optimal educational policies in dynamic setting.