The theory of instantaneous power in three-phase four-wire systems: a comprehensive approach

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The Theory of Instantaneous Power in Three-Phase Four-Wire Systems: A Comprehensive Approach

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***Abstract*— This paper describes a holistic approach to the theory of instantaneous power in three-phase four-wire systems, focusing on the original theory created in 1983 and a modified theory presented in 1994. The two theories are perfectly identical if no zero-sequence voltage is included in a three-phase three-wire system. However, they are different in the instantaneous active and reactive power in each phase if a zero-sequence voltage and current are included in a three-phase four-wire system. Theory and computer simulations in this paper lead to the following conclusions: An active filter without energy storage components can fully compensate for the neutral current even in a three-phase four-wire system including a zero-sequence voltage and current, when a proposed control strategy based on the original theory is applied. However, the active filter cannot compensate for the neutral current fully, when an already-proposed control strategy based on the modified theory is applied.**

I. Introduction

A. Background

In 1983, the theory of instantaneous power in three-phase circuits was originated by one of the authors and his former colleagues [1] [2]. The theory is applicable to three-phase four-wire systems, as well as to three-phase three-wire systems. In addition, it is characterized by allowing us to define the instantaneous reactive power in each phase as a unique value for arbitrary three-phase voltage and current waveforms without any restriction, and by yielding a lucid explanation of the physical meaning of instantaneous reactive power. Since the emergence of the theory, sixteen years have passed, and it has been realized that the concept of instantaneous reactive power has become firmly established among electrical engineers, in particular, power electronics researchers and engineers. For example, Willems has made the following descriptions of the theory [8]:

> "Their concept is very interesting for practical purposes, in particular to analyze the instantaneous compensation of reactive power without energy storage. · · · · · Akagi's imaginary power concept hence exactly shows to what extent compensators without energy storage can be used to reduce line losses. This result is precisely the extremely interesting contribution realized by Akagi and his co-authors."

Penetration of voltage-source PWM inverters into practical applications has spurred interest in expansion of the theory into polyphase circuits and applications of the theory to power electronics equipment [3]-[19].

Three-phase three-wire circuits are exclusively used for utility power distribution systems of 6.6 kV and industrial power distribution systems of low-voltage classes in Japan. On the other hand, three-phase four-wire circuits are widely used for industrial power distribution systems of low-voltage classes in many countries, for instance, in the United States. For this reason, research on active filters intended for installation on three-phase four-wire systems has been carried out in many countries other than Japan.

Recent broadcasting equipment used in Japan has required a large-capacity single-phase 100-V power supply, and therefore a three-phase four-wire system with a phase-to-neutral voltage rated at 100 V has been applied to such special cases in Japan. Meiden has developed active filters for three-phase four-wire systems of low-voltage classes. The active filters in commercial use range from 75 to 500 kVA. It is reported in [12] that the 300-kVA active filter achieves excellent compensation of the neutral current and harmonic currents flowing in a three-phase four-wire system.

B. The Original Theory and The Modified Theory

In 1994, a modified theory of instantaneous power was formulated by Nabae, et. al. [13] [14], and then it was established by Peng and Lai [16] [19]. Hereinafter, the theory originated in 1983 is referred to as "the original theory" in this paper, in order to avoid confusion and to distinguish it from "the modified theory." The modified theory is also applicable to three-phase four-wire systems, and it allows us to define the instantaneous reactive power in each phase as a unique value. However, the instantaneous reactive power defined by the modified theory is different from that defined by the original theory under three-phase four-wire systems including a zero-sequence voltage and current. Therefore, the modified theory would be different in filtering characteristics from the original theory when the two theories are applied to an active filter without energy storage components.

This paper focuses on similarity and difference between the original theory and the modified theory. The difference exists in how to deal with the zero-sequence circuit in a three-phase four-wire circuit, thus resulting in a significant difference in their "mapping matrices." Moreover, this paper proposes a control strategy of an active filter with-

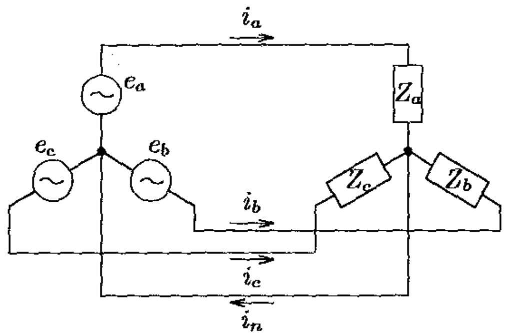

Fig. 1. Three-phase four-wire system.

out energy storage components for compensation of both the zero-sequence current and the instantaneous reactive current in each phase.

II. The Theory of Instantaneous Power in a Three-Phase Four-Wire System

Fig. 1 shows a basic circuit configuration of a three-phase four-wire system including a zero-sequence voltage and current. For the sake of simplicity, the distribution line impedance and the neutral line impedance existing between the supply and the load are discarded from Fig. 1. However, the theory developed in this paper is applicable to such a practical three-phase four-wire system where a non-negligible amount of impedance exists in the distribution and neutral lines. The reason is that attention is paid to three-phase instantaneous voltages and currents on any bus, for instance, at the beginning terminals, on an intermediary bus or at the end terminals.

Three-phase instantaneous voltages and currents on the a-b-c coordinates can be transformed into those on the 0-α-β coordinates by the following equations:

$$\begin{bmatrix} e_0 \\ e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}. \tag{2}$$

The following relation exists between the neutral current i_n in Fig. 1 and the zero-sequence current i_0 in (2).

$$i_n = i_a + i_b + i_c \tag{3}$$
$$i_0 = \frac{1}{\sqrt{3}}(i_a + i_b + i_c) = \frac{1}{\sqrt{3}} i_n \tag{4}$$

The instantaneous real power in the three-phase four-wire system, p is given by

$$\begin{aligned} p &= e_a i_a + e_b i_b + e_c i_c \\ &= e_0 i_0 + e_\alpha i_\alpha + e_\beta i_\beta. \end{aligned} \tag{5}$$

A. The Original Theory

The original theory in [1] and [2] defines two instantaneous real powers p_0 and $p_{\alpha\beta}$, and an instantaneous imaginary power $q_{\alpha\beta}$ in the three-phase four-wire system as follows:

$$\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}. \tag{6}$$

Equation (6) suggests that $p_0(= e_0 i_0)$ obviously implies the instantaneous real power in the zero-sequence circuit, and that $e_\alpha i_\alpha$ and $e_\beta i_\beta$ also mean instantaneous power because they are defined by the product of the instantaneous voltage in one phase and the instantaneous current in the same phase. Therefore, $p_{\alpha\beta}$ is the instantaneous real power in the α and β-phase circuit, and so its dimension is [W]. Conversely, $e_\alpha i_\beta$ and $e_\beta i_\alpha$ are not instantaneous power because they are defined by the product of the instantaneous voltage in one phase and the instantaneous current in another phase. Accordingly, $q_{\alpha\beta}$ in the α and β-phase circuit is not the instantaneous real power but a new electrical quantity defined in [1], and so a new dimension should be introduced to $q_{\alpha\beta}$, because its dimension is not [W], [VA] or [var]. The authors think that "Imaginary Watt" [IW] is a good candidate for the dimension.

Since the utility power supply in (6) is assumed as three-phase voltage sources, e_0, e_α and e_β, which are a set of three already known values, equation (6) can be interpreted as "mapping" of a three-dimensional current space to a three-dimensional power space, and vice versa. There are many mapping matrices from a theoretical point of view, whereas few matrices can offer a lucid physical meaning from a practical point of view. The mapping matrix shown in (6) was defined in 1983 for the first time in the world. The inverse transformation of (6) is given by

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{e_0 e_{\alpha\beta}^2} \begin{bmatrix} e_{\alpha\beta}^2 & 0 & 0 \\ 0 & e_0 e_\alpha & -e_0 e_\beta \\ 0 & e_0 e_\beta & e_0 e_\alpha \end{bmatrix} \cdot \begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix}, \tag{7}$$

where

$$e_{\alpha\beta}^2 = e_\alpha^2 + e_\beta^2.$$

The instantaneous currents on the 0-α-β coordinates can be obtained from (7):

$$i_0 = \frac{1}{e_0} p_0 \; (= \frac{1}{e_0} e_0 i_0 = i_0) \tag{8}$$

$$\begin{aligned} i_\alpha &= \frac{1}{e_{\alpha\beta}^2} e_\alpha p_{\alpha\beta} + \frac{1}{e_{\alpha\beta}^2}(-e_\beta q_{\alpha\beta}) \\ &\equiv i_{\alpha p} + i_{\alpha q} \end{aligned} \tag{9}$$

$$\begin{aligned} i_\beta &= \frac{1}{e_{\alpha\beta}^2} e_\beta p_{\alpha\beta} + \frac{1}{e_{\alpha\beta}^2}(e_\alpha q_{\alpha\beta}) \\ &\equiv i_{\beta p} + i_{\beta q}, \end{aligned} \tag{10}$$

where

i_0 : zero-sequence instantaneous current
$i_{\alpha p}$: α-phase instantaneous active current
$i_{\beta p}$: β-phase instantaneous active current
$i_{\alpha q}$: α-phase instantaneous reactive current
$i_{\beta q}$: β-phase instantaneous reactive current.

The theoretical derivation of (7)–(10) from (6) is performed under an assumption of $e_0 \neq 0$, because it is impossible to calculate the inverse matrix of (6) when $e_0 = 0$. However, i_α in (9) and i_β in (10) exclude e_0 even if $e_0 \neq 0$. This implies that the original theory deals with the zero-sequence circuit as a single-phase circuit being independent of the α-phase circuit and the β-phase circuit. Therefore, it is practically acceptable to substitute $e_0 = 0$ into final equations or results derived under the assumption of $e_0 \neq 0$, when the original theory is applied to a three-phase four-wire system without zero-sequence voltage. In general, the derivation of the final equations or results might not be complicated, because the zero-sequence circuit is independent of the α-phase circuit and the β-phase circuit.

The first and second terms on the right-hand side of (9) and (10) correspond to the instantaneous active current and the instantaneous reactive current in the α-phase and the β-phase, respectively. Thus, the following relations exist:

$$\begin{aligned} & e_0 i_0 + e_\alpha i_{\alpha p} + e_\beta i_{\beta p} \\ &= e_0 i_0 + e_\alpha\left(\frac{1}{e_{\alpha\beta}^2} e_\alpha p_{\alpha\beta}\right) + e_\beta\left(\frac{1}{e_{\alpha\beta}^2} e_\beta p_{\alpha\beta}\right) \\ &= p_0 + p_{\alpha p} + p_{\beta p} = p \end{aligned} \tag{11}$$

$$\begin{aligned} & e_\alpha i_{\alpha q} + e_\beta i_{\beta q} \\ &= e_\alpha\left\{\frac{1}{e_{\alpha\beta}^2}(-e_\beta q_{\alpha\beta})\right\} + e_\beta\left(\frac{1}{e_{\alpha\beta}^2} e_\alpha q_{\alpha\beta}\right) \\ &= p_{\alpha q} + p_{\beta q} = 0. \end{aligned} \tag{12}$$

The instantaneous active and reactive powers in each phase are defined as follows:

$p_0 = e_0 i_0$: zero-sequence instantaneous power
$p_{\alpha p} = e_\alpha i_{\alpha p}$: α-phase instantaneous active power
$p_{\beta p} = e_\beta i_{\beta p}$: β-phase instantaneous active power
$p_{\alpha q} = e_\alpha i_{\alpha q}$: α-phase instantaneous reactive power
$p_{\beta q} = e_\beta i_{\beta q}$: β-phase instantaneous reactive power

B. The Modified Theory

The modified theory in [13] [14] defines an instantaneous real power p and three instantaneous imaginary powers, q_0, q_α and q_β as follows:

$$\begin{bmatrix} p \\ q_0 \\ q_\alpha \\ q_\beta \end{bmatrix} = \begin{bmatrix} e_0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \\ e_\beta & 0 & -e_0 \\ -e_\alpha & e_0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}. \tag{13}$$

The following equations concerning q_0, q_α, and q_β can be derived from (13):

$$\begin{aligned} e_0 q_0 &= -e_0 e_\beta i_\alpha + e_0 e_\alpha i_\beta \\ e_\alpha q_\alpha &= e_\alpha e_\beta i_0 - e_0 e_\alpha i_\beta \\ e_\beta q_\beta &= -e_\alpha e_\beta i_0 + e_0 e_\beta i_\alpha. \end{aligned}$$

The sum of all terms on the right-hand side of the above equations is always zero, thus leading to

$$e_0 \cdot q_0 + e_\alpha \cdot q_\alpha + e_\beta \cdot q_\beta = 0. \tag{14}$$

The rank of the mapping matrix in (13) is three, so that the number of independent variables among q_0, q_α and q_β is not three but two. As a result, this leads to a three-dimensional power space in the modified theory, like that in the original theory. However, the two mapping matrices defined by (6) and (13) are different in formulation. The inverse transformation of (13) is performed as follows:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{e_{0\alpha\beta}^2} \begin{bmatrix} e_0 & 0 & e_\beta & -e_\alpha \\ e_\alpha & -e_\beta & 0 & e_0 \\ e_\beta & e_\alpha & -e_0 & 0 \end{bmatrix} \cdot \begin{bmatrix} p \\ q_0 \\ q_\alpha \\ q_\beta \end{bmatrix}, \tag{15}$$

where

$$e_{0\alpha\beta}^2 = e_0^2 + e_\alpha^2 + e_\beta^2.$$

The instantaneous current in each phase can be obtained from (15):

$$\begin{aligned} i_0 &= \frac{1}{e_{0\alpha\beta}^2} e_0 p + \frac{1}{e_{0\alpha\beta}^2}(e_\beta q_\alpha - e_\alpha q_\beta) \\ &\equiv i_{0p} + i_{0q} \end{aligned} \tag{16}$$

$$\begin{aligned} i_\alpha &= \frac{1}{e_{0\alpha\beta}^2} e_\alpha p + \frac{1}{e_{0\alpha\beta}^2}(e_0 q_\beta - e_\beta q_0) \\ &\equiv i_{\alpha p} + i_{\alpha q} \end{aligned} \tag{17}$$

$$\begin{aligned} i_\beta &= \frac{1}{e_{0\alpha\beta}^2} e_\beta p + \frac{1}{e_{0\alpha\beta}^2}(e_\alpha q_0 - e_0 q_\alpha) \\ &\equiv i_{\beta p} + i_{\beta q}, \end{aligned} \tag{18}$$

where

i_{0p} : zero-sequence instantaneous active current
i_{0q} : zero-sequence instantaneous reactive current.

Equation (16) implies that the zero-sequence instantaneous current i_0 can be divided into two instantaneous currents; i_{0p} and i_{0q}. Thus, the following relations exist:

$$\begin{aligned} & e_0 i_{0p} + e_\alpha i_{\alpha p} + e_\beta i_{\beta p} \\ &= e_0\left(\frac{1}{e_{0\alpha\beta}^2} e_0 p\right) + e_\alpha\left(\frac{1}{e_{0\alpha\beta}^2} e_\alpha p\right) + e_\beta\left(\frac{1}{e_{0\alpha\beta}^2} e_\beta p\right) \\ &= p_{0p} + p_{\alpha p} + p_{\beta p} = p, \end{aligned} \tag{19}$$

$$\begin{aligned} & e_0 i_{0q} + e_\alpha i_{\alpha q} + e_\beta i_{\beta q} \\ &= e_0\left\{\frac{1}{e_{0\alpha\beta}^2}(e_\beta q_\alpha - e_\alpha q_\beta)\right\} + e_\alpha\left\{\frac{1}{e_{0\alpha\beta}^2}(e_0 q_\beta - e_\beta q_0)\right\} \\ &\quad + e_\beta\left\{\frac{1}{e_{0\alpha\beta}^2}(e_\alpha q_0 - e_0 q_\alpha)\right\} \\ &= p_{0q} + p_{\alpha q} + p_{\beta q} = 0. \end{aligned} \tag{20}$$

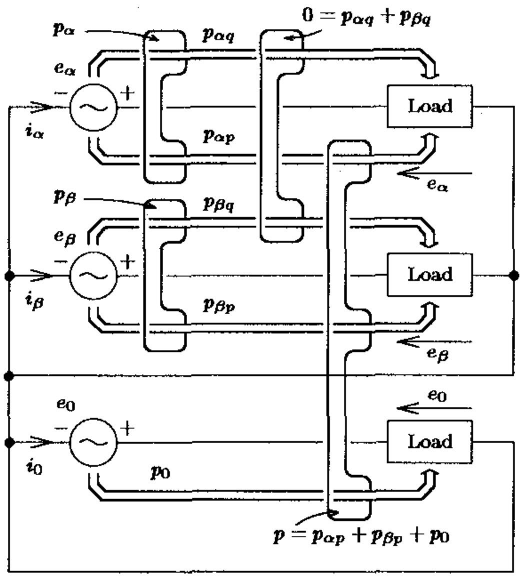

Fig. 2. Power flow based on the original theory.

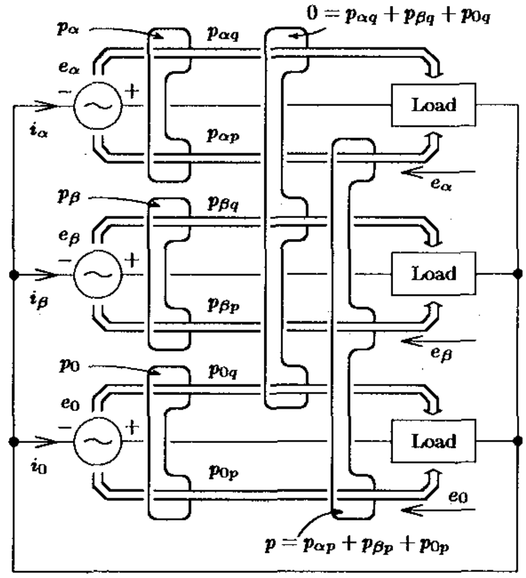

Fig. 3. Power flow based on the modified theory.

Here, $p_{0p}(= e_0 i_{0p})$ and $p_{0q}(= e_0 i_{0q})$ are referred as to

p_{0p} : zero-sequence instantaneous active power,
p_{0q} : zero-sequence instantaneous reactive power.

III. Similarity and Difference between the Original Theory and the Modified Theory

A. In the case of $e_0 \neq 0$ and $i_0 \neq 0$

Fig. 2 depicts power flow based on the original theory in a three-phase four-wire system. Note that the original theory considers the zero-sequence circuit as a single-phase circuit independent of the α-phase circuit and the β-phase circuit, just as the so-called "symmetrical coordinate method" splits a three-phase four-wire system into a zero-sequence circuit, a positive-sequence circuit and a negative-sequence circuit, in which the zero-sequence circuit is considered as a single-phase circuit independent of the positive-sequence circuit and the negative-sequence circuit.

In a single-phase circuit, it is possible to define instantaneous active power as the product of instantaneous voltage and current, whereas it would be impossible to uniquely define instantaneous reactive power from only the instantaneous voltage and current at that instant of time. Therefore, the original theory deals with zero-sequence current i_0 as instantaneous active current because the zero-sequence circuit forms instantaneous active power, $p_0 = e_0 i_0$, when $e_0 \neq 0$. This concludes that no instantaneous reactive current exists in the zero-sequence circuit.

Equation (12) means that the sum of α-phase instantaneous reactive power $p_{\alpha q}$ $(= e_\alpha i_{\alpha q})$ and β-phase instantaneous reactive power $p_{\beta q}$ $(= e_\beta i_{\beta q})$ is always zero. This implies that $p_{\alpha q}$ and $p_{\beta q}$ make no contribution to energy transfer within the α and β-phase circuit. However, $p_{\alpha q}$ makes a significant contribution to energy transfer within the α-phase circuit, and $p_{\beta q}$ does the same in the β-phase circuit, so that the instantaneous reactive power in each phase has the same dimension of [W] as the instantaneous active power in terms of physical science. A new dimension, which is not [var], can be introduced to the instantaneous reactive power in terms of electrical engineering. The original theory introduces $q_{\alpha\beta}$ as the instantaneous imaginary power which determines $p_{\alpha q}$ and $p_{\beta q}$, so that the original theory defines the two independent instantaneous real powers p_0 and $p_{\alpha\beta}$, in order to form the three-dimensional power space.

Fig. 3 depicts power flow based on the modified theory. It is clear from (16)–(18) that the modified theory is characterized by equally dealing with the zero-sequence circuit, the α-phase circuit, and the β-phase circuit. This implies that zero-sequence current i_0 can be divided into zero-sequence instantaneous active current i_{0p} and zero-sequence instantaneous reactive current i_{0q}. However, equation (20) exists because the original theory is the same as the modified theory in the physical meaning of instantaneous reactive power in each phase. That is, p_{0q}, $p_{\alpha q}$ and $p_{\beta q}$ make no contribution to energy transfer in the 0-α-β circuit.

Equation (14) suggests that the number of independent instantaneous imaginary powers are two, so that the number of independent instantaneous real powers must be one in the three-dimensional power space.

Substituting (16)–(18) into (6) produces the following relations among p_0, $p_{\alpha\beta}$ and $q_{\alpha\beta}$ defined by the original theory and p, q_0, q_α and q_β defined by the modified theory:

$$p_0 = \frac{e_0^2}{e_{0\alpha\beta}^2} p + \frac{e_0}{e_{0\alpha\beta}^2}(e_\beta q_\alpha - e_\alpha q_\beta) \tag{21}$$

$$p_{\alpha\beta} = (1 - \frac{e_0^2}{e_{0\alpha\beta}^2})p - \frac{e_0}{e_{0\alpha\beta}^2}(e_\beta q_\alpha - e_\alpha q_\beta) \quad (22)$$

$$q_{\alpha\beta} = q_0. \quad (23)$$

The following relationship exists between the two independent instantaneous real powers in the original theory and the instantaneous real power in the modified theory:

$$p_0 + p_{\alpha\beta} = p. \quad (24)$$

B. In the case of $e_0 = 0$ and $i_0 = 0$

Substitution of $e_0 = i_0 = 0$ into (6) and (13) yields $p_0 = 0$ from (6) and $q_\alpha = q_\beta = 0$ from (13), thus leading to $p_{\alpha\beta} = p$ and $q_{\alpha\beta} = q_0$. As a result, equations (6) and (13) are simplified as follows:

$$\begin{bmatrix} p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} p \\ q_0 \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \quad (25)$$

The above equation concludes that the original theory is identical to the modified theory in a three-phase three-wire system excluding zero-sequence voltage.

C. In the case of $e_0 = 0$ and $i_0 \neq 0$

The constraint of $e_0 = 0$ produces the relationship of $p_0 = p_{0p} = p_{0q} = 0$, so that no difference exists in power flow between Figs. 2 and 3. In other words, substitution of $e_0 = 0$ into (21) and (22) brings (25) into existence. This implies that $i_{\alpha p}$ and $i_{\alpha q}$ in the original theory, which are defined by (9), are identical to $i_{\alpha p}$ and $i_{\alpha q}$ in the modified theory, which are defined by (17), respectively. Moreover, $i_{\beta p}$ and $i_{\beta q}$, which are defined by (10), are identical to $i_{\beta p}$ and $i_{\beta q}$, which are defined by (18), respectively.

Interpretation is different in i_0 between the two theories. Whenever $e_0 = 0$, p_0 is always zero, irrespective of i_0. The original theory, therefore, reaches such an understanding that i_0 is simply the "instantaneous current" in the zero-sequence circuit when $e_0 = 0$, although i_0 should be referred to as the "instantaneous active current" in the zero-sequence circuit when $e_0 \neq 0$. On the other hand, the modified theory[1] can divide i_0 into i_{0p} and i_{0q}. Whenever $e_0 = 0$, it is clear from (16) that $i_{0p} = 0$, thus resulting in $i_0 = i_{0q}$. Therefore, the modified theory keeps such an understanding that i_0 is instantaneous reactive current when $e_0 = 0$. As shown in section V, however, no energy storage component is required to an active filter based on either the original theory or the modified theory for compensation of i_0, as long as $e_0 = 0$.

D. In the case of $e_0 \neq 0$ and $i_0 = 0$

Application of $e_0 \neq 0$ and $i_0 = 0$ to (6) produces $p_0 (= 0)$, and $p_{\alpha\beta}$ and $q_{\alpha\beta}$ which are identical to those defined by (25) in the original theory. The reason is that the original theory deals with the zero-sequence circuit as a single-phase circuit independent of the others. Substitution of the above conditions to (13) uniquely yields p, q_0, q_α and q_β in the modified theory[2]. As a result, the following equations can be obtained from (16):

$$i_0 = i_{0p} + i_{0q} = 0$$

$$i_{0p} = \frac{1}{e_{0\alpha\beta}^2} e_0(e_\alpha i_\alpha + e_\beta i_\beta) = -i_{0q}.$$

These equations suggest that the modified theory can split i_0 into i_{0p} and i_{0q}, even in a three-phase three-wire system including zero-sequence voltage. However, it is impossible to control i_{0p} and i_{0q} independently because no zero-sequence current flows in any three-phase three-wire system.

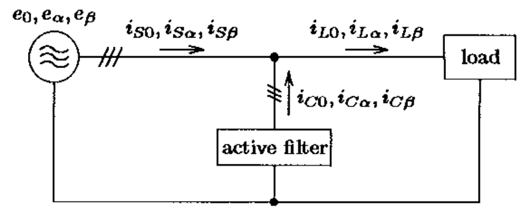

Fig. 4. System configuration on the 0-α-β coordinates.

IV. Control Strategy of an Active Filter without Energy Storage Components

Fig. 4 shows a basic system configuration of an active filter for compensation of both zero-sequence current and instantaneous reactive current on the 0-α-β coordinates in a three-phase four-wire system.

A. Control Strategy Based on the Original Theory

The original theory yields compensating currents, i_{C0}, $i_{C\alpha}$ and $i_{C\beta}$ on the 0-α-β coordinates as follows:.

$$\begin{bmatrix} i_{C0} \\ i_{C\alpha} \\ i_{C\beta} \end{bmatrix} = \frac{1}{e_0 e_{\alpha\beta}^2} \begin{bmatrix} e_{\alpha\beta}^2 & 0 & 0 \\ 0 & e_0 e_\alpha & -e_0 e_\beta \\ 0 & e_0 e_\beta & e_0 e_\alpha \end{bmatrix} \cdot \begin{bmatrix} p_{C0} \\ p_{C\alpha\beta} \\ q_{C\alpha\beta} \end{bmatrix}. \quad (26)$$

Since p_{C0} and $p_{C\alpha\beta}$ in the above equation are independent, it is possible to impose the following constraint on p_{C0} and $p_{C\alpha\beta}$:

$$p_C = p_{C0} + p_{C\alpha\beta} = 0. \quad (27)$$

This constraint suggests that no instantaneous real power flows into or out of the active filter even when $e_0 \neq 0$. In other words, power exchange of $e_0 i_{C0}$ happens between the zero-sequence circuit and the α and β-phase circuit.

The following constraint that the active filter compensates for the instantaneous imaginary power on the load side is imposed on $q_{C\alpha\beta}$:

$$q_{C\alpha\beta} = q_{L\alpha\beta}. \quad (28)$$

[1]Discussion on (14): The conditions that $e_0 = 0$ and $i_0 \neq 0$ yield $q_\alpha = e_\beta \cdot i_0 \neq 0$ and $q_\beta = -e_\alpha \cdot i_0 \neq 0$, thus resulting in the following equation; $e_\alpha \cdot q_\alpha + e_\beta \cdot q_\beta = 0$. It is clear from this equation and $e_0 = 0$ that equation (14) always exists, irrespective of q_0.

[2]Discussion on (14): The conditions that $e_0 \neq 0$ and $i_0 = 0$ yield $q_\alpha = -e_0 \cdot i_\beta \neq 0$ and $q_\beta = e_0 \cdot i_\alpha \neq 0$. The combination of these equations and $q_0 = e_\alpha \cdot i_\beta - e_\beta \cdot i_\alpha$ brings (14) into existence.

Equations (27) and (28) give the following relations:

$$p_{C0} = p_{L0} = e_0 i_{L0} \tag{29}$$
$$p_{C\alpha\beta} = -p_{L0} = -e_0 i_{L0} \tag{30}$$
$$q_{C\alpha\beta} = e_\alpha i_{L\beta} - e_\beta i_{L\alpha}. \tag{31}$$

Substitution of (29)–(31) into (26) yields the compensating currents.

$$i_{C0} = i_{L0} \tag{32}$$
$$i_{C\alpha} = \frac{1}{e_{\alpha\beta}^2}(-e_0 e_\alpha i_{L0} + e_\beta^2 i_{L\alpha} - e_\alpha e_\beta i_{L\beta}) \tag{33}$$
$$i_{C\beta} = \frac{1}{e_{\alpha\beta}^2}(-e_0 e_\beta i_{L0} - e_\alpha e_\beta i_{L\alpha} + e_\alpha^2 i_{L\beta}) \tag{34}$$

The relationship of $i_{C0} = i_{L0}$ in (32) exists irrespective of e_0, and therefore the active filter without energy storage components can compensate for the zero-sequence current fully, even when $e_0 \neq 0$. Equations (32)–(34) are derived from (29) under the assumption of $e_0 \neq 0$. As described in section II, direct substitution of $e_0 = 0$ into (33) and (34) offers the compensating currents when $e_0 = 0$.

B. Control Strategy Based on the Modified Theory

Application of the modified theory to the active filter gives the compensating currents.

$$\begin{bmatrix} i_{C0} \\ i_{C\alpha} \\ i_{C\beta} \end{bmatrix} = \frac{1}{e_{0\alpha\beta}^2} \begin{bmatrix} e_0 & 0 & e_\beta & -e_\alpha \\ e_\alpha & -e_\beta & 0 & e_0 \\ e_\beta & e_\alpha & -e_0 & 0 \end{bmatrix} \cdot \begin{bmatrix} p_C \\ q_{C0} \\ q_{C\alpha} \\ q_{C\beta} \end{bmatrix} \tag{35}$$

Since the modified theory defines p_C as instantaneous real power, the active filter without energy storage components requires imposing a constraint of $p_C = 0$. On the other hand, the modified theory defines two independent instantaneous imaginary powers, and therefore two degrees of freedom exist among q_{C0}, $q_{C\alpha}$ and $q_{C\beta}$. However, a constraint of $q_{C0} = q_{C\alpha} = q_{C\beta} = 0$ is meaningless because it corresponds to no installation of any active filter. The control strategy proposed in [14] is taken in the following. This stems from both the constraint of $p_C = 0$ and the constraint that the active filter compensates for the instantaneous imaginary powers on the load side. The control strategy is given as

$$p_C = 0 \tag{36}$$
$$q_{C0} = q_{L0} \tag{37}$$
$$q_{C\alpha} = q_{L\alpha} \tag{38}$$
$$q_{C\beta} = q_{L\beta}. \tag{39}$$

Substitution of (36)–(39) into (35) yields the compensating currents.

$$i_{C0} = \frac{1}{e_{0\alpha\beta}^2}\{(e_\alpha^2 + e_\beta^2) i_{L0} - e_0 e_\alpha i_{L\alpha} - e_0 e_\beta i_{L\beta}\} \tag{40}$$
$$i_{C\alpha} = \frac{1}{e_{0\alpha\beta}^2}\{-e_0 e_\alpha i_{L0} + (e_0^2 + e_\beta^2) i_{L\alpha} - e_\alpha e_\beta i_{L\beta}\} \tag{41}$$
$$i_{C\beta} = \frac{1}{e_{0\alpha\beta}^2}\{-e_0 e_\beta i_{L0} - e_\alpha e_\beta i_{L\alpha} + (e_0^2 + e_\alpha^2) i_{L\beta}\} \tag{42}$$

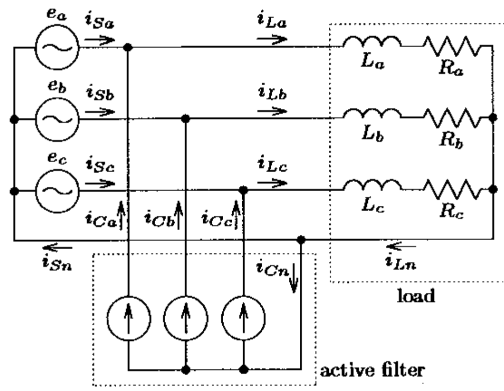

Fig. 5. Simulation model on the a-b-c coordinates.

When the condition of $e_0 = 0$ is substituted into (33)–(34) and (40)–(42), it is easy to find out that their corresponding equations become equal. Therefore, the control strategy based on the modified theory is identical to that based on the original theory, thus resulting in the same filtering characteristics, as long as $e_0 = 0$. However, when $e_0 \neq 0$, the two control strategies are different, so that their filtering characteristics are not identical. In particular, the control strategy based on (40)–(42) cannot compensate for the zero-sequence current fully, because $i_{C0} \neq i_{L0}$.

V. Discussion By Simulation

This section performs computer simulation under ideal conditions for the purpose of clarifying the similarity and difference between the two theories.

A. Simulation Conditions

Fig. 5 depicts a simulation model on the *a-b-c* coordinates, and Table I summarizes simulation conditions. The three-phase supply is assumed as the following sinusoidal voltage sources:

$$e_a = \sqrt{2}\, E_a \sin(\omega t)$$
$$e_b = \sqrt{2}\, E_b \sin(\omega t - 2\pi/3)$$
$$e_c = \sqrt{2}\, E_c \sin(\omega t + 2\pi/3).$$

Each phase-to-neutral rms voltage is shown in Table I. Note that three-phase sinusoidal voltages under conditions I and III exclude any zero-sequence voltage, while three-phase sinusoidal voltages under conditions II and IV include a zero-sequence voltage. For the purpose of extracting a distinct difference between the two theories, the unbalanced three-phase load shown in Fig. 5 is assumed to be a set of three linear *R-L* circuits, excluding any harmonic-producing load. The parameters of the load are given by

$$R_a = 1.06\ \Omega, \quad R_b = R_c = 1.32\ \Omega$$
$$L_a = 3.36\ \text{mH}, \ L_b = L_c = 4.20\ \text{mH}.$$

In the following simulation, the power circuit of the active filter is assumed to be three-phase ideal current sources, as shown in Fig. 5.

TABLE I

SIMULATION CONDITIONS.

Condition		I	II	III	IV
Waveforms		Fig. 6	Fig. 7	Fig. 8	Fig. 9
Voltage [V]	E_a	115	115	115	115
	E_b	115	115	115	115
	E_c	115	92	115	92
Theory		Original		Modified	

TABLE II

THE RATIO [%] OF THE 3RD AND 5TH HARMONIC COMPONENTS CONTAINED IN THE COMPENSATING CURRENT TO THE FUNDAMENTAL COMPONENT IN EACH PHASE.

Condition		I	II	III	IV
Waveforms		Fig. 6	Fig. 7	Fig. 8	Fig. 9
i_{Ca}	3rd	4.8	4.3	4.8	11.3
	5th	0	0.3	0	0.7
i_{Cb}	3rd	5.9	4.3	5.9	13.8
	5th	0	0.3	0	0.9
i_{Cc}	3rd	5.5	3.7	5.5	12.7
	5th	0	0.3	0	0.9

Figs. 6–9 show simulated waveforms, corresponding to conditions I to IV. Table II summarizes in percentage units the third and fifth-order harmonic currents contained in the compensating current with respect to the fundamental current in each phase under conditions I to IV.

B. Control Strategy Based on the Original Theory

Figs. 6 and 7 show simulated waveforms when the control strategy based on the original theory is applied to the active filter. In the case of $e_0 = 0$ (Fig. 6), no energy storage components are required to the active filter, because $p_{C0} = p_{L0} = e_0 \cdot i_{L0} = 0$. However, i_{Ca}, i_{Cb} and i_{Cc} are non-sinusoidal waveforms including an amount of the third-order harmonic current, as shown in Table II. The reason is as follows; $q_{C\alpha\beta}$ consists of both a dc component $\bar{q}_{C\alpha\beta}$ and an ac component $\tilde{q}_{C\alpha\beta}$ with a frequency twice as high as the line frequency. The active filter fully compensates for $q_{C\alpha\beta}$ including $\tilde{q}_{C\alpha\beta}$. Substitution of $p_{C0} = p_{C\alpha\beta} = 0$ and $q_{C\alpha\beta} = \tilde{q}_{C\alpha\beta}$ into (24) makes it clear to produce the third-order harmonic current in $i_{C\alpha}$ and $i_{C\beta}$. If $\bar{q}_{C\alpha\beta}$ is extracted from $q_{C\alpha\beta}$ by using a low-pass filter and the active filter compensates for $\bar{q}_{C\alpha\beta}$, no third-order harmonic current would appear in i_C and i_S.

Fig. 7 shows simulated waveforms in the case of $e_0 \neq 0$, where the active filter also compensates for the neutral current fully. This stems from controlling the active filter in such a way as to cancel p_{C0} with $p_{C\alpha\beta}$. Thus, no energy storage component is required to the active filter. However, an amount of fifth-order harmonic current, along with an amount of third-order harmonic current, is included in i_S and i_C. The reason is as follows: The denominator of the right-hand side in (33) and (34), $e_{\alpha\beta}^2$ is constant in Fig. 6 because $E_a = E_b = E_c$, whereas it varies at twice the line frequency in Fig. 7 because $E_a = E_b \neq E_c$.

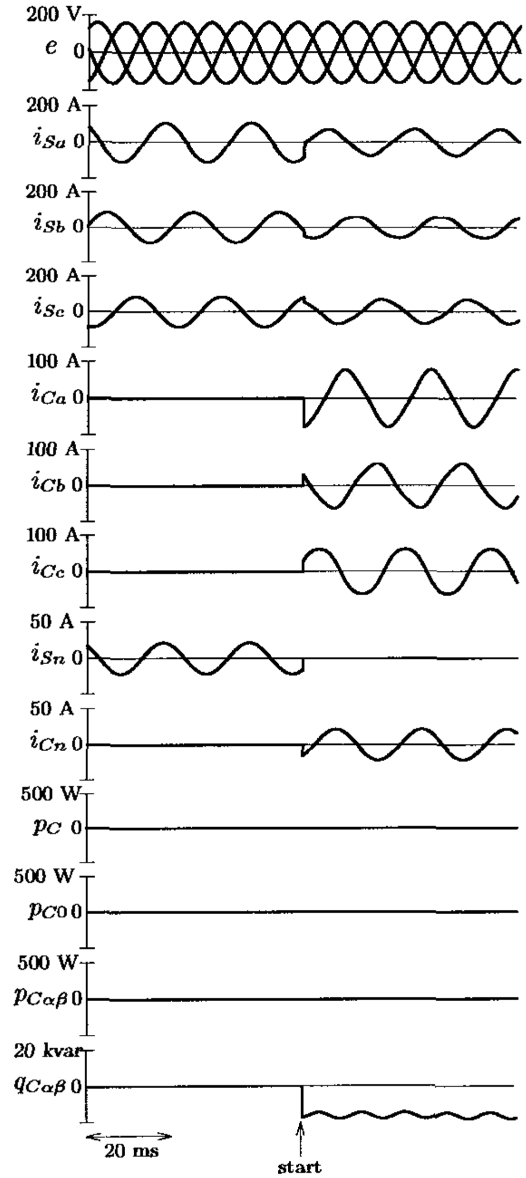

Fig. 6. Simulated waveforms under condition I.

C. Control Strategy Based on the Modified Theory

Figs. 8 and 9 show simulated waveforms when the control strategy based on the modified theory is applied to the active filter. As described in the previous section, the waveforms of voltage and current in Fig. 6 are identical to their corresponding waveforms in Fig. 8, because $e_0 = 0$. Moreover, the waveform of $q_{C\alpha\beta}$ in Fig. 6 is identical to that of q_{C0} in Fig. 8. When $e_0 \neq 0$, it is clear from the waveform of i_S in Fig. 9 that the active filter cannot compensate for the neutral current fully. The remaining neutral current corresponds to i_{0p}, which is the first term on the right-hand side of (16).

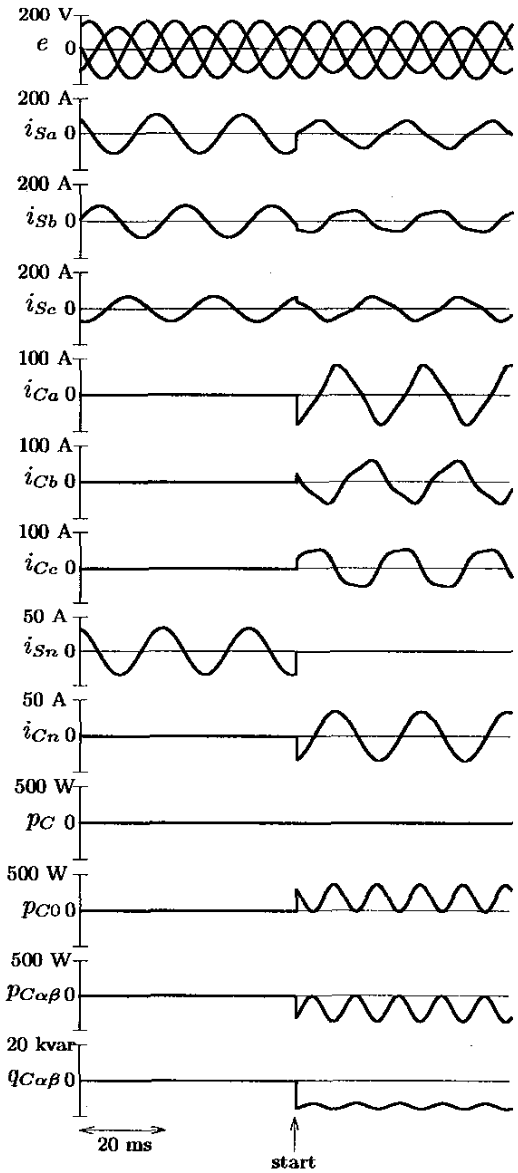

Fig. 7. Simulated waveforms under condition II.

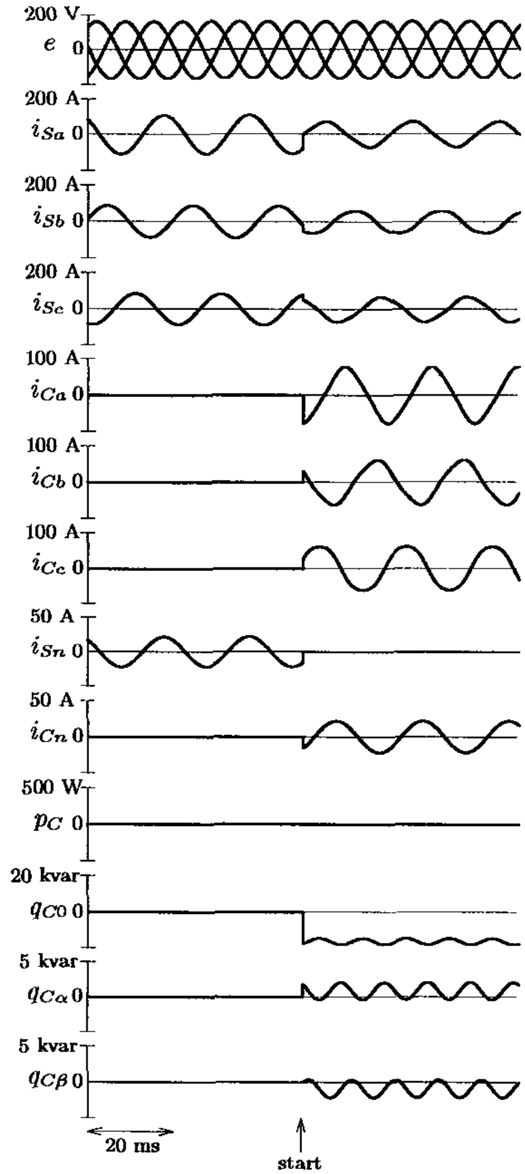

Fig. 8. Simulated waveforms under condition III.

If the authors are allowed to correct the control strategy based on the modified theory, an "improved" control strategy based on the modified theory provides the same filtering characteristics as does the control strategy based on the original theory, even when $e_0 \neq 0$. The control strategy given by (36)–(39) is corrected as follows:

$$p_C = 0 \tag{43}$$

$$q_{C0} = q_{L0} \tag{44}$$

$$i_{C0} = i_{L0}. \tag{45}$$

Here, it should be noted that the following constraint stemming from (14) is imposed on q_{C0}, $q_{C\alpha}$ and $q_{C\beta}$:

$$e_0 \cdot q_{C0} + e_\alpha \cdot q_{C\alpha} + e_\beta \cdot q_{C\beta} = 0. \tag{46}$$

The improved compensating currents i_{C0}, $i_{C\alpha}$ and $i_{C\beta}$, which can be theoretically derived from (35) and (43)–(46), are identical to (32), (33) and (34), respectively, thus resulting in producing the same waveforms as those shown in Fig. 7. It is, therefore, concluded that no essential difference exists between the original theory and the modified theory except for a distinct difference between their mapping matrices.

VI. Conclusions

This paper has focused on similarity and difference between the original theory and the modified theory in three-phase four-wire systems. A notable difference exists in mapping matrix between the two theories. Thus, the following questions might be asked: "Which is right, the original theory or the modified theory?" and "What is wrong with the two theories?" The answer to the questions is that both theories are right, and so they are applicable

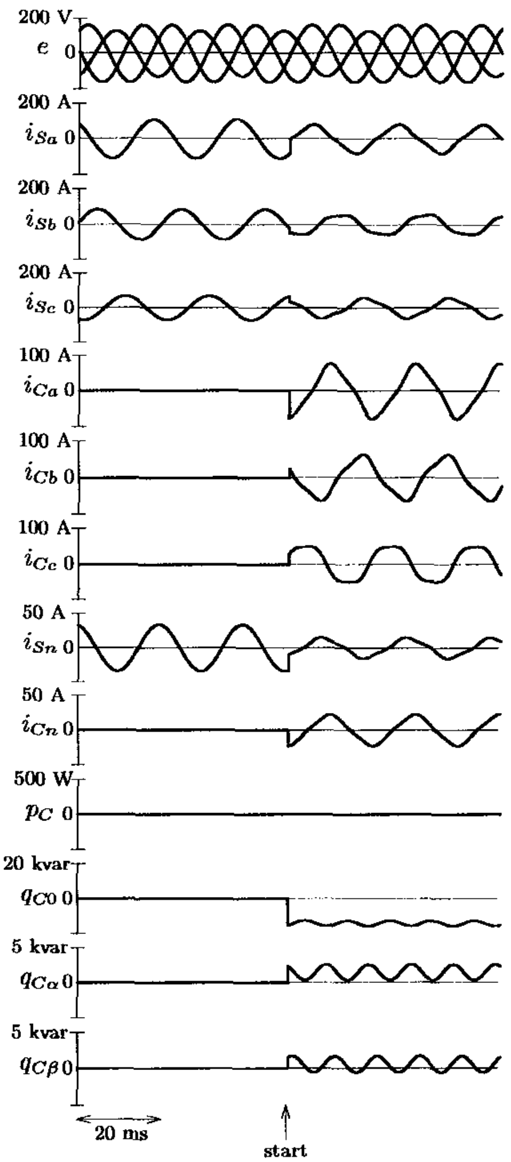

Fig. 9. Simulated waveforms under condition IV.

to a three-phase four-wire system including zero-sequence voltage and current. The difference and features of the two theories are summarized as follows:

- The original theory considers the zero-sequence circuit as a single-phase circuit, "independently" of the α-phase circuit and the β-phase circuit, and therefore it would be suitable for applications on the 0-α-β coordinates.
- The modified theory deals with the zero-sequence circuit, the α-phase circuit and the β-phase circuit "equally," and therefore it would be suitable for applications on the a-b-c coordinates.

Moreover, this paper has proposed a control strategy of an active filter without energy storage components, which is characterized by fully compensating for both the zero-sequence current and the instantaneous reactive power in each phase.

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