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A theorem on semi-centralizing derivations of prime rings

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A THEOREM ON SEMI-CENTRALIZING DERIVATIONS OF PRIME RINGS

Dedicated to Professor Hisao Tominaga on his 60th birhday

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Let R be an (associative) ring with center C, and S a subset of R. A derivation $d: x \mapsto x'$ of R is said to be *centralizing* (resp. *skew-centralizing*) on S if $s's - ss' \in C$ (resp. $s's + ss' \in C$) for every $s \in S$. More generally, d is defined to be *semi-centralizing* on S if $s's - ss' \in C$ or $s's + ss' \in C$ for every $s \in S$.

The following has been proved in [1, Theorem 1 (2)] and [2, Theorem 2].

Theorem 1. *Let d be a non-zero derivation of a prime ring R, and S a non-zero ideal of R.*

(1) *If d is centralizing or skew-centralizing on S, then R is commutative.*

(2) *If d is semi-centralizing on R, then R is commutative.*

In this very brief note, we improve the above theorem as follows :

Theorem 2. *Let d be a non-zero derivation of a prime ring R, and S a non-zero ideal of R. If d is semi-centralizing on S, then R is commutative.*

Proof. Suppose, to the contrary, that R is not commutative. In view of Theorem 1 (1), d is not centralizing on S and R is of characteristic not 2. Then, by [1, Lemma 4], $S \cap C = 0$ and there exists $t \in S$ such that $t^2 \neq 0$ but $(t^2)' = 0$. Since R is a prime ring, so is the non-zero ideal $T = Rt^2R$ of R. Moreover, by [1, Lemma 1 (3)], $0 \neq T' \subseteq R't^2R + Rt^2R' \subseteq T$. Hence d induces a non-zero derivation of T which is semi-centralizing on T. Thus, T is commutative by Theorem 1 (2), and therefore R itself is commutative by [1, Lemma 1 (1)]. This is a contradiction.

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