Some useful techniques in analysis and design of electric machines

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Some Useful Techniques in Analysis and Design of Electric Machines (Invited)

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Abstract — Some new techniques in the analysis and design of electric machines are discussed, namely, the time-periodic analysis of magnetic field in non-oriented materials taking into account hysteresis characteristics, 3-D transient analysis of capacitor-discharge impulse magnetizer and 3-D optimization method of electric machines considering the nonlinearity and eddy current.

The usefulness of the techniques is shown by applying to the actual machines and comparing with measured results.

I. INTRODUCTION

By the appearance of faster workstations with larger memory, complicated 3-D magnetic fields in practical electric machines is trying to be solved. In order to analyze such fields, techniques for solving magnetic field under special conditions, such as the time-periodic analysis of magnetic field and the analysis of the impulse magnetizer of permanent magnet. 3-D optimal design of electric machines is also trying to be carried out, because it became possible for the designer to use fast and cheap workstations.

In this paper, recent developments focusing the time-periodic analysis[1], magnetization of permanent magnet motor and optimal design of electric machines using both 3-D FEM and stochastic optimization method are discussed.

II. TIME-PERIODIC ANALYSIS OF MAGNETIC FIELD IN NON-ORIENTED MATERIAL

A. Method of Analysis

1) Formulation: In the nonlinear magnetic field analysis using the initial B-H curve, the magnetic vector potential $\boldsymbol{A}$ and the reluctivity ν are usually treated as unknown variables. On the other hand, in the case when the hysteresis characteristics are taken into account, the magnetization $\boldsymbol{M}$ should be used instead of ν due to the discontinuity of ν at the point where the flux density $\boldsymbol{B}$ is equal to zero[2]. The fundamental equation is given by:

$$\nu_o\{\mathrm{rot}(\mathrm{rot}\boldsymbol{A}-\boldsymbol{M})\}=\boldsymbol{J_o}-\sigma\frac{\partial \boldsymbol{A}}{\partial t} \tag{1}$$

where $\boldsymbol{J_o}$ is the current density vector in the magnetizing winding. ν_o and σ are the reluctivity in vacuum and the conductivity, respectively.

The term $\partial Mx/\partial A_j$ which appears in the formulation of the Newton-Raphson method, for example, is represented as follows:

$$\frac{\partial Mx}{\partial A_j}=\frac{\partial Mx}{\partial|\boldsymbol{B}|}\bullet\frac{\partial|\boldsymbol{B}|}{\partial A_j} \tag{2}$$

where $\partial Mx/\partial|\boldsymbol{B}|$ cannot be directly obtained from M-B loop because the direction of $\boldsymbol{M}$ is not given in M-B loop for non-oriented material. In this paper, $\partial Mx/\partial|\boldsymbol{B}|$ is determined as follows under the assumption that the direction of $\boldsymbol{M}$ is the same as that of $\boldsymbol{B}$:

$$\frac{\partial Mx}{\partial|\boldsymbol{B}|}=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\frac{\partial Mx}{\partial|\boldsymbol{M}|}=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\frac{\partial|\boldsymbol{M}|\cos\theta}{\partial|\boldsymbol{M}|}$$
$$=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\cos\theta=\frac{\partial|\boldsymbol{M}|}{\partial|\boldsymbol{B}|}\bullet\frac{Bx}{|\boldsymbol{B}|} \tag{3}$$

where θ is the angle between the $\boldsymbol{M}$ vector and the x-axis. Bx is obtained directly from the FEM calculation.

The time-periodic finite element method[3] is used for analyzing 3-D stationary nonlinear magnetic field. The flowchart is shown in Fig.1. Using this method, the time-periodic waveform can be obtained directly without transient calculation. The relaxation factor[4] is introduced to improve the convergence characteristics of Newton-Raphson method. The 1-st order brick edge element is used in the 3-D analysis[5].

Typical dc hysteresis loops, which are measured, are stored in a computer. The hysteresis loop used is interpolated by the loops stored.

2) Position of Flux Density on Hysteresis Loop: When the flux density $\boldsymbol{B}_t^k$ at the instant t of the k-th nonlinear iteration is obtained, the position of the flux density $\boldsymbol{B}_t^k$ on hysteresis loop cannot be determined uniquely (point α or β) from only the absolute value $|\boldsymbol{B}_t^k|$ as shown in Fig.2(a). Then, the following methods for determining the position of $\boldsymbol{B}_t^k$ are proposed[1]:

Method 1: If the angle θm between $\boldsymbol{B}_{max}^k$ and $\boldsymbol{B}_t^k$ is less than 90°, the position of $\boldsymbol{B}_t^k$ is regarded as α in the positive

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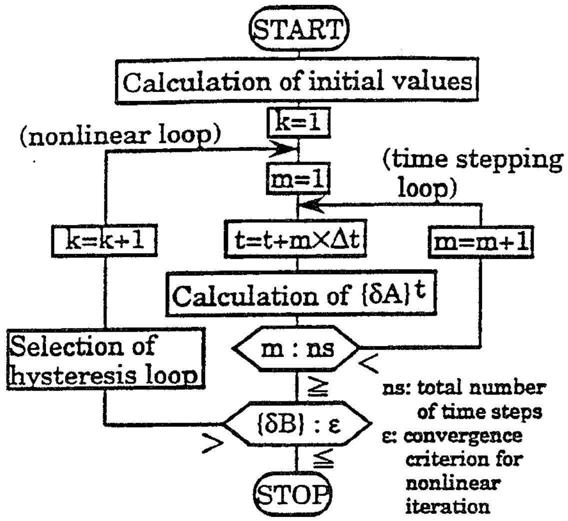


Fig.1 Flowchart of time-periodic finite element method taking into account hysteresis characteristics.

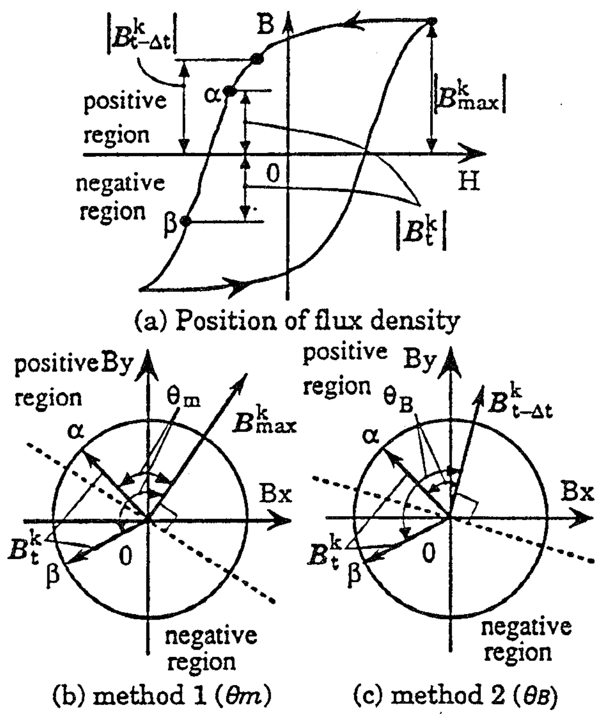


Fig.2 Position of flux density on hysteresis loop.

region as shown in Fig.2(b). If θm is larger than 90°, B_t^k is β in the negative region.

Method 2: If the angle θ_B between $B_{t-\Delta t}^k$ and B_t^k is less than 90°, the position of B_t^k is regarded as α in the positive region as shown in Fig.2(c). If θ_B is larger than 90°, B_t^k is regarded as β in the negative region.

Method 3: The following condition is added to Method 2 so that the position of flux density can be moved from the positive region to the negative region(vice versa) even if there is some numerical error: When $|B_t^k|$ is larger than $|B_{t-\Delta t}^k|$, the position is moved to the negative region.

B. Results and Discussion

1) Analyzed Model and Flux Density Waveforms: Fig.3 shows the analyzed model. The core is laminated infinitely in the z-direction. The current density in the coil is 3.1×10^4A/m^2 (ac, 50Hz). The analyzed region is subdivided into the 1-st order brick edge elements. The number of elements is 400. Half a period of the waveform is divided into 6 steps.

The waveforms of the average flux densities in the core obtained from the calculations using the initial curve and the hysteresis loop are shown in Fig.4. The eddy current in the steel is considered. The effect of hysteresis characteristics is remarkable in this model.

2) Convergence Characteristics: The effect of the methods for determining the position of flux density on the

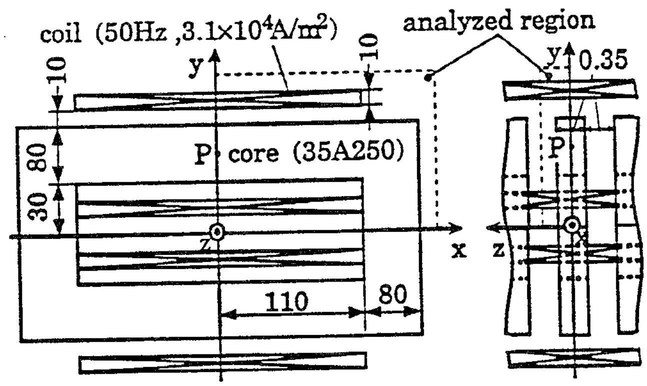


Fig.3 Analyzed model.

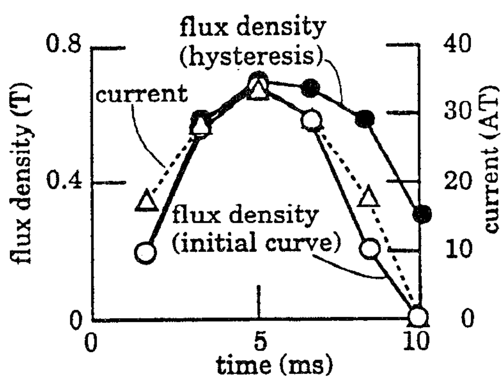


Fig.4 Waveform of flux density.

hysteresis loop, which are denoted in Section A, on the convergence characteristics is shown in Fig.5 and Table I. Fig.5 shows the position of the flux density at the point P shown in Fig.3 on the hysteresis loop when the current is the maximum. The variation of the position in the case of the method 2 is spurious, because the positive and negative regions cannot be evaluated correctly due to a numerical error near the $\boldsymbol{B}=0$ region. The method 1 is not acceptable from a physical point of view, when the flux density vector rotates. Therefore, the method 3 is preferable because the direction of flux density $\boldsymbol{B}_t^k$ should be estimated by comparing with $\boldsymbol{B}_{t-\Delta t}^k$ at previous step.

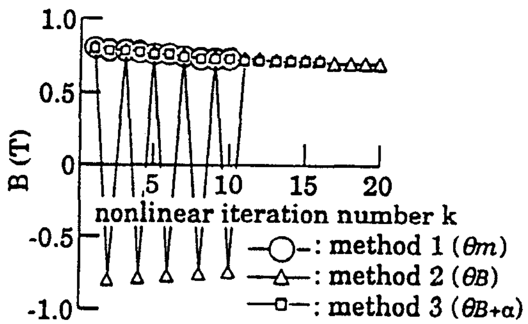


Fig.5 Position of flux density when current is the maximum.

Table I Effect of methods for determining position of flux density

method	method 1 (θm)	method 2 (θB)	method 3 ($\theta B+\alpha$)
total CPU time(s)	68 (10)	149 (20)	120 (16)

( ) : number of nonlinear iterations
Convergence criterion for Newton-Raphson method : 0.01T
Workstation: IBM3AT (49.7MFLOPS)

III. 3-D MAGNETIZATION OF MULTIPOLE PERMANENT MAGNET

A method for analyzing the magnetic field in a capacitor-discharge impulse magnetizer is established by modifying the finite element method. As the detailed distribution of the flux density can be obtained, the optimum design of the magnetizer which produce desired magnet will be possible using the new method.

A. Outline

Magnetic fields with eddy currents are denoted by the following equation:

$$\mathrm{rot}(\nu\,\mathrm{rot}\boldsymbol{A}) = \boldsymbol{J}_0 - \sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\,\phi\right) \tag{4}$$

$$\mathrm{div}\left\{-\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\,\phi\right)\right\} = 0 \tag{5}$$

where ν, σ and ϕ are the reluctivity, the conductivity and the electric potential respectively.

As the exciting current $\mathbf{J}_0$ of impulse magnetizer in Eq.(4) is unknown, it is difficult to analyze magnetic fields in such a device using a conventional finite element method[6].

A new method for calculating currents and flux distributions in impulse magnetizer has been developed. In this method, Eqs.(4) and (5) in conjunction with Kirchhoff's equation for the exciting circuit are used. The vector potential, scalar potential and the exciting current are treated as unknown variables.

Fig.6 shows and equivalent circuit of the magnetizer. The finite element region enclosed by the broken line corresponds to the magnetizer. C, R and L are the capacitance, the resistance and the leakage inductance outside the finite element region. Rc is the resistance of the winding in the finite element region.

The equation obtained from the Kirchhoff's law in the exciting circuit is as follows:

$$\frac{\partial \Phi}{\partial t} + (R + Rc)I_0 + L\frac{\partial I_0}{\partial t} - \frac{1}{C}\left(Q_0 - \int I_0 dt\right) = 0 \tag{6}$$

where, Φ is the interlinkage flux to the winding and Q_0 is the initial charge of the capacitor. Solving Eqs.(4), (5) and (6) simultaneously while treating vector potential A and charge Q as unknown variables, the transient magnetic field during the discharge of the capacitor can be analyzed[1].

B. Analysis of Magnetizer

1) Analyzed Model: Fig.7 shows the analyzed capacitor-discharge impulse magnetizer. The magnetizer is used to magnetize the four-pole permanent magnet for flat motor. The yoke is made of steel (SS400), and its conductivity σ is 0.7505×10^6S/m. The capacitance C in Fig.7 is 3000μF, the total resistance (R+Rc) is 0.035Ω and the external inductance L is 7μH. The charging voltage is 2000V.

2) Results and Discussion: Fig.9 shows the flux and eddy current distributions. Although the large eddy current flows at the beginning of discharge, the eddy current is reduced when

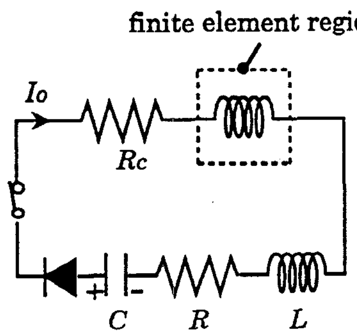


Fig.6 Equivalent circuit of magnetizer.

the discharging current becomes a maximum (t=0.2ms). Fig.9 shows the distribution of the maximum flux density along the line a–b (y=0, z=7.5mm). These figures illustrate the fairly good agreement between the calculation and measurement.

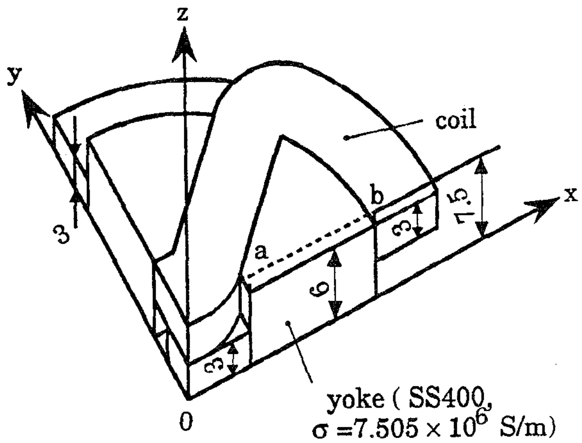


Fig.7 Magnetizer.

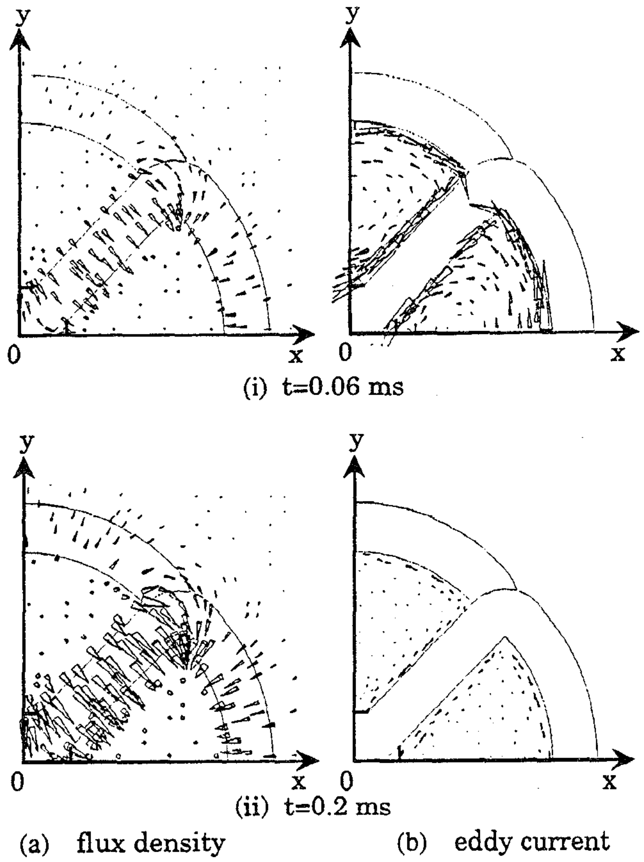


Fig.8 Flux and eddy current distributions.

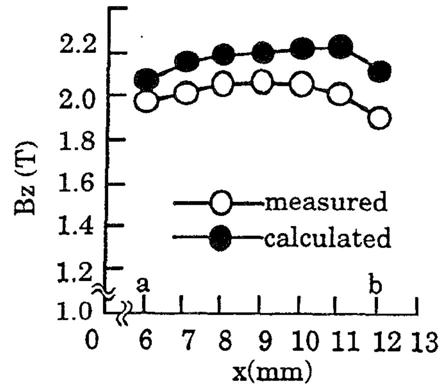


Fig.9 Maximum flux density along the line a-b.

IV. 3-D OPTIMIZATION METHOD OF ELECTRIC MACHINES

Many researches of new optimization method are recently reported, but these are mainly theoretical and two-dimensional[7, 8]. In order to apply the optimization method to the design of actual machines, it is necessary to develop the optimization method using the 3-D finite element method and the stochastic method for searching the minimum value of the objective function.

A. Outline

Many techniques, such as the simulated annealing and genetic algorithm, are used in the optimization. As extremely large number of iterations (1000-10000 iterations) are necessary by using these techniques, it is not practical to use such stochastic method. If the Rosenbrock's method[9], which is a kind of direct search method, is applied, the number of iterations can be reduced (nearly 100 iterations), and the CPU time is acceptable order from the viewpoint of practical application.

In order to take into account the nonlinearity and 3-D eddy current, the 3-D finite element method using hexahedral elements should be used. The practical problem of applying the 3-D finite element method to the optimal design of electric machines is how to modify the hexahedral mesh according to the change of shape of electric machine or how to construct the automatic mesh generator corresponding to the changed contour lines of machine.

Then, a technique of modifying the hexahedral mesh is conceived, and a 3-D optimization method is developed by utilizing this modification technique of 3-D mesh and the Rosenbrock's method.

B. Modification of 3-D Mesh

There is no robust mesh generator for hexahedral element which generates a mesh for the obtained shape. Then, the modification of the shape is carried out by shifting nodes of hexahedral elements in a specified region. Let us explain the process using the model in which the lines g-h and e-f are rotated by the angles θ_1 and θ_2 (design variables) as shown in Fig.10.

(a) The region is divided to α, β and γ regions as shown in Fig.10.

(b) The nodes on the lines g-h and e-f are rotated by θ_1 and θ_2. In this case, the nodes on the lines a-d and b-c are fixed.

(c) The rotation angles of other nodes in each region (α, β and γ) are linearly interpolated as shown in Fig.10.

C. Examples of Optimization

1) Linear dc Motor: In a linear dc motor (LDM) for magneto-optical storage apparatus[9], large thrust is required in order to reduce the access time. Therefore, an optimal design method should be applied to LDM, so that the thrust becomes larger. Then, the optimal design of LDM with large thrust is carried out using the finite element method and the Rosenbrock's method. The effectiveness of the method is examined by comparing the thrust between the initial shape and optimal shape.

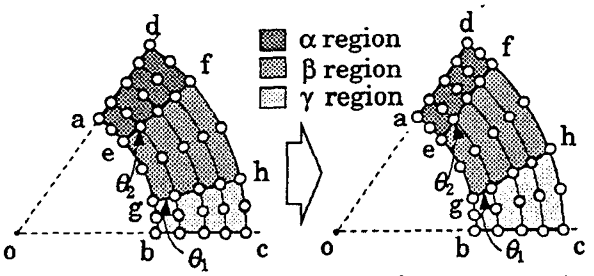
Fig.11 shows the initial shape of the moving coil type of LDM for magneto-optical storage apparatus. The coil is moved at the range of $-8.5 \le x_c \le 8.5$ (x_c: x-coordinate of the center of the coil) by the thrust which is generated by the flux of the magnet and the current in the coil. The core is made of carbon steel (S10C). The magnetization of the magnet ($SmCo_5$) is 1.03T. The current density of the coil is 24.6A/mm^2 (dc).

The cross-sectional area, the length (resistance) and the current of the coil are constant. The gap length G_1 (=0.4mm) is the margin for the manufacturing error. The gap length G_2 (≥0.5mm) is the margin to avoid the contact between the coil and the core when the coil is moved to the end. Dimensions L_1 to L_5 shown in Fig.11 are chosen as design variables for the optimal design. The other dimensions L_6 to L_8 are determined based on L_1 to L_5[10].

Fig.12 shows the flux distributions for initial and optimal shapes. The number of iterations of Rosenbrock's method is 61. The thrust (=1.49N) for the optimal shape is about 1.7 times larger than that (=0.86N) for the initial shape. The CPU time is about 7.2 hours using HP735 (45 MFLOPS).

2) Permanent Magnet Type Retarder: The 3-D optimization method is applied to the design of a permanent magnet type retarder shown in Fig.13 which is used as an auxiliary breaking system in a heavy vehicle[11]. A breaking torque is produced by the flux and eddy current in the rotor. The outer rotor rotates with a constant speed (=1000rpm) The outer rotor and yoke are made of carbon steel (S10C). The permanent magnet is assumed to be magnetized in parallel direction and the magnetization is 1T. In the dc steady state analysis, eddy currents flow only in the outer rotor.

The method of dc steady state eddy current analysis using a moving coordinate system is used[12]. The nonlinearity of iron is taken into account. The analyzed region (1/24 region) is subdivided into 3456 1-st order hexahedral nodal elements.



(a) before movement (b) after movement

Fig. 10 Method for modifying mesh.

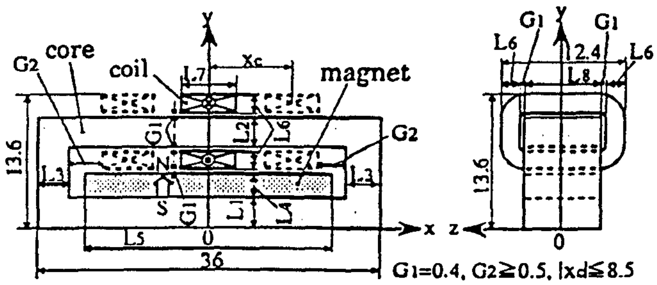
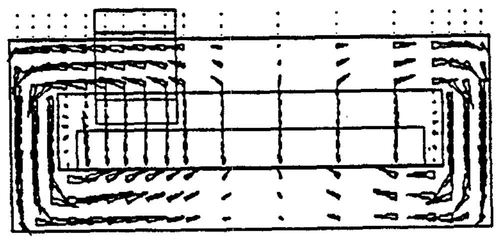
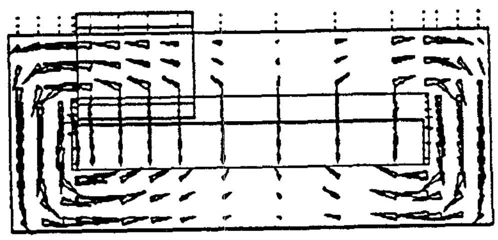


Fig.11 Linear motor.



(a) initial



(b) optimal

Fig.12 Flux distributions.

The angles θ_1 and θ_2 (design variables shown in Fig.13) which produce a maximum breaking torque are searched by using the Rosenbrock's method.

Fig.14 shows the flux distributions of initial and optimal shapes. Table II shows the obtained design variables and the breaking torques. The breaking torque can be increased by about 20%. The number of iterations of the Rosenbrock's method is 21, and the CPU time using IBM 37T (25.9 MFLOPS) is about 15.6 hours.

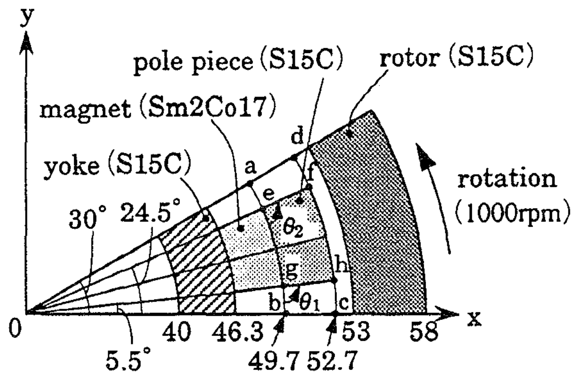
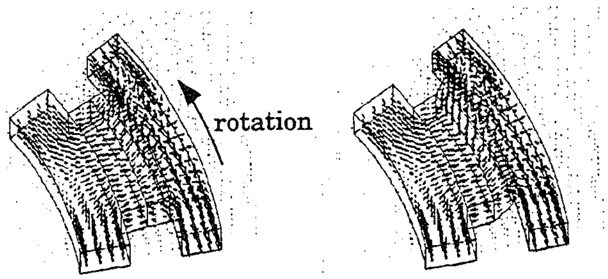


Fig. 13 Permanent magnet type of retarder (thickness: 16mm).



(a) initial (b) optimal

Fig. 14 Flux distributions.

Table II Design variables and breaking torques

shape	design variable (deg)		breaking torque (N · m)
	θ_1	θ_2	
initial	90	90	0.886
optimal	120	120	1.097

V. CONCLUSIONS

The major features discussed in this paper can be summarized as follows:

(1) The formulation for the ac steady state analysis of magnetic field, taking into account the eddy current and the hysteresis characteristics in non-oriented material, is shown.

(2) The technique for determining the position of flux density on hysteresis loop of non-oriented material under time-periodic condition is investigated.

(3) The transient magnetic field in a capacitor-discharge impulse magnetizer can be analyzed by solving both the Maxwell's equation and the Kirchhoff's equation.

(4) 3-D optimal design method of an electric machine is developed using the hexahedral finite element and the Rosenbrock's method.

(5) By using the Rosenbrock's method, the 3-D optimal design of a practical machine may be possible within an acceptable CPU time.

There are many requirements for the practical analysis of electric machines, such as the modeling of minor loops in hysteresis characteristics in 3-D eddy current problem, more efficient optimal design method which can treat a complicated practical machine etc. New methods or techniques which can solve these problems should be investigated in the future.

REFERENCES

[1] N.Takahashi, T.Nakata, K.Muramatsu, M.Nakano and Y.Ejiri, "3-D time-periodic finite element analysis of magnetic field in non-oriented materials taking into account hysteresis characteristics," *IEEE Trans. on Magn.*, vol.33, no.2, March 1997.

[2] T.Nakata, N.Takahashi and Y.Kawase, "Finite element analysis of magnetic fields taking into account hysteresis characteristics," *ibid.*, vol.21, no.5 pp.1856-1858, September 1985.

[3] T.Nakata, N.Takahashi, K.Fujiwara, K.Muramatsu, H.Ohashi and H.L.Zhu, "Practical analysis of 3-D dynamic nonlinear magnetic field using time-periodic finite element method," *ibid.*, vol.31, no.3, pp.1416-1419, May 1995.

[4] K.Fujiwara, T.Nakata, N.Okamoto and K.Muramatsu, "Method for determining relaxation factor for modified Newton-Raphson method," *ibid.*, vol., no.29, pp.1862-1865, March 1993.

[5] A.Kameari, "Calculation of transient 3D eddy current using edge-elements," *ibid.*, vol.26, no.2, pp.466-469, March 1990.

[6] T.Nakata and N.Takahashi, "Numerical analysis of transient magnetic field in a capacitor-discharge impulse magnetizer," *ibid.*, vol.22, no.5, pp.526-528, September 1990.

[7] A.A.Arkadan, S.Subramaniam-Sivanesan and N.A.O.Demerdash, "Shape optimization of PM devices using constrained gradient based inverse problem methodology," *ibid.*, vol.32, no.3, pp.1222-1225, May 1996.

[8] P.Alotto, A.V.Kuntsevitch, Ch.Magele, G.Molinari, C.Paul, K.Preis, M.Repetto and K.R.Richter, "Multiobjective optimization in magnetostatics: a proposal for benchmark problems," *ibid.*, vol.32, no.3, pp.1238-1241, May, 1996.

[9] D.M.Himmelblau, *Applied Nonlinear Programming*, McGraw-Hill, 1972.

[10] N.Takahashi, K.Muramatsu, T.Nakata, H.L.Zhu and K.Uehara, "Optimal design of linear dc motor using finite element method," *Proceedings of the LDIA'95*, pp.331-334, May, 1995.

[11] K.Muramatsu, N.Takahashi, T.Hashio, C.Yamada, S.Kobayashi and T.Kuwahara, "3-D eddy current analysis in moving conductors of permanent magnet type retarders using moving coordinate system," Proceedings of IEEE IEMDC '97 Conference, no.219, 1997.

[12] K.Muramatsu, T.Nakata, N.Takahashi and K.Fujiwara, "Linear ac steady-state eddy current analysis of high speed conductor using mofing coodinate system," IEEE Trans. on Magn., vol.32, no.3, pp.749-752, May 1996.