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COMPUTATION ACCURACIES OF BOUNDARY ELEMENT METHOD AND FINITE ELEMENT METHOD  
IN TRANSIENT EDDY CURRENT ANALYSIS

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**Abstract** - In this paper, computation accuracies of boundary element method and finite element method in transient eddy current problems are compared by using a slot-embedded conductor model and a diffusion model which can be solved theoretically. The computation accuracies of vector potential or magnetic flux density by BEM and FEM were evaluated and it was clear that larger stepwidth of time can be chosen in BEM compared with FEM for the same accuracy.

### INTRODUCTION

Transient eddy current analysis is one of urgent requirements in the design of electrical equipment. The finite element method (FEM) and boundary element method (BEM) are mainly used as numerical methods of transient eddy current analysis [1],[2]. Several works for transient eddy current analysis have been done [3]-[4], and the authors have received many invaluable suggestions from these works.

In this paper, computation accuracies of the BEM and FEM for two-dimensional transient problems are compared. In the BEM, triangular elements with constant vector potential were used for domain integrals and unknown vector potential or flux was assumed to be constant only on each boundary element. In the FEM, triangular elements with quadratically varying vector potential were used. A slot-embedded conductor model and a diffusion model which can be solved theoretically are chosen as computation model, and computation accuracies of vector potential or magnetic flux density by BEM and FEM are evaluated.

### FORMULATION

#### Boundary Element Method

Using the magnetic vector potential and the scalar potential, the differential equation to be solved is given from Maxwell's equations in two-dimensional problems as follows:

$$\frac{1}{\mu\sigma} \nabla^2 A = \frac{\partial A}{\partial t} + \frac{\partial \phi}{\partial z} \quad (1)$$

where  $A$  is the  $z$ -component of vector potential,  $\phi$  is the scalar potential,  $\mu$  is the permeability and  $\sigma$  is the conductivity. Here, the time differentiation in Eq. (1) is approximated by the difference:

$$\frac{\partial A}{\partial t} = \frac{A_{t+\Delta t} - A_t}{\Delta t} \quad (2)$$

From Eq. (2), Eq. (1) is rewritten as

$$k \nabla^2 A_{t+\Delta t} - \frac{1}{\Delta t} A_{t+\Delta t} = \frac{1}{\Delta t} A_t + \frac{\partial \phi_{t+\Delta t}}{\partial z} \quad (3)$$

where  $k=1/(\mu\sigma)$ .

The fundamental solution of Eq. (3), which is the Helmholtz type, is given by

$$A_{t+\Delta t}^* = \frac{1}{2\pi k} K_0 \left( \frac{r}{\sqrt{k\Delta t}} \right) \quad (4)$$

where  $K_0$  is the modified Bessel function of the second kind of zero order. From Eq. (3), one can obtain the basic equation of BEM for the field point on the boundary surface as follows:

$$\begin{aligned} & \frac{1}{2} A_{t+\Delta t} + k \int_{\Gamma} A_{t+\Delta t} \frac{\partial A_{t+\Delta t}^*}{\partial n} d\Gamma \\ &= k \int_{\Gamma} \frac{\partial A_{t+\Delta t}}{\partial n} A_{t+\Delta t}^* d\Gamma + \frac{1}{\Delta t} \int_{\Omega} \left( A_t - \frac{\partial \phi_{t+\Delta t}}{\partial z} \right) A_{t+\Delta t}^* d\Omega \end{aligned} \quad (5)$$

When applying the BEM for transient problems, the region to be analyzed is divided into triangular elements and the boundary surface is divided into segments which are called boundary elements. Here, vector potentials on the triangular element and boundary element are assumed both to be constant. The triangular elements are used only for domain integral appeared in Eq. (5), and unknown vector potential or unknown magnetic flux density is defined on each boundary element. By using Eq. (5), the final simultaneous equations are set up for unknown vector potentials and magnetic flux densities on the boundary elements.

#### Finite Element Method

The simultaneous equations of FEM for magnetic vector potentials which satisfy Eq. (1) are given by

$$[S]\{A\} = \left(-\frac{\partial \phi}{\partial z}\right)\{g\} - [G] \frac{\partial \{A\}}{\partial t} \quad (6)$$

where  $\{A\}$  is the unknown vector potential,  $[G]$ ,  $[S]$  and  $\{g\}$  are defined by triangular mesh and the physical constant of material.

Approximating the time differentiation in Eq. (6) by central differences, the following equation is obtained

$$\begin{aligned} & \left( \frac{1}{\Delta t} [G] + \frac{1}{2} [S] \right) A_{t+\Delta t} \\ &= \left( \frac{1}{\Delta t} [G] - \frac{1}{2} [S] \right) A_t + \frac{1}{2} \left( -\frac{\partial \phi_{t+\Delta t}}{\partial z} - \frac{\partial \phi_t}{\partial z} \right) \{g\} \end{aligned} \quad (7)$$

When applying the FEM, the region to be analyzed is divided into triangular elements. Then, the vector potential on the triangular element is approximated by a quadratic function. The final simultaneous equations are set up by using Eq. (7).

#### Initial-Value Problem

The transient eddy current problem is solved as a initial-value problem by using Eq. (5) for the BEM or Eq. (7) for the FEM. Therefore, the vector potential  $\{A\}_{t+\Delta t}$  is solved using a known vector potential  $\{A\}_t$ , which is the initial value, at each time step. At first time step, computations of the coefficients of Eq. (5) or (7) are completed, and at each time step, some matrix calculations are done in order to obtain  $\{A\}_{t+\Delta t}$ .



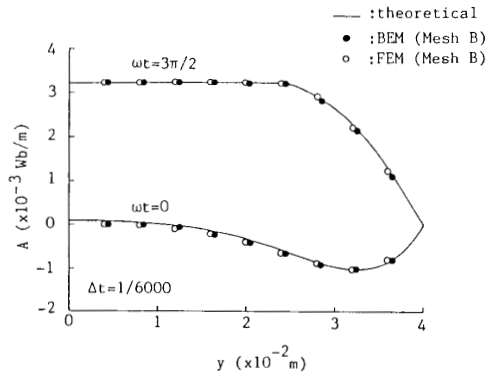


Fig. 4 Distributions of the vector potential in the slot-embedded conductor model.

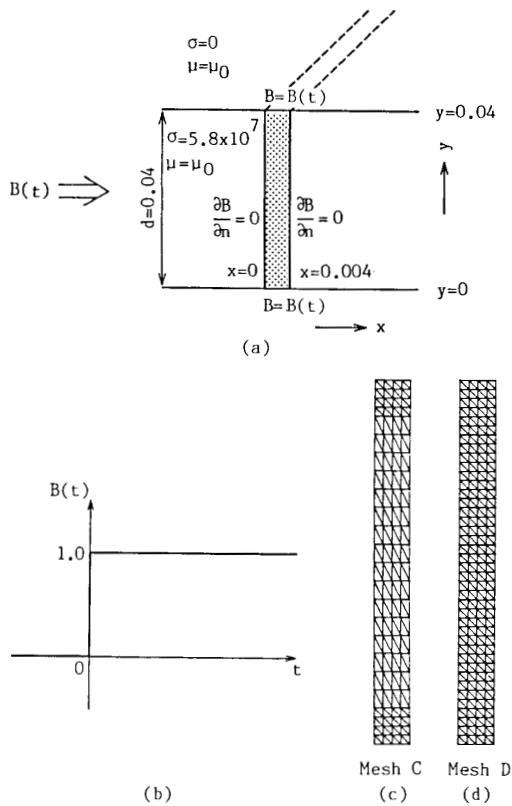


Fig. 5 Diffusion model, (a) computation model, (b) impressed magnetic flux density, (c) mesh C, (d) mesh D.

#### CONCLUSION

Computation accuracies of the BEM and the FEM in the slot-embedded conductor model and the diffusion model were evaluated. As the result, it was clear that larger stepwidth of time can be chosen in the BEM compared with the FEM for the same accuracy; however the BEM and the FEM provide similar results, in the case of no current source, as the diffusion model.

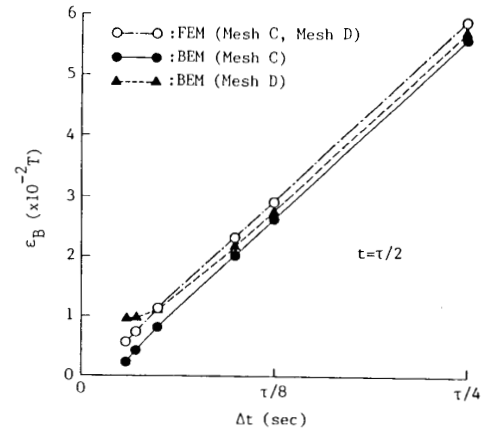


Fig. 6 Maximum difference,  $\epsilon_B$ , between computation and theoretical values of vector potential.

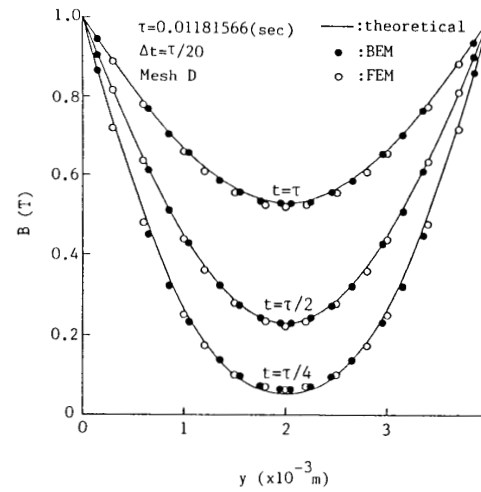


Fig. 7 Distributions of the magnetic flux density in the diffusion model.

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