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### 3-D MAGNETIC FIELD ANALYSIS BY USING SPECIAL ELEMENTS

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#### Introduction

We have already developed the "gap element"[1], which has energy but has no area, for 2-D magnetic field analysis. As this element has no area, it is very easy to put new gaps on the optional positions of the mesh, to remove gaps or to modify the gap length freely. Moreover, the "shield element"[2], for analyzing 2-D eddy currents in a thin conducting plate, has also been developed.

In this paper, these special elements are expanded into 3-D analysis of magnetic fields using the  $A-\phi$  method.

#### Special Elements

##### (1) 3-D gap element

A gap element is shown by the thick line 1-2-3-4-1 in Fig.1(b). This element corresponds to an air gap between high permeability materials as shown in Fig.1(a). The gap element has no area, but has nearly the same energy as the gap. The gap element has the following advantages:

- (a) As this element has no area, it is easy to put new gaps on the optional positions of the mesh or to remove gaps.
- (b) As the modification of the gap length  $D$  is also easy, the influence of  $D$  on the magnetic characteristics can be easily examined by using this element.
- (c) When the gap becomes narrow enough, the accuracy of solution is better than that obtained by using flat ordinary elements.

As the flux in the air gap is perpendicular to the boundary of the steel, the following equations can be assumed among each component of vector potential  $A$  at the nodes 1 to 4 and 1' to 4' in Fig.1(a):

$$A_{xi} = A_{xi}', \quad A_{yi} = A_{yi}', \quad A_{zi} = A_{zi}' \quad (i=1, \dots, 4, \quad i'=1', \dots, 4') \quad (1)$$

As the flux in the air gap flows in the  $z$ -direction,  $A_{zi}$  and  $A_{zi}'$  can be set to zero. Then, the vector potentials  $A_{xi}$ ,  $A_{xi}'$ ,  $A_{yi}$  and  $A_{yi}'$  can be eliminated from the unknown variables.

The Rayleigh-Ritz equation for 3-D gap element can be given by minimizing the energy as follows[1]:

$$\frac{\partial W}{\partial A_{xi}} = \frac{\partial W}{\partial A_{xi}'} + \frac{\partial}{\partial A_{xi}} (D \iint \frac{\nu_0 B_z^2}{2} dx dy) \quad (2)$$

$\partial W / \partial A_{yi}$  can be obtained in the same way.  $\nu_0$  is the reluctivity and  $B_z$  is the  $z$ -component of the flux density.  $W_t$  is the total energy of the whole region. The second term in Eq.(2) is the derivative of the energy of the gap element.

##### (2) 3-D shield element

A shield element is shown in Fig.2. This element corresponds to the highly conductive non-magnetic material such as an electromagnetic shielding plate. It has no area, but has nearly the same energy as the eddy current in the shield. This element has the same advantages as the gap element.

As the shield is very thin, it can be assumed that the eddy current does not flow in the  $z$ -direction, and the flux is perpendicular to the shield. Therefore, the relationships among vector potentials  $A$  shown in Eq.(1) and the following relationship among the scalar potentials  $\phi$  can be satisfied in the shield element:

$$\phi_1 = \phi_1' \quad (i=1, \dots, 4, \quad i'=1', \dots, 4') \quad (3)$$

The finite element formulation for the shield element can be obtained by the similar way as the gap element.

#### Examples of Application

Figure 3 shows a model to verify the effectiveness of the gap element. Figure 4 shows a model to investigate the effects of an aluminum shield. The calculated and measured results will be shown in the full paper.

#### References

- [1] T. Nakata, Y. Ishihara and N. Takahashi: "Some Useful Techniques on Implementing the Finite Element Method for Computation of Electromagnetic Fields in Electrical Machinery", Interdisciplinary Finite Element Analysis (book, Edited by J.F. Abel et al.), 545 (1981) Cornell University.
- [2] T. Nakata, N. Takahashi, Y. Kawase, T. Masuyama and K. Fujiwara: "Development of Special Elements for Magnetic Field Analysis", Papers of Technical Meeting on Rotating Machines and Static Apparatus, IEE of Japan, RM-81-37, SA-81-27 (1981).

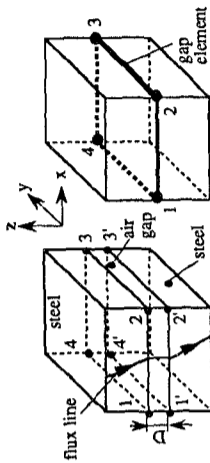


Fig.1 Gap element.

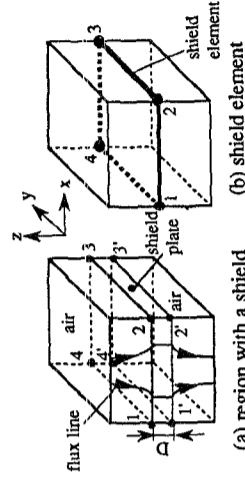


Fig.2 Shield element.

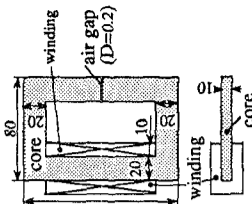


Fig.3 Transformer model.

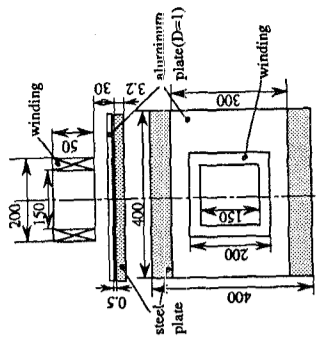


Fig.4 Shield model.