

Physics

Electricity & Magnetism fields

Okayama University

Year 1992

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INVESTIGATION OF EFFECTIVENESS OF EDGE ELEMENTS

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Abstract - The effectiveness of the edge element is investigated by comparing systematically the number of unknown variables with that of the nodal element. It is shown that the edge element is superior to the nodal element from the standpoints of the computer storage and the CPU time.

The 3-D periodic boundary condition for the edge element is also derived in order to reduce the computer storage and the CPU time.

I. INTRODUCTION

Recently, the edge element has drawn the attention of many researchers[1-5]. The total number of unknown variables which affects the computer storage and the CPU time is changed by the kinds of the element (nodal or edge element) and the unknown variable ($A-\phi$ or $T-\Omega$ method) and the volume ratio of the conductor region.

Though the 3-D periodic boundary condition for the nodal element has been examined[6], the periodic boundary condition for the edge element has not previously been investigated.

In this paper, two kinds of element and two methods of analysis are compared in terms of the number of unknown variables and the number of non-zero entries in the coefficient matrix. The most suitable element and method of analysis for the respective problems are discussed. The periodic boundary condition for the edge element is also examined.

II. COMPARISON OF NUMBER OF UNKNOWN VARIABLES FOR OR VARIOUS ELEMENTS AND METHODS OF ANALYSIS

Only brick elements are used for the comparison. It is assumed that the number of elements is so large that the decrease of unknown variables by applying boundary conditions can be neglected. The gauge condition is ignored[3].

A. Number of Unknown Variables and Non-zero Entries

1) *Nodal element*: Since the number of nodes in one nodal element is equal to 8 and the number of elements which share one node is equal to 8 as shown in Fig.1(a), the average number of nodes per element is equal to unity ($=8/8$) when the number of elements is extremely large. Therefore, the

relationship between the total number, ne , of the elements and the total number, nt , of the nodes can be represented as follows:

$$nt = ne \quad (1)$$

In the $A-\phi$ method using the nodal element, 4 kinds of unknown variables (ϕ and three components of A) are defined per node in the conductor region R_j and three kinds of unknown variables (three components of A) in the air region R_0 as shown in Table I. Therefore, the total number, $nu(nodal, A-\phi)$, of unknown variables can be expressed as

$$nu(nodal, A-\phi) = 4 \alpha nt + 3 (1 - \alpha) nt \\ = (\alpha + 3) ne \quad (2)$$

where α is the ratio of the number of elements in the conductor region R_j to that of the whole region (R_j+R_0). Since the number of nodes related to a node i is equal to 27

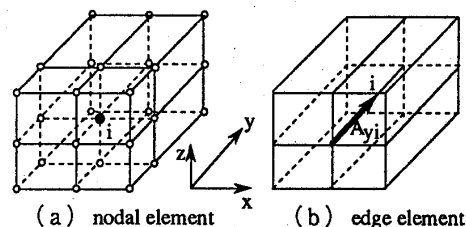


Fig.1 Elements related to node i or edge i

Table I Numbers of unknown variables and non-zero entries

element	method	variables	number of unknown variables(nu)	number of non-zero entries(nz)
nodal	$A-\phi$	$R_0 A(3), R_j A(3), \phi(1)$	$(\alpha + 3) ne$	$27(7\alpha + 9) ne$
	$T-\Omega$	$\Omega(1), T(3), \Omega(1)$	$(3\alpha + 1) ne$	$27(15\alpha + 1) ne$
edge	A	$A(1), A(1)$	$3 ne$	$9 \cdot 9 ne$
	$T-\Omega$	$\Omega(1), T(1), \Omega(1)$	$(3\alpha + 1) ne$	$9 \cdot 23\alpha + 3) ne$

() : number of variables defined on node or edge

ne : number of total elements

α : volume ratio of conductor region to whole region

Manuscript received July 7, 1991, revised October 31, 1991. This work was supported in part by the Grant-in-Aid for Co-operative Research(A) from the Ministry of Education, Science and Culture in Japan (No.01302031).

as shown in Fig.1(a), the number of unknown variables related to the node i is 27×4 in R_j and 27×3 in R_o . As a result, the total number, $nz(\text{nodal}, A-\phi)$, of non-zero entries can be given by

$$\begin{aligned} nz(\text{nodal}, A-\phi) &= 4 \alpha n t \times 27 \times 4 \\ &\quad + 3(1-\alpha) n t \times 27 \times 3 \\ &= 27(7\alpha + 9) n e \end{aligned} \quad (3)$$

In the T- Ω method, four kinds of unknown variables (Ω and three components of T) are defined per node in R_j and one unknown variable (Ω) in R_o as shown in Table I. Therefore, the total number, $nu(\text{nodal}, T-\Omega)$, of unknown variables and the total number, $nz(\text{nodal}, T-\Omega)$ of non-zero entries for the T- Ω method can be given by

$$\begin{aligned} nu(\text{nodal}, T-\Omega) &= 4 \alpha n t + 27 + (1-\alpha) n t \times 27 \\ &= (3\alpha + 1) n e \quad (4) \\ nz(\text{nodal}, T-\Omega) &= 4 \alpha n t \times 27 \times 4 + (1-\alpha) n t \times 27 \\ &= 27(15\alpha + 1) n e \quad (5) \end{aligned}$$

2) *Edge element*: Since the number of edges in one element is equal to 12 and the number of elements which share one edge is equal to 4 as shown in Fig.1(b), the average number of edges per element is equal to $3(=12/4)$. Therefore, the relationship between the total number, ne , of the elements and the total number, nh , of the edges can be represented as follows:

$$nh = 3ne \quad (6)$$

In the A- ϕ method using the edge element, ϕ can be set at zero[3]. The only one component of A is defined along each edge. For example, only the y-component A_{yi} is defined along the edge i as shown in Fig.1(b). Therefore, the total number, $nu(\text{edge}, A)$, of unknown variables can be expressed as:

$$nu(\text{edge}, A) = nh = 3ne \quad (7)$$

Since the number of edges related to an edge i is equal to 33 as shown in Fig.1(b), the total number, $nz(\text{edge}, A)$, of non-zero entries can be given by

$$nz(\text{edge}, A) = nu(\text{edge}, A) \times 33 = 99ne \quad (8)$$

In the T- Ω method, one component of T is defined along each edge in R_j , and Ω is defined on each node in (R_j+R_o). Therefore, the total number, $nu(\text{edge}, T-\Omega)$, of unknown variables is given by

$$\begin{aligned} nu(\text{edge}, T-\Omega) &= \alpha nh + \alpha n t + (1-\alpha) n t \\ &= (3\alpha + 1) n e \end{aligned} \quad (9)$$

The numbers of edges and nodes related to the edge i in R_j in Fig.1(b) are 33 and 18 respectively. The numbers of edges and nodes related to the node i in R_j in Fig.1(a) are equal to 54 and 27 respectively. The number of nodes related to the node i in R_o in Fig.1(a) is equal to 27. Therefore, the total number, $nz(\text{edge}, T-\Omega)$, of non-zero entries for the T- Ω method can be given by

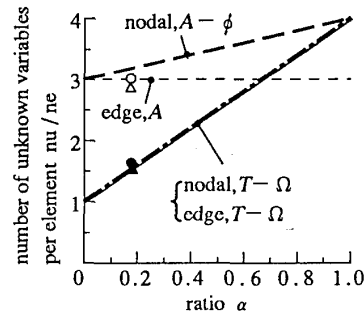
$$\begin{aligned} nz(\text{edge}, T-\Omega) &= \alpha nh \times (33 + 18) \\ &\quad + \alpha n t \times (54 + 27) + (1-\alpha) n t \times 27 \end{aligned}$$

$$= 9(23\alpha + 3) n e \quad (10)$$

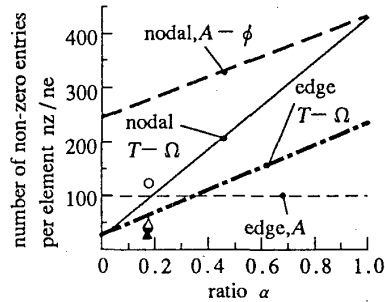
B. Comparisons

Fig.2(a) shows the effects of the ratio α on the total number, nu/ne , of unknown variables per element. The figure can be obtained from (2), (4), (7) and (9). When α is nearly equal to zero (the conductor region is very small in the analyzed region), nu/ne for the T- Ω method is about 1/3 of nu/ne for the A- ϕ method. When α is greater than 0.7, the A method using the edge element has the smallest value of nu/ne . Fig.2(b) shows the effects of the ratio α on the total number, nz/ne , of non-zero entries per element. The figure can be obtained from (3), (5), (8) and (10). nz/ne of the T- Ω method is smaller than that of the A- ϕ method using the nodal element. nz/ne for the edge element is smaller than that for the nodal element. When α is greater than 0.4, nz/ne for the A method using the edge element is the smallest of all.

In order to verify the features illustrated in Fig.2, the numbers of unknown variables and non-zero entries are examined using a real model. Fig.3 shows a model proposed by the IEE of Japan. The features of the model are described in the reference[7]. An ac current of which the effective value is 1000AT (frequency: 50Hz) is applied. The ratio α for the model without hole is equal to 0.18 and that with hole is



(a) unknown variables



(b) non-zero entries

○ : nodal, A - ϕ
 △ : edge, A
 ● : nodal, T - Ω
 ▲ : edge, T - Ω

IEEJ model
(Fig.3)

Fig.2 Effects of ratio α on number of unknown variables and number of non-zero entries.

$$B_x = -B_0x \quad (11)$$

$$B_y = -B_0y \quad (12)$$

$$B_z = -B_0z \quad (13)$$

The vector potential A is perpendicular to the flux density B on the periodic boundaries[5] as shown in Fig.5. Then, the relationship between A_x and A_0 is represented by the following equations:

$$A_x = -A_0x \quad (14)$$

$$A_y = -A_0y \quad (15)$$

$$A_z = -A_0z \quad (16)$$

1) *Edge element*: Since only components parallel to the edges are defined in the edge element[1], (14) is ignored on the periodic boundaries in the case of Fig.4.

Fig.6 shows the spatial distribution of average flux density in the steel plate. The result which is obtained by analyzing the 1/4 region with the periodic boundary condition coincides with the result which is obtained by analyzing the 1/2 region without the periodic boundary condition. Therefore, we can conclude that one condition of (14)-(16) can be ignored for the edge element.

2) *Nodal element*: In the nodal element, all of three conditions ((14)-(16)) mentioned above can be satisfied on the periodic boundaries[5]. Even in the nodal element, however, it can be proved that only the components parallel to the periodic boundaries are sufficient to satisfy the above-mentioned conditions. The comparison between the flux distribution which is obtained using all of three conditions and that which is obtained ignoring one condition is also shown in Fig.6. As both results using the nodal element are the same, we can conclude that one condition of (14)-(16) can be ignored even for the nodal element. It is better, however,

to impose three conditions in order to reduce the number of unknown variables.

The results of edge element are different from those of nodal element due to the difference of the accuracy of both types of element[9].

IV. CONCLUSIONS

The results obtained can be summarized as follows:

(1) When the ratio α of the number of elements in the conductor region to that of the whole region is small, the number, n_z , of non-zero entries, which affects the CPU time, for the T- Ω method using the edge element is the smallest of all. When α is large, n_z for the A method using the edge element is the smallest.

(2) The periodic boundary condition can also be used for the edge element in order to reduce the analyzed region.

The reason why only two components of A parallel to the periodic boundary are sufficient for both nodal and edge elements should be examined in future.

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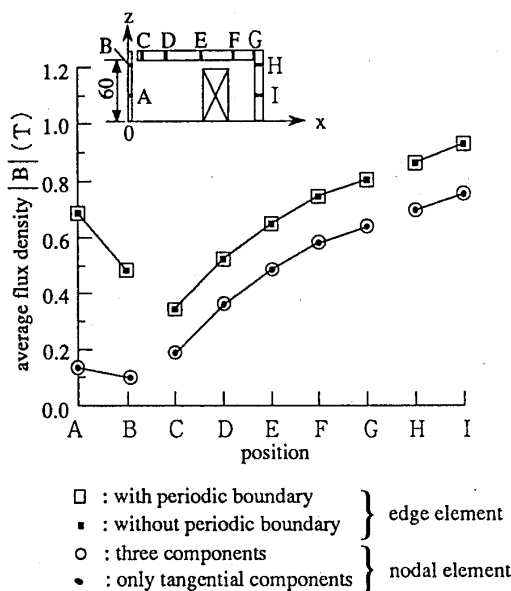


Fig.6 Distributions of flux densities along steel plates.