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### 3-D NON-LINEAR EDDY CURRENT ANALYSIS USING THE TIME-PERIODIC FINITE ELEMENT METHOD

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#### ABSTRACT

The 3-D finite element method using the  $A-\phi$  formulation for analyzing time-periodic non-linear magnetic fields with eddy currents has been developed. The CPU time of the new method can be reduced than that of the conventional step-by-step method, because the new method calculates the time-periodic phenomena directly not through the transient phenomena.

#### 1. INTRODUCTION

When periodic non-linear fields are analyzed taking into account eddy currents and voltage sources[1], it is not practical to apply the so-called step-by-step method[2] to such problems. Because a number of iterations are necessary until periodic solutions are obtained and the very long CPU time is consumed. The improved 2-D finite element method for such problems has already been developed[3,4]. The improved method is called the "time-periodic finite element method". In this method, a time-periodic waveform can be directly calculated not through the transient phenomena by efficiently using the relationship between the vector potentials at the instants  $t$  and  $t+T/2$  ( $T$ :period) of the periodic waveform. The matrix equation is constructed in half a period and the vector potentials in half a period are solved at the same time. The CPU time can be reduced compared with the conventional step-by-step method without much increase of the computer storage. This advantage is especially important in 3-D magnetic field analysis.

In this paper, the method is expanded into 3-D analysis of non-linear magnetic fields with eddy currents using the  $A-\phi$  formulation. The finite element discretization of the expanded method is described in detail. As an example of application, a loaded transformer is analyzed by using the new method and the conventional step-by-step method, and the CPU times and the computer storages of both methods are compared with each other.

#### 2. METHOD OF ANALYSIS

##### 2.1 Fundamental Equations

In the  $A-\phi$  method, the following equations are discretized in order to analyze 3-D magnetic fields with eddy currents[1]:

$$\begin{aligned} G_{oi} = & - \int_{\Omega} \text{grad} N_i \times (\nu \text{rot} A) dV \\ & + \int_{\Omega_e} N_i \sigma \left( \frac{\partial A}{\partial t} + \text{grad} \phi \right) dV \\ & - \int_{\Omega_c} N_i J_o dV = 0 \end{aligned} \quad (1)$$

$$G_{di} = \int_{\Omega_e} \text{grad} N_i \cdot \sigma \left( \frac{\partial A}{\partial t} + \text{grad} \phi \right) dV = 0 \quad (2)$$

where  $A$  and  $\phi$  are the magnetic vector potential and the electric scalar potential respectively.  $J_o$  is the current density in the exciting winding.  $\nu$  and  $\sigma$  are the reluctivity and the conductivity respectively.  $N_i$  is the interpolation function.  $\Omega$  denotes the analyzed region.  $\Omega_e$  and  $\Omega_c$  are the region of conductors with

eddy currents and that of windings where eddy currents are neglected.

When electrical machines excited from voltage sources are analyzed, not only Eqs.(1) and (2) but also the following equation derived from Kirchhoff's second law should be discretized[1]:

$$\eta = V_o - \frac{d\Psi}{dt} - (R_o + R_c) I_o - L_o \frac{dI_o}{dt} = 0 \quad (3)$$

where  $\Psi$  is the interlinkage flux of the winding.  $V_o$  is the terminal voltage of the power source.  $I_o$  is the current in the exciting winding.  $R_c$  is the dc resistance of the winding.  $R_o$  and  $L_o$  are the resistance and the inductance of the lead wire and the load which are not included in the finite element region as shown in Fig.1.

The current density  $J_o$  in Eq.(1) is represented as follows[1]:

$$J_o = \frac{nc}{Sc} I_o (i \sin \theta \cos \psi + j \sin \theta \sin \psi + k \cos \theta) \quad (4)$$

where  $Sc$  and  $nc$  are the cross-sectional area and the number of turns of the winding respectively.  $i, j$  and  $k$  are the unit vectors in the  $x$ -,  $y$ - and  $z$ -directions respectively.  $\psi$  and  $\theta$  are the angles from the  $x$ - and  $z$ -axes as shown in Fig.2.

The interlinkage flux  $\Psi$  in Eq.(3) can be denoted by the  $x$ -,  $y$ - and  $z$ -components  $A_x, A_y$  and  $A_z$  of  $A$  in the winding as follows[1]:

$$\Psi = \frac{nc}{Sc} \int_{\Omega_c} (A_x \sin \theta \cos \psi + A_y \sin \theta \sin \psi + A_z \cos \theta) dV \quad (5)$$

##### 2.2 Time-Periodic Finite Element Method

When the waveform of a vector potential is symmetric and periodic with time as shown in Fig.3, the following relationship is hold between vector potentials  $A^t$  and  $A^{t+T/2}$  at the instants  $t$  and  $t+T/2$  ( $T$ :period):

$$A^t = -A^{t+T/2} \quad (6)$$

The same relationship is hold on the current  $I_o$  as follows:

$$I_o^t = -I_o^{t+T/2} \quad (7)$$

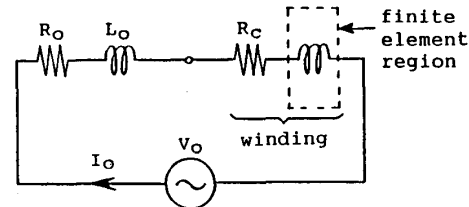
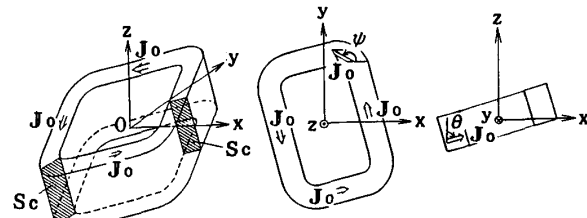


Fig.1 Equivalent circuit.



(a) bird's eye view (b) xy plane (c) zx plane  
Fig.2 Winding.

In the time-periodic finite element method, the vector potentials  $A^t$ ,  $A^{t+\Delta t}$ , ...,  $A^{t+T/2-\Delta t}$  ( $\Delta t$ : time interval) and the currents  $I_0^t$ ,  $I_0^{t+\Delta t}$ , ...,  $I_0^{t+T/2-\Delta t}$  are treated as unknown variables, and they are calculated simultaneously taking into account the relationships of Eqs.(6) and (7).

When the potential and the current at each instant are treated as unknown variables, the following equations for the non-linear analysis are obtained from Eqs.(1) to (5)[1]:

$$\left. \begin{aligned} & [C^t] \begin{Bmatrix} \delta u_i^{t-\Delta t} \\ \delta I_{0l}^{t-\Delta t} \end{Bmatrix} + [H^t] \begin{Bmatrix} \delta u_i^t \\ \delta I_{0l}^t \end{Bmatrix} \\ & = \begin{Bmatrix} -\{G_i^t\} \\ -\{\eta_k^t\} \end{Bmatrix} \\ & [C^{t+\Delta t}] \begin{Bmatrix} \delta u_i^t \\ \delta I_{0l}^t \end{Bmatrix} + [H^{t+\Delta t}] \begin{Bmatrix} \delta u_i^{t+\Delta t} \\ \delta I_{0l}^{t+\Delta t} \end{Bmatrix} \\ & = \begin{Bmatrix} -\{G_i^{t+\Delta t}\} \\ -\{\eta_k^{t+\Delta t}\} \end{Bmatrix} \\ & \vdots \\ & [C^{t+T/2-\Delta t}] \begin{Bmatrix} \delta u_i^{t+T/2-\Delta t} \\ \delta I_{0l}^{t+T/2-\Delta t} \end{Bmatrix} \\ & + [H^{t+T/2-\Delta t}] \begin{Bmatrix} \delta u_i^{t+T/2-\Delta t} \\ \delta I_{0l}^{t+T/2-\Delta t} \end{Bmatrix} \\ & = \begin{Bmatrix} -\{G_i^{t+T/2-\Delta t}\} \\ -\{\eta_k^{t+T/2-\Delta t}\} \end{Bmatrix} \end{aligned} \right\} \quad (8)$$

(i, j = 1, 2, ---nu)

(k, l = 1, 2, ---ni)

where nu is the number of unknown nodes of which the potentials are unknown. ni is the number of currents.  $\delta$  means the increment of the unknown variable.  $\{ui\}$  is denoted as follows:

$$\{ui\} = \{A_{xi}, A_{yi}, A_{zi}, \phi_i\}^T \quad (9)$$

Increments of potentials  $\{\delta u_i\}$  and currents  $\{\delta I_{0l}\}$  at the instants  $t-\Delta t$ ,  $t$ , ...,  $t+T/2-\Delta t$  are unknown variables.  $[C]$  and  $[H]$  are nearly the same as those of the conventional step-by-step method[1,5].

By applying the relationships of Eqs.(6) and (7) to  $\{\delta u_i^{t-\Delta t}\}$  and  $\{\delta I_{0l}^{t-\Delta t}\}$  in Eq.(8), the following matrix equation is obtained:

$$\begin{bmatrix} [H^t] & 0 & \dots & 0 & -[C^t] \\ [C^{t+\Delta t}] & [H^{t+\Delta t}] & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & [C^{t+T/2-\Delta t}] & [H^{t+T/2-\Delta t}] & 0 \end{bmatrix} \times \begin{Bmatrix} \begin{Bmatrix} \delta u_i^t \\ \delta I_{0l}^t \end{Bmatrix} \\ \begin{Bmatrix} \delta u_i^{t+\Delta t} \\ \delta I_{0l}^{t+\Delta t} \end{Bmatrix} \\ \vdots \\ \begin{Bmatrix} \delta u_i^{t+T/2-\Delta t} \\ \delta I_{0l}^{t+T/2-\Delta t} \end{Bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{Bmatrix} -\{G_i^t\} \\ -\{\eta_k^t\} \end{Bmatrix} \\ \begin{Bmatrix} -\{G_i^{t+\Delta t}\} \\ -\{\eta_k^{t+\Delta t}\} \end{Bmatrix} \\ \vdots \\ \begin{Bmatrix} -\{G_i^{t+T/2-\Delta t}\} \\ -\{\eta_k^{t+T/2-\Delta t}\} \end{Bmatrix} \end{Bmatrix} \quad (10)$$

As the coefficient matrix in Eq.(10) is very large and non-symmetric, considerable CPU time and computer

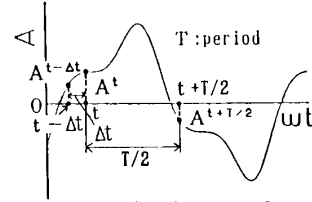


Fig.3 Periodic waveform.

storage are required using the conventional method of solution. Therefore, the iteration technique is introduced by dividing Eq.(10) into the following m equations:

$$\begin{aligned} & [H^{t+k\Delta t}] \begin{Bmatrix} \delta u_i^{t+k\Delta t} \\ \delta I_{0l}^{t+k\Delta t} \end{Bmatrix} \\ & = -\alpha\beta_k [C^{t+k\Delta t}] \begin{Bmatrix} \delta u_i^{t+(k-1)\Delta t} \\ \delta I_{0l}^{t+(k-1)\Delta t} \end{Bmatrix} \\ & + \begin{Bmatrix} -\{G_i^{t+k\Delta t}\} \\ -\{\eta_k^{t+k\Delta t}\} \end{Bmatrix} \quad (11) \end{aligned}$$

(k = 0, 1, ---, m-1)

where m is the number of time steps in half a period.  $\beta_k$  is equal to -1 (k=0) and 1 (k>0).  $\alpha$  is the relaxation factor and is chosen to be equal to 0. The iteration is carried out until u and I<sub>0</sub> are converged. Using this iteration technique, the non-linear steady state magnetic fields can be obtained within smaller CPU time than the conventional step-by-step method.

### 3. AN EXAMPLE OF APPLICATION

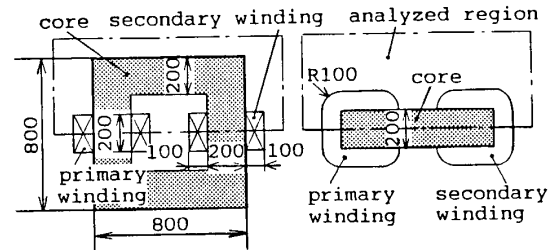
The currents in the primary and the secondary windings of a loaded transformer shown in Fig.4 are analyzed. The core is made of non-oriented silicon steel M-15. The effective voltage and the frequency of the power source are 200V and 50Hz respectively. The numbers of turns of the primary and the secondary windings are both equal to 120. L<sub>0</sub> shown in Fig.1 is assumed to be zero and 4Ω or 20Ω is selected for the load R<sub>0</sub>.

Figures 5 and 6 show the current waveforms obtained by the step-by-step method and the time-periodic method respectively.

Figure 7 shows the error of the current. The error  $\varepsilon_1^{(k)}$  is defined as follows:

$$\varepsilon_1^{(k)} = \frac{I_m^{(k)} - I_m}{I_m} \times 100\% \quad (12)$$

where  $I_m$  is the peak value of the current calculated by the time-periodic method.  $I_m^{(k)}$  is the peak value of the current in the k-th half a period of the waveform calculated by the step-by-step method. The curves for  $\varepsilon_1^{(k)} > 0$  and  $\varepsilon_1^{(k)} < 0$  denote the errors for the even and the odd half a period



(a) front view (b) plan view  
Fig.4 Analyzed model.

respectively. Assuming that the exact solution is obtained when the error  $\varepsilon_1(k)$  becomes less than 1%, the number of iterations required for the step-by-step method is 3 to 6 periods in those examples.

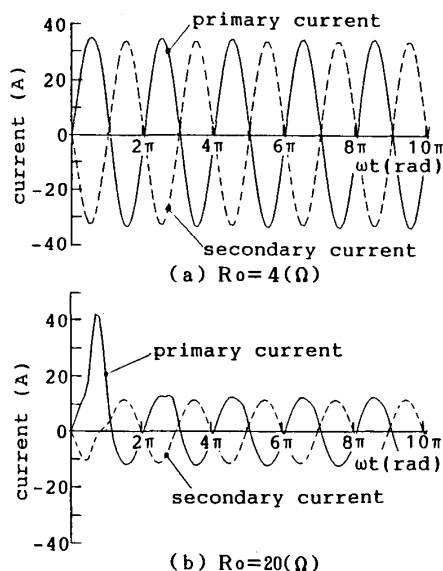


Fig.5 Current waveforms (step-by-step method).

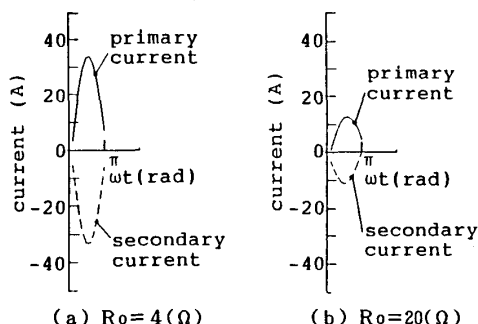


Fig.6 Current waveforms (time-periodic method).

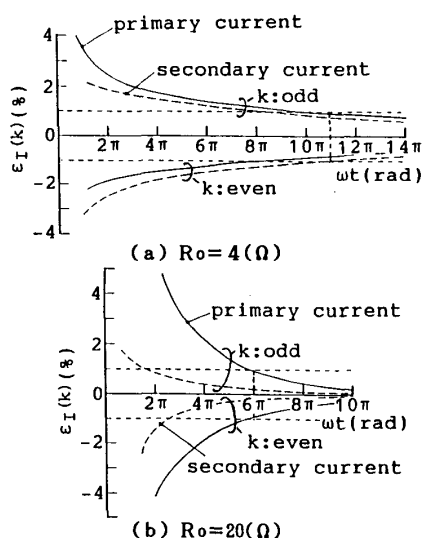


Fig.7 Error of the current.

Table 1 denotes the comparison of the CPU time. The calculation of the step-by-step method is stopped when the error  $\varepsilon_1(k)$  becomes less than 1%. The CPU time of the time-periodic method can be reduced to about 1/3 of the step-by-step method.

Table 2 denotes the computer storage M. M is defined as the size of the dimensions declared in the computer code. The computer storage of the time-periodic method is increased than that of the step-by-step method, because all unknown variables in half a period are memorized in the time-periodic method. However, the increase can be negligible.

Table 1 Comparison of CPU time

method	CPU time (sec)	
	$R_0=4(\Omega)$	$R_0=20(\Omega)$
step-by-step method	446	298
time-periodic method	139	103

Table 2 Comparison of computer storage

method	M (MB)
step-by-step method	0.44
time-periodic method	0.50

#### 4. CONCLUSIONS

The method for analyzing 3-D time-periodic non-linear magnetic fields with eddy currents, in the case when electrical machines are excited from voltage sources, has been developed. It is shown that the CPU time can be reduced to, for example, about 1/3 of the step-by-step method.

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