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EDDY CURRENT AND DEFLECTION ANALYSES OF A THIN PLATE IN TIME-CHANGING MAGNETIC FIELD

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Abstract- **Eddy current and deflection analysis of thin plate model in time-changing magnetic field are described. The model is solved as a coupled problem in which the time-changing magnetic field induces eddy currents and the eddy currents cause deflection of the thin plate by the Lorentz force. The eddy current analysis and deflection analysis are performed by a integro-differential method using a current vector potential and a structural finite element method using beam elements, respectively. The formulations of the motional electromotive force and the Lorentz force for the thin plate model are presented. Furthermore, the applicability of the proposed method is verified by using a cantilevered beam model.**

INTRODUCTION

Many useful methods for eddy current analysis have been presented. For thin conducting plates, the integro-differential method using a current vector potential is practical[1]. In this paper, an approach for calculating a coupled problem of the thin conducting plates, in which the time changing magnetic field and the motion of the conductor induce eddy currents and the eddy currents cause deflection of the thin plate by the Lorentz force, is described. Eddy current distribution in the thin conductor is solved by the integro-differential method and deflection of the conductor is solved a finite element method using beam elements[2]. Since eddy currents are induced by the time-changing magnetic field and the motion of the conductor and the Lorentz force is produced by the eddy currents with the constant magnetic field, the integro-differential method and the finite element method taking account of the interactions are solved alternately. Formulations of the proposed methods and solutions for the cantilevered beam[3] are described.

FORMULATIONS

Eddy current analysis

Taking the motional electromotive force into account, the eddy current in the thin plate is solved by the integro-differential method using the current vector potential. The governing equation of the current vector potential **T** is given by

$$\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{T}\right) = -\frac{\partial \mathbf{B}}{\partial t} - \nabla\times(\mathbf{B}\times\mathbf{v}) \qquad (1)$$

where σ is the conductivity, **B** is the magnetic flux density and **v** is the velocity of the thin plate. The eddy current density $\mathbf{J}_e$ is given by

$$\mathbf{J}_e=\nabla\times\mathbf{T} \qquad (2)$$

When the conductor can be approximated by a thin plate, one component of the current vector potential is used for the analysis. The integro-differential equation for the normal component of the current vector potential is obtained as follows:

$$\frac{1}{\sigma}\nabla^2 T = \frac{\mu_0 h}{4\pi}\frac{\partial}{\partial t}\iint_S \frac{\{\nabla\times(\mathbf{n}'T)\}\times\mathbf{r}\cdot\mathbf{n}}{r^3}\,ds + \frac{\partial \mathbf{B}_0\cdot\mathbf{n}}{\partial t} + \nabla\times(\mathbf{B}\times\mathbf{v})\cdot\mathbf{n} \qquad (3)$$

where h is the thickness of the thin plate, **r** is the vector from the source point to the field point, **n**' is the unit normal vector at the source point, **n** is the unit normal vector at the field point and $\mathbf{B}_0$ is the external magnetic flux density.

After discretization, a matrix equation for unknown current vector potential {T} is obtained as follows:

$$[R]\{T\}+[L]\,\partial\{T\}/\partial t=\partial\{B_y\}/\partial t+\{k_1 B_x v_y\} \qquad (4)$$

where [R] is the resistance matrix, [L] is the inductance matrix, B_x and B_y are the x and y components of the external magnetic flux density, respectively, and k_1 is the constant. In order to determine k_1, the motional electromotive force is calculated for the element *e* shown in Fig. 1 by

$$\int_e \nabla\times(\mathbf{B}\times\mathbf{v})\cdot\mathbf{n}\,ds = \int_L \mathbf{B}\times\mathbf{v}\cdot d\mathbf{l} = \mathbf{a}\cdot(\mathbf{B}\times\mathbf{v})_a+\mathbf{b}\cdot(\mathbf{B}\times\mathbf{v})_b+\mathbf{c}\cdot(\mathbf{B}\times\mathbf{v})_c \qquad (5)$$

Equation (4) is solved by using the backward difference with respect to time.

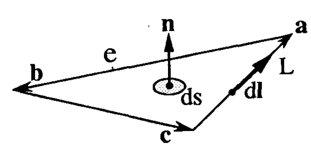

Fig. 1 Triangular element to calculate the motional electromotive force.

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Deflection analysis

For the deflection analysis, the finite element method using axial-flexural beam elements[2] is applied. The thin plate is approximated by the beam elements with uniform cross section. Following matrix equation for the unknown displacement {u} is obtained by the formulation of the finite element method.

$$[M]\partial^2\{\mathbf{u}\}/\partial t^2+[C]\partial\{\mathbf{u}\}/\partial t+[K]\{\mathbf{u}\}=\{k_2(\mathbf{J}_e\times\mathbf{B}_x)\cdot\mathbf{n}\} \quad (6)$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix, k_2 is the constant to discretize the Lorentz force, and the displacement **u** at the node consists of the axial displacement Δx, the flexural displacement Δy and the rotation angle θ. In order to determine k_2, the Lorentz force is calculated for the element *e* shown in Fig. 2 by

$$\begin{aligned}\int_V \nabla\times(\mathbf{J}_e\times\mathbf{B})dv\cdot\mathbf{n} &= h\int_S \mathbf{J}_e\times\mathbf{B}\cdot\mathbf{n}ds\\ &= h\,S_e\mathbf{J}_e\times\mathbf{B}\cdot\mathbf{n}\\ &= h\,S_eJ_zB_x \qquad (7)\end{aligned}$$

where S_e is the area of the element.

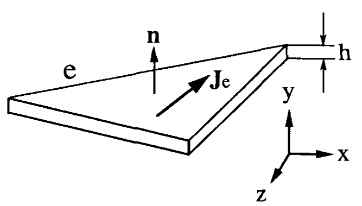

Fig. 2 Triangular element to calculate the Lorentz force.

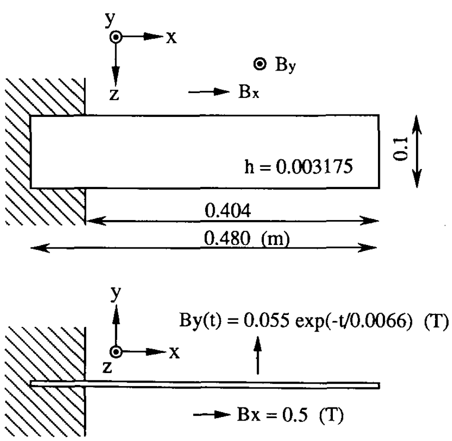

Fig. 3 Cantilevered beam model.

Equation (6) is solved by using the Newmark's β method using the developed code. Furthermore, Eqs. (4) and (6) are solved alternately as initial value problems taking account of the interactions.

COMPUTATION RESULTS

The cantilevered beam model which is proposed in the TEAM workshop[3] is shown in Fig. 3. The thin conductor was divided into 224 triangular elements for eddy current analysis and 5 beam elements for deflection analysis as shown in Fig. 4[4],[5]. The computation results of the modal analysis is shown in Fig. 5. Eddy current distribution is shown in Fig. 6 by the equipotential lines of the current vector potential. The comparisons between computation results and the experimental results[6] are shown in Fig. 7. The computation results agree with the experimental results[6].

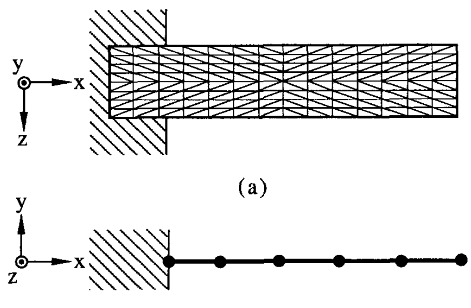

Fig. 4 Arrangement of elements, (a) triangular elements, (b) beam elements.

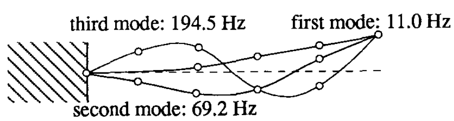

Fig. 5 Results of the modal analysis.

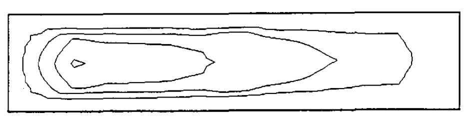

Fig. 6 Distribution of the eddy current at t=0.01 sec.

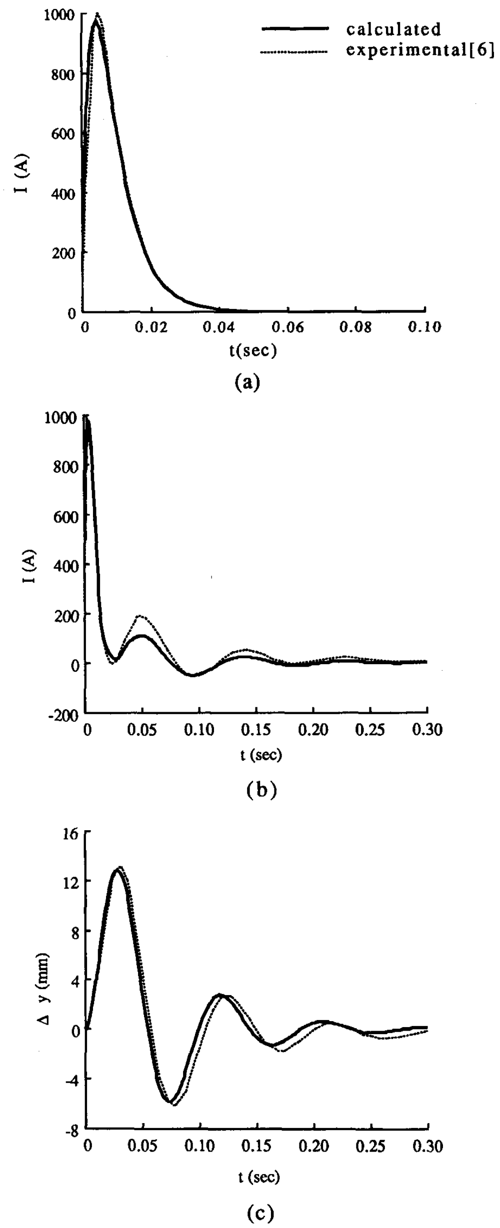

Fig. 7 Computation results of the cantilevered beam model(B_x=0, 0.5 T), (a) total eddy current, Bx=0, (b) total eddy current, B_x=0.5, (c) deflection at the free end, B_x=0.5.

CONCLUSION

The method for calculating the eddy-current problem of thin conducting plates coupled with mechanical problem was presented. The eddy current analysis taking account of the motion electromotive force and the deflection analysis taking account of the Lorentz force is performed by the integro-differential method and the finite element method, respectively. Furthermore, the formulations of the motion electromotive force and the Lorentz force for the thin plate model were proposed. The computation results for the cantilevered beam model agreed with the experimental results and so the applicability of the proposed method was verified.

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