Physics

Electricity & Magnetism fields

Okayama University *Year* 1990

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FINITE ELEMENT ANALYSIS OF INDUCED CURRENTS IN AXISYMMETRIC MULTI-CONDUCTORS CONNECTED IN PARALLEL TO VOLTAGE SOURCES

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ABSTRACT

A method for analyzing induced currents, including both eddy currents and circulating currents in axisymmetric multi-conductors connected in parallel to voltage sources, has been developed using the finite element method. The method has the advantage that the non-linear analysis of induced currents in conductors connected to voltage sources with distorted waveforms (for example, an inverter-fed transformer) is possible. Effectiveness of the method is illustrated by applying it to a transformer model.

1. INTRODUCTION

Windings of large electric machines are composed of many stranded conductors to avoid the concentration of the current. In order to obtain an accurate estimate of the losses, it is necessary to calculate the eddy current distribution and also the circulating current. Although various methods of evaluation have already been developed[1-6], it is difficult to assess the current distribution throughout the conductors, given only the applied voltage[7-9]. In conventional models the total current driven by the voltage source is specified and, although the voltage sources can be taken into account, it is not possible to calculate accurately the effects of the non-linearity[10,11].

In this paper, the conventional method[1] is improved so that the above-mentioned difficulties can be overcome. The voltage sources and external impedances can be taken into account by combining the loop equations obtained from Kirchhoff's law with Maxwell's equations for the magnetic field analysis. The validity of the method is examined experimentally.

2. METHOD OF ANALYSIS

2.1 Fundamental Equation

Axisymmetric magnetic fields with induced currents are governed by the following partial differential equation:

$$\frac{\partial}{\partial r}\left\{\frac{\nu}{r}\frac{\partial(rA_\theta)}{\partial r}\right\}+\frac{\partial}{\partial z}\left(\nu\frac{\partial A_\theta}{\partial z}\right)=\sigma\left(\frac{\partial A_\theta}{\partial t}+\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right) \quad (1)$$

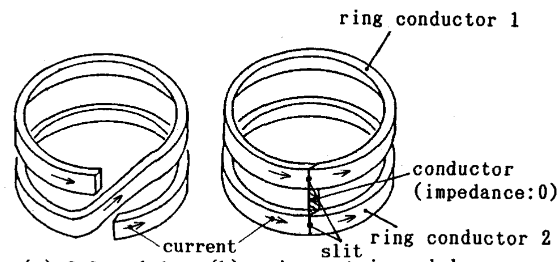
where A_θ and ϕ are the θ-component of the magnetic vector potential and the electric scalar potential respectively. ν and σ are the reluctivity and the conductivity respectively. The term on the right-hand side of Eq.(1) represents the induced current density in the conductor.

In this calculation, not only the magnetic vector potential A_θ but also the electric scalar potential ϕ are treated as unknown variables. If the electric scalar potential ϕ is directly treated as an unknown variable instead of $\partial\phi/\partial\theta$[1,8], the coefficient matrix of the linear equations becomes symmetric[11].

2.2 Modelling of Conductors in Windings

The method for modelling conductors in windings is explained using a simple coil with 2 turns as shown in Fig.1.

In the actual coil where the current has three components shown in Fig.1(a), a long CPU time is required to carry out a 3-D analysis. Therefore, the coil is approximated by a group of ring conductors with equal currents as shown in Fig.1(b). These rings have slits, and they are short-circuited by a conductor which has both zero impedance and zero current density. This approximation enables us to calculate the induced currents in the coil using the axisymmetric method.



(a) 3-D model (b) axisymmetric model

Fig.1 Coil (2 turns).

2.3 Relationship among Potentials, Currents and Applied Voltage

In order to take into account the connections of conductors, the continuity condition of current is used. This is illustrated with the multi-conductor model shown in Fig.2. Figure 3 shows an equivalent circuit corresponding to the model. m and n denote the numbers of strands and turns respectively. ϕ_{ij} ($i=1,\cdots,m$, $j=1,\cdots,n-1$) and ϕ_n are the electric scalar potentials at points P_{ij} and P_n respectively as shown in Fig.3. I_i is the current in each strand. I_o is the total current. V_o is the applied voltage and Z_o is the external impedance.

At points P_{ij} ($i=1,\cdots,m$, $j=1,\cdots,n-1$), the following equation is obtained from the continuity condition of current between the ijth conductor and the i(j+1)th conductor.

$$g_{ij}\cdot(\phi_{i(j-1)}-\phi_{ij})-\overline{I_{ij}} = g_{i(j+1)}\cdot(\phi_{ij}-\phi_{i(j+1)})-\overline{I_{i(j+1)}} \quad (2)$$

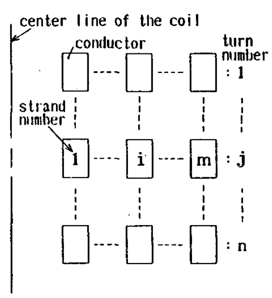


Fig.2 Cylindrical winding with multi-conductors.

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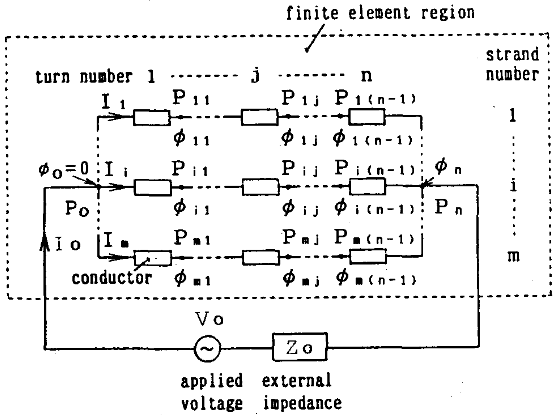

Fig.3 Equivalent circuit of winding.

where g_{ij} and $\overline{I_{ij}}$ are denoted as follows:

$$g_{ij} = \frac{\sigma}{2\pi}\iint_{\Omega ij} \frac{1}{r}\, drdz \qquad (3)$$

$$\overline{I_{ij}} = \sigma\iint_{\Omega ij} \frac{\partial A_\theta}{\partial t}\, drdz \qquad (4)$$

where Ω_{ij} is the cross section of the ijth conductor.

At the point P_n, the following equation is obtained:

$$V_o + \phi_n = R_o \cdot I_o + L_o \frac{\partial I_o}{\partial t} \qquad (5)$$

where R_o and L_o are the resistance and the inductance of the impedance Z_o, which cannot be included in the finite element region. The total current I_o is represented as follows:

$$I_o = \sum_{i=1}^{m} \{ g_{in}\cdot(\phi_{i(n-1)} - \phi_n) - \overline{I_{in}} \} \qquad (6)$$

Both the vector potential A_θ and the electric scalar potentials ϕ_{ij} and ϕ_n can be directly obtained by solving Eqs.(1),(2) and (5) simultaneously. The "time-periodic finite element method"[12,13] is introduced in order to analyze periodic fields taking into account the eddy current, the voltage source and the nonlinearity of the core.

3. EXPERIMENTAL VERIFICATION

An air-core winding with 4 strands and 4 turns shown in Fig.4 is analyzed so as to verify the method. In order to simplify the experimental model, the conductors are not transposed. The conductivity σ, the applied voltage V_o and the external impedance Z_o are 5.689×10^7 S/m, 8.9V and 0.685Ω (pure resistance) respectively. The frequency f of the power source is 60Hz. The induced current densities on the surfaces of respective strands shown in Fig.4 are compared.

Figure 5 shows time variations of the applied voltage and the currents. I_1 to I_4 denote the currents in the respective strands. Because the conductors are not transposed, there is big imbalance between the currents.

Table 1 shows the comparison between the measured induced current density J_m and the calculated one J_a.

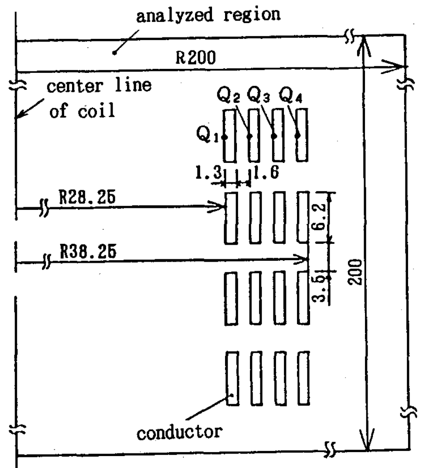

Fig.4 Air-coil winding.

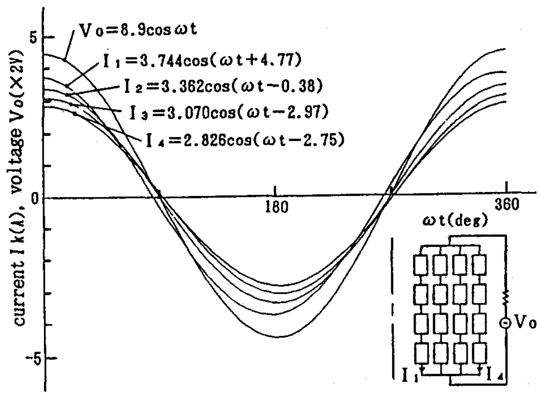

Fig.5 Time variations of applied voltage and currents.

Table 1 Comparison of induced current density

position	measured $J_m(A/m^2)$	calculated $J_a(A/m^2)$	error ε (%)
Q_1	4.514×10^5	4.751×10^5	5.25
Q_2	4.197×10^5	4.265×10^5	1.62
Q_3	3.836×10^5	3.884×10^5	1.25
Q_4	3.612×10^5	3.571×10^5	−1.14

The error ε is defined by

$$\varepsilon = \frac{J_a - J_m}{J_m} \times 100\ (\%) \qquad (7)$$

The calculated results are in good agreement with the measured ones.

4. AN EXAMPLE OF APPLICATION

A transformer model with 4 strands and 4 turns shown in Fig.6 is analyzed in order to illustrate effectiveness of the method. σ, V_o Z_o and f are 5.841×10^7 S/m, 10V, 0.001Ω (pure resistance) and 50Hz respectively. The core is made of non-oriented silicon steel (grade : AISI-78 M-15) and the eddy current in it is neglected. Flux and current distributions for two modes of connection shown in

Fig.7 are analyzed. The number in each conductor denotes the strand number.

Figure 8 shows time variations of the applied voltage and the currents. Io denotes the total current. If the conductors are transposed, the balance of the currents between I_1 to I_4 is improved. The imbalance of the currents of the non-linear model is bigger than that of the linear one shown in Fig.5.

5. CONCLUSIONS

An improved method has been developed for computing induced currents, including both eddy currents and circulating currents in multi-conductors connected in parallel to the voltage source. Although the case with only one voltage source is treated here, the method can be expanded into cases with multiple voltage sources.

Because the method can take into account the voltage sources, the external impedances and the nonlinearity of the core, it becomes possible to analyze flux and current distributions under actual operating conditions (for example, an inverter-fed transformer) and to design windings which produce the minimum power loss.

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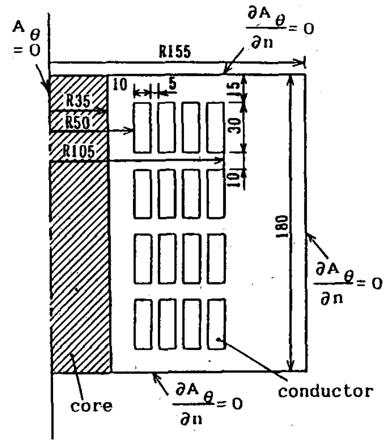

Fig.6 Analyzed model.

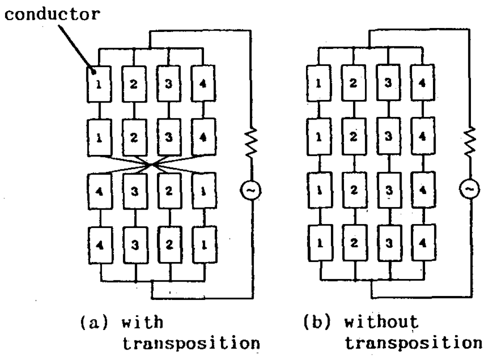

Fig.7 Connection diagrams.

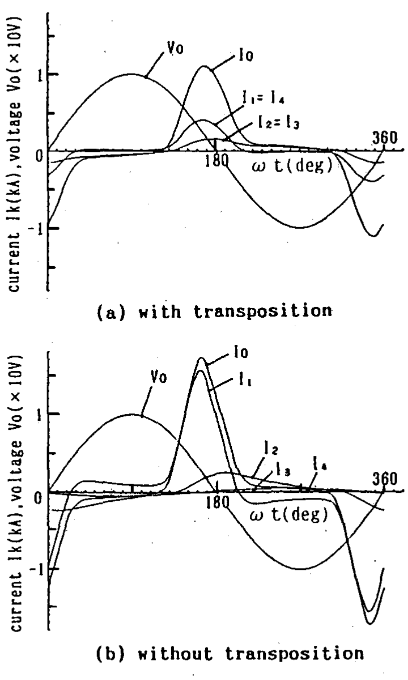

Fig.8 Time variations of applied voltage and currents.