# Practical analysis of 3-D dynamic nonlinear magnetic field using time-periodic finite element method

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# Nonlinear Transient Analysis of Electric Field Coupled with Temperature at the Joint of a Power Cable

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*Abstract* – Distribution of temperature and electric field at the joint of a dc power cable is analyzed taking into account the nonlinearity of electric resistance which is a function of temperature and electric field strength.

## I. INTRODUCTION

The dc electric field distribution is affected by not the capacitance but the resistance. The electric field strength can be computed by iterative calculations taking into account the nonlinearity of resistance which is a function of temperature and electric field strength. As the convergence of such a calculation is not easy, only a few analyses have been reported.

In this paper, nonlinear transient analysis of temperature and electric field distribution at the joint of a power cable is carried out by introducing a relaxation factor [1] in the nonlinear iterations.

## II. METHOD OF ANALYSIS

### A. Analysis of Temperature Distribution

The basic equation for the axi-symmetric analysis of heat transfer with time t is given by [2]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) = -\frac{\partial Q}{\partial t} + C\frac{\partial T}{\partial t} , \qquad (1)$$

where r and z are the radius and axial distance. T and Q are the temperature and heat generation, $\lambda$ and C are the thermal conductivity and heat capacity, respectively.

The temperature distribution in the power cable is affected considerably by the heat transfer on the boundary between the cable and the surrounding air. Thus, the heat transfer is considered in the analysis of the temperature distribution. The heat transfer is taken into account by introducing the heat flux q on the surface which is given by the following equation:

$$q = h(T - T_c) , \qquad (2)$$

where h is the heat transfer coefficient, and $T_c$ is the ambient temperature.

### B. Analysis of Electric Field Distribution

The basic equation for the axi-symmetric analysis of the electric field is given by [3]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{1}{\rho}\frac{\partial\phi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial\phi}{\partial z}\right) = 0 , \qquad (3)$$

where $\phi$ is the electric scalar potential.

The dc electric field is affected by the resistivity $\rho$ of insulator which is a function of temperature T and electric field strength E. It can be denoted as follows :

$$\rho = \rho_0 \bullet e^{-\alpha(T-T_0)} \bullet (E / E_0)^{-p} , \qquad (4)$$

where $\rho_0$ is the resistivity at the specified temperature $T_0$ and electric field strength $E_0$. $\alpha$ and p are the temperature and electric field coefficients, respectively.

### C. Calculation Procedure

Firstly, the variation of temperature distribution with time is calculated using (1). The heat transfer shown in (2) is taken into account by the boundary integral term of the finite element method. Secondly, the distribution of electric field is obtained taking into account the nonlinearity of resistivity shown in (4). The modified Newton-Raphson method which uses the relaxation factor [1] is applied for the nonlinear iterations, because a final solution cannot be obtained without this method due to a considerable nonlinearity.

## III. ANALYZED MODEL

Fig.1 shows the analyzed model for the joint of a dc transmission cable. The copper conductor for the cable is covered with insulating paper and immersed in insulation oil. The epoxy and PPLP (Polypropylene Laminated Paper) are used in order to reduce the electric field strength. The sheath and case are used as the shield for the electric field and protection, respectively. a-b-c-d is an adiabatic boundary. a-d is a boundary for heat transfer. In the thermal analysis, the initial value of temperature is assumed to be 20°C which is the same as the ambient temperature. In the electric field analysis, the potentials on the conductor and screen (sheath and case) of the cable are assumed to be 250kV and 0V, respectively. The heat transfer coefficient h is chosen to be 11.1(W/m$^2$°C) assuming that the surrounding air flows slowly.

## IV. RESULTS AND DISCUSSIONS

The temperature and electric field distribution for a load current of 2000A is analyzed.

### A. Distribution of Temperature

Fig.2 shows the distribution of temperature. Fig.3 shows variations of temperature distribution with time. The temperature gradient increases with time.

### B. Distribution of Electric Field

Fig.4 shows the equi-potential lines. The potential of the cable conductor is 250kV, and those of sheath and case are all equal to zero. The number of iterations of the modified Newton-Raphson method was around ten.

Fig.5 shows the distribution of the absolute value of electric field strength E along the boundary e-f between the epoxy and PPLP regions shown in Fig.1. Fig.6 shows the distribution of E along the boundary g-h between the epoxy and paper. The maximum electric field strength $E_{mp}$ at t=1 hour on the PPLP side is normalized to unity.

The electric field strength E in the PPLP along the boundary e-f is larger than that in epoxy, because there is a remarkable difference between the resistivities of epoxy and PPLP. On the contrary, the amplitude of E in the epoxy along the boundary g-h is not much different from that in paper, because the difference between the resistivities of epoxy and paper is small. The maximum electric field strength $E_{me}$ on the epoxy side increases and $E_{mp}$ on the
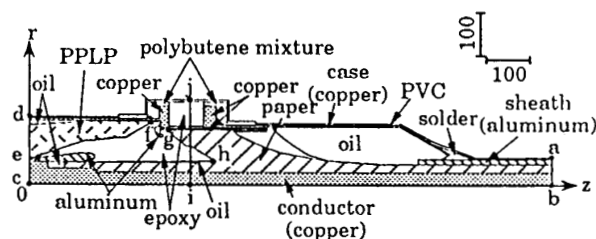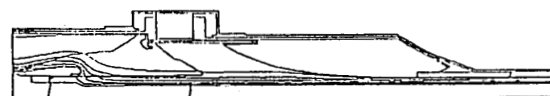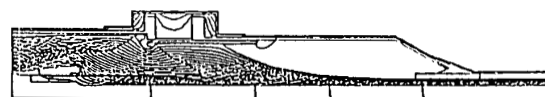


Fig.1 Analyzed model.



(a) 1 hour after



(b) steady state
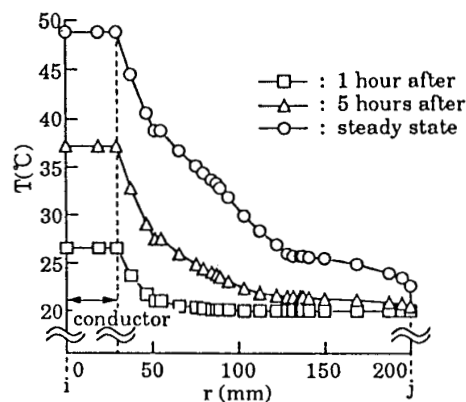
Fig.2 Equi-temperature lines.
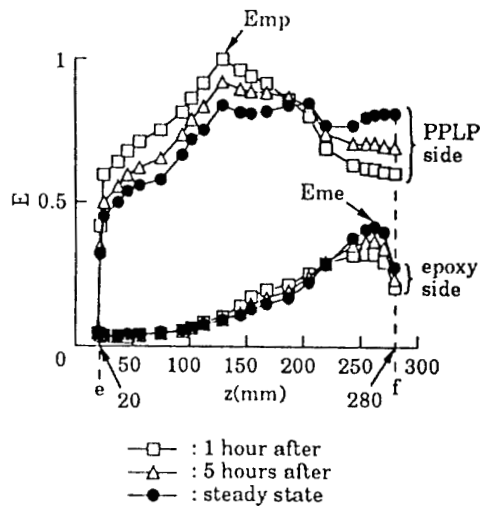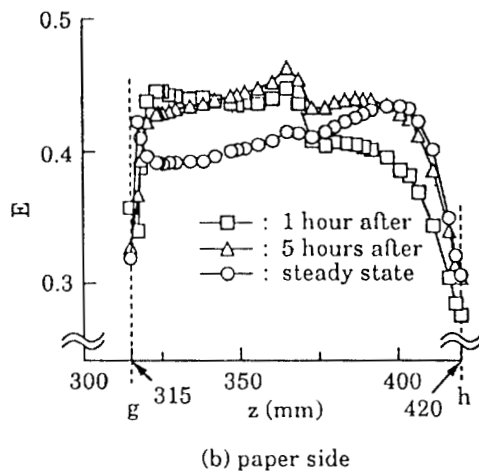


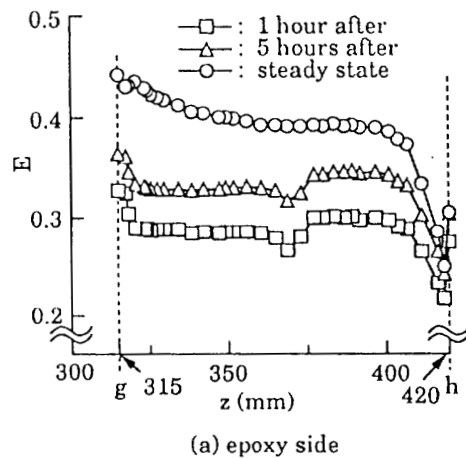Fig.3 Variation of temperature distribution with time.



(a) 1 hour after



(b) steady state

Fig.4 Equi-potential lines.

Fig.5 Distributions of electric field strength E along e-f.



(a) epoxy side



(b) paper side

Fig.6 Distributions of electric field strength E along g-h.

PPLP side decreases with time. The amplitudes of $E_{mp}$ and $E_{me}$ are important, because the breakdown begins when the maximum value of the electric field strength exceeds the breakdown voltage.

## C. Factors Affecting Temperature and Electric Field Distribution

### (1) Heat transfer coefficient

The heat transfer coefficient h depends upon the surface condition, the surrounding medium (air, oil, etc.) and the velocity of flow. Fig.7 shows the effect of h on the temperature of the cable conductor at the origin 0. When h becomes larger, the temperature rises more slowly, reaching a lower final value at the steady state.

Fig.8 shows the effect of h on the temperature distribution. The figure shows that the temperature
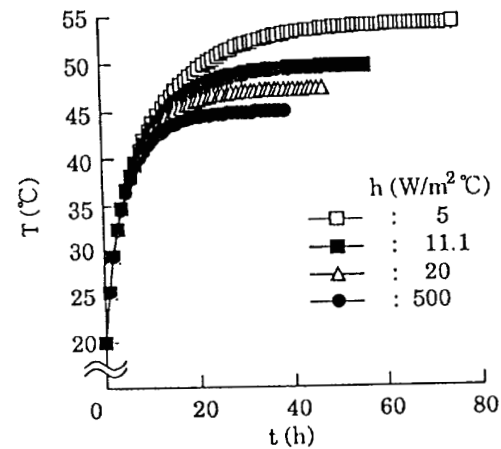


Fig.7 Effect of h on temperature rise of cable conductor.
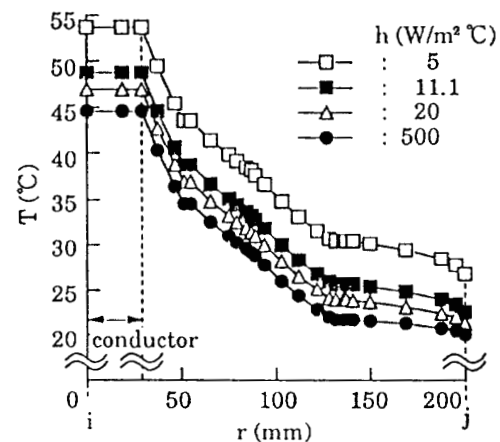


Fig.8 Effect of h on temperature distribution at steady state.

gradient is scarcely affected by h. The distribution of E is also scarcely affected by h. This is because the resistivity gradient changes little due to the nearly constant gradient of the temperature distribution as shown in Fig.8.

**(2) Load current**

Fig.9 shows the effect of the amplitude of load current I on the distribution of electric field strength E. The distribution of E is considerably affected by the load current I, because the temperature rise is almost proportional to $I^2$. Therefore, the cable should be designed so that breakdown does not occur under an overload current.
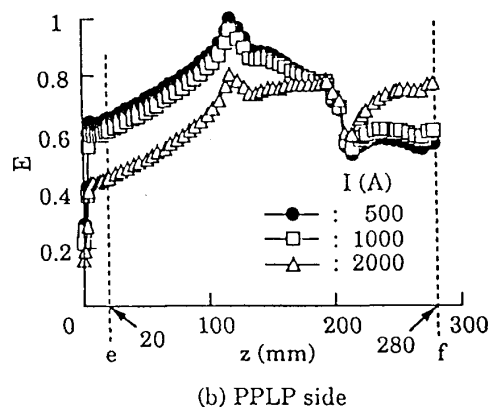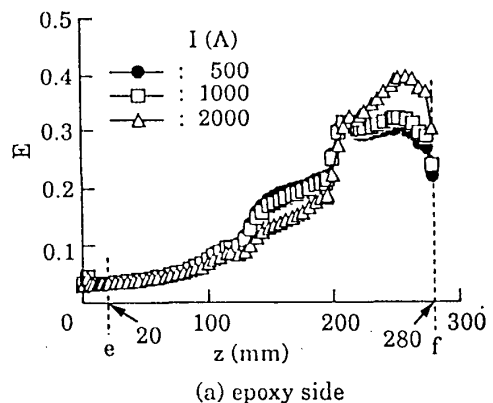


(a) epoxy side



(b) PPLP side

Fig.9 Effect of load current I on distribution of electric field strength E along e-f.

## V. CONCLUSIONS

The results obtained can be summarized as follows:

(1) The analysis of temperature and electric field distribution taking into account the nonlinearity of

electric resistance is possible by introducing the modified Newton-Raphson method which uses the relaxation factor.

(2) It is shown that although the distribution of electric field is scarcely affected by the heat transfer coefficient, it is considerably affected by the amplitude of load current.

The 3-D analysis taking into account convection will be reported in the future.

## REFERENCES

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[3] T. Nakata and N. Takahashi, "Finite element method in electrical engineering (2nd Edition)", Morikita Publishing Co., 1986.