Linear AC steady-state eddy current analysis of high speed conductor using moving coordinate system

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***Abstract* – A method for linear analysis of ac steady state eddy currents in a moving conductor using a moving coordinate system is investigated. It is shown that the moving coordinate system is superior to a fixed coordinate system for the analysis of a high speed conductors from the standpoint of stability of the solution. The applicable extent of the moving coordinate system is also discussed.**

I. INTRODUCTION

When magnetic fields in electrical machines with moving conductors, such as the linear induction motor[1], are analyzed, eddy currents due to the movement of conductors should be taken into account. Both the moving coordinate system[2–5] and the fixed coordinate system[1–3] are used in the analysis with a moving conductor. The effectiveness of the moving coordinate system for analysis of transient and dc steady state eddy currents has been shown in previous studies[2,3]. When the moving coordinate system is applied to linear analysis of ac steady state eddy currents, the time differential term $\partial \boldsymbol{A}/\partial t$ cannot be represented by $j\omega\boldsymbol{A}$, because the coordinates are moving. Therefore, the discretization method for $\partial \boldsymbol{A}/\partial t$ using a moving coordinate system should be investigated for linear analysis of the ac steady state.

In this paper, the method for linear analysis of ac steady state eddy currents using moving coordinate system, which is a combination of the phasor and the finite difference methods, is proposed. The applicable extent of the moving coordinate system is also investigated. The results obtained by using this method and the conventional method using a fixed coordinate system are compared.

II. INVESTIGATION OF EDDY CURRENT TERM

A. Discretization of Eddy Current Term

Let us examine the terms corresponding to the eddy current density in the $\boldsymbol{A}$–ϕ method ($\boldsymbol{A}$: magnetic vector potential, ϕ: electric scalar potential) using the moving coordinate system (X, Y, Z) shown in Fig.1. For simplicity, a 1–D model, moving with a constant velocity υ in the x–direction, shown in Fig.2, is examined without consideration of the gradϕ term[2]. A magnetic field which varies sinusoidally with time is applied to the conductor.

In the transient analysis, the time derivative $\partial A/\partial t$ for the moving coordinate system can be discretized by the backward difference method which is superior to the others[3] as follows[2]:

$$\frac{\partial A(X_1)^{t+\Delta t}}{\partial t} \cong \frac{A^*(X_1)^{t+\Delta t} - A(X_1)^t}{\Delta t} \cong \frac{A^*(x_2)^{t+\Delta t} - A(x_1)^t}{\Delta t} \tag{1}$$

$$(A^*(X_1)^{t+\Delta t} = A^*(x_2)^{t+\Delta t},\ A(X_1)^t = A(x_1)^t)$$

where Δt is the time interval. For example, $A(x_2)^{t+\Delta t}$ shows the vector potential of the point P (fixed coordinate: x_2) at the instant ($t+\Delta t$). The superscript (*) indicates the unknown variable.

In the linear analysis of the ac steady state, the magnetic vector potential A(x) varies sinusoidally with time at a fixed point. Therefore the following equation using the phasor method can be obtained from (1) :

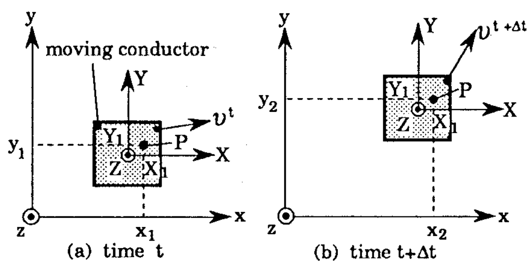

Fig. 1. Moving and fixed coordinate systems.

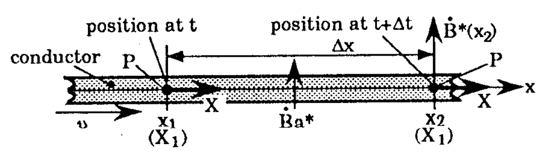

Fig. 2. Moving conductor with constant velocity.

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$$\frac{\partial \dot{A}(X_1)}{\partial t} \cong \frac{\dot{A}^*(x_2) - \dot{A}^*(x_1) \bullet e^{-j\omega\Delta t}}{\Delta t} \quad (2)$$

where superscript (•) indicates the complex variable. $e^{-j\omega\Delta t}$ means the previous value of Δt. The coefficient matrix becomes unsymmetric as $\dot{A}^*(x_1)$ is treated as the unknown variable.

For comparison, the eddy current term using a fixed coordinate system is given as follows :

$$\frac{\partial \dot{A}(x_2)}{\partial t} - \upsilon \bullet \dot{B}(x_2) = j\omega \dot{A}^*(x_2) - \upsilon \bullet \dot{B}^*(x_2) \quad (3)$$

where υ is the velocity vector of the moving conductor, and $\dot{B}$ is the flux density ($=-\partial\dot{A}/\partial x$). The matrix using the fixed coordinate system is unsymmetric due to the velocity term[3] as in the case of the moving coordinate system.

In 2–D and 3–D analysis, the same discretization mentioned above is carried out for all directions.

B. Physical Meaning of the Eddy Current Term

In order to understand the physical meaning of $\partial\dot{A}(X_1)/\partial t$ in the case of a moving coordinate system, the velocity term is derived from (2) as follows:

$$\begin{aligned}\frac{\partial \dot{A}(X_1)}{\partial t} &\cong \frac{\dot{A}^*(x_2) - \dot{A}^*(x_2) \bullet e^{-j\omega\Delta t}}{\Delta t} + \frac{\dot{A}^*(x_2) - \dot{A}^*(x_1)}{\Delta x} \bullet e^{-j\omega\Delta t} \bullet \frac{\Delta x}{\Delta t} \\ &\cong \frac{\dot{A}^*(x_2) - \dot{A}^*(x_2) \bullet e^{-j\omega\Delta t}}{\Delta t} - \dot{B}a^* \bullet e^{-j\omega\Delta t} \bullet \upsilon \quad (4)\end{aligned}$$

where Δx is the distance between x_1 and x_2, and $\dot{B}a$ is the average flux density between x_1 and x_2 ($=-\{\dot{A}(x_2)-\dot{A}(x_1)\}/\Delta x$). In the moving coordinate system, when Δt is too large, an error occurs due to the first term of the right–hand side of (4) which should be equal to $j\omega\dot{A}^*(x_2)$. Consequently, the effect of the time interval Δt on the accuracy should be examined.

III. Investigation of the Time Interval

A. Analysis Model and Method

The effect of the time interval Δt on the accuracy is investigated using the model shown in Fig.3. It is assumed that the moving conductor is infinitely long in the x–direction and moves with a constant velocity υ_x (=10, 100, 1000m/s). The conductivity σ is 10^7S/m. The ac exciting current of 2000AT at 500Hz is applied to the coil. The relative permeability μ_s of the pole pieces is 1000.

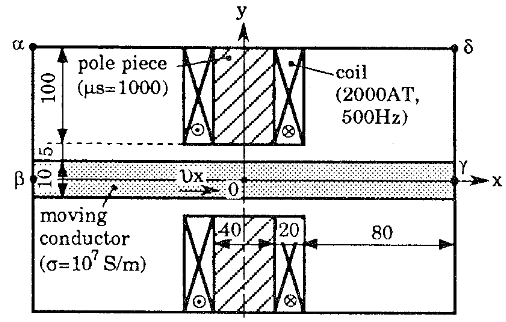

Fig. 3. Analyzed model.

Region α–β–γ–δ–α is analyzed. It is subdivided into 24 and 4 divisions in the x– and y–directions using 1–st order rectangular elements. The division in the x–direction is uniform. Neumann boundary conditions are imposed on all boundaries. Δt is chosen to be 2Te, Te, 0.5Te and 0.1Te (Te: the time required for the conductor to move the distance equal to the length of one element at each velocity). The Peclet numbers Pe ($=\mu\sigma\upsilon L/2$, μ : permeability, σ : conductivity, L : length of one element)[6] corresponding to the respective velocities are 0.6, 6 and 60.

B. Results and Discussion

Fig.4 shows the flux distributions on the line β–γ at the instant when the current in the coil is maximum. The results obtained from the fixed coordinate system using the Galerkin finite element method are also shown. At low velocity (υ_x=10m/s), the result of the moving coordinate system with small Δt is almost the same as those obtained from the fine mesh which is subdivided into 240 divisions in the x–direction. This is because the first term of the right–hand side of (4) becomes predominant at low velocity. On the other hand, at high velocity (υ_x=100, 1000m/s), Δt should be chosen to be equal to Te. The reason is that Ba in (4) coincides with the average flux density within one element when 1–st order finite elements are used. Therefore, Ba is more accurate at Δt=Te, because it is evaluated as the average value of flux density in one element.

When the analysis of a low speed moving conductor of which the Peclet number Pe is smaller than unity is carried out, the fixed coordinate system, which does not have any problem such as the first term of (4), should be used as shown in Fig.4(a). However, the result obtained using the fixed coordinate system has some oscillations in the high speed region as shown in Figs.4(b) and (c). Therefore, at the high speed, the proposed method using the moving coordinate system which does not cause oscillation is effective.

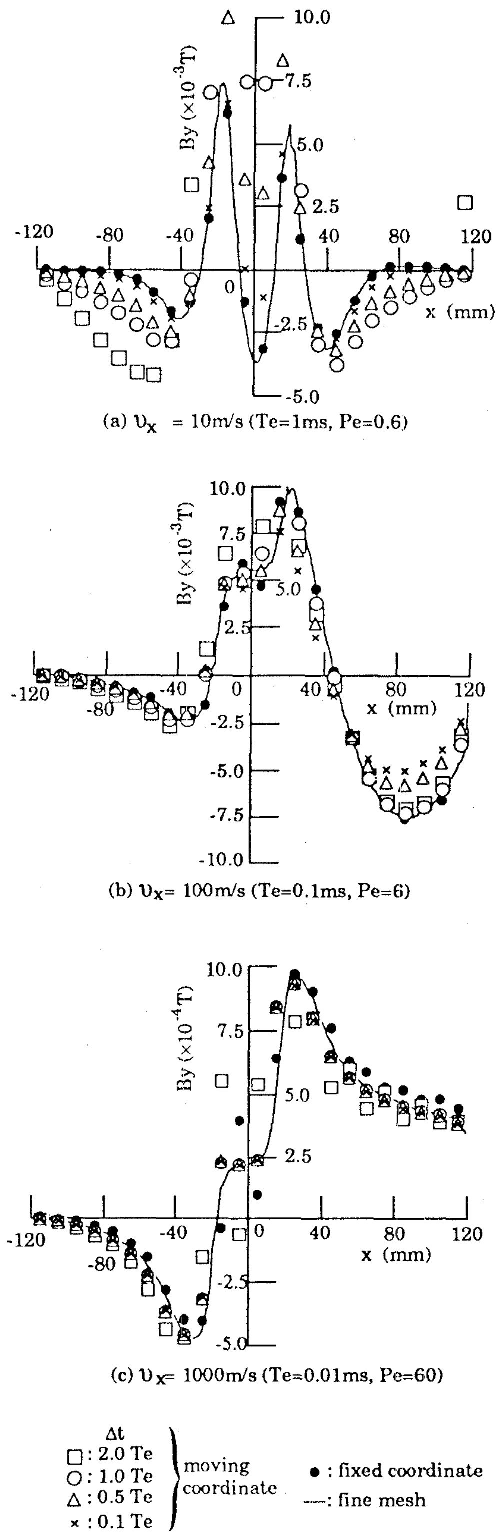

Fig.4. Flux distributions.

C. Applicable Extent of the Moving Coordinate System

In this section, the applicable extent of the moving coordinate system from the standpoint of the accuracy is discussed. Firstly, the error of the first term of the right-hand side of (4) is examined using the following function:

$$A(t) = -\cos(\omega t) \tag{5}$$

For simplicity, ω is assumed to be unity. The time difference $\partial A/\partial t$ calculated using the first term of (4) which Δt is chosen to be T/4 (T : a period) is compared with the analytical value (sin(t)) in Fig.5. Fig.6 shows the errors ε_a and ε_ϕ of the amplitude and phase calculated using the first term of (4) as shown in the following equations:

$$\varepsilon_a = |a_{err}/\sin(T/4)| \times 100\ (\%) \tag{6}$$

$$\varepsilon_\phi = |\phi_{err}/T| \times 100\ (\%) \tag{7}$$

where a_{err} and ϕ_{err} are differences of the amplitude and phase between the calculation and the analytical solution as shown in Fig.5. When the time interval Δt is smaller than T/20, the errors ε_a and ε_ϕ are smaller than 3%. Therefore, the time interval Δt should be smaller than T/20 (Condition A). Secondly, from the standpoint of the second term on the right–

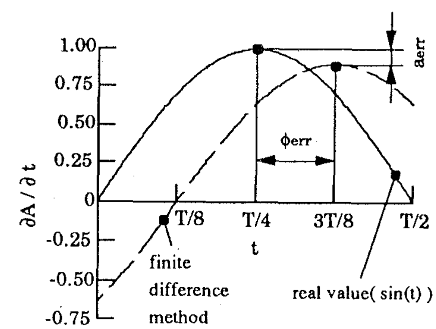

Fig. 5. Definition of errors of amplitude and phase.

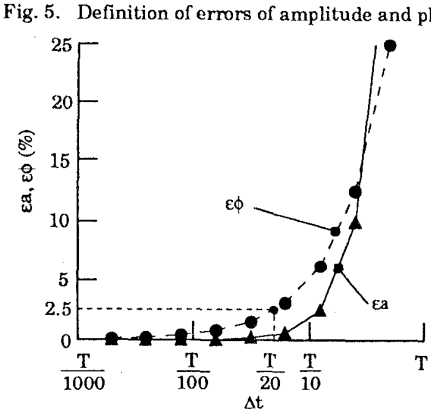

Fig. 6. Relationship between Δt and error.

hand side of (4), the time interval Δt should be chosen to be Te for the reason mentioned in Section III. B (Condition B). Both conditions A and B should be satisfied in order to obtain accurate results. When elements with different sizes are used, the value of time interval Δt should be determined in each element. In the model shown in Fig.3, the values of the time intervals Δt determined by these conditions becomes 1.0Te at the velocities υ_x of 100 and 1000m/s. Fig.4 shows that the time intervals Δt (=1.0Te) give accurate results at each velocity. If the time interval Δt determined by Condition B cannot be satisfied with Condition A such as the case when υ_x is 10m/s, a more investigation is necessary. This will be reported in future.

IV. Three Dimensional Application

In order to understand the stability of the solution clearly, 3-D flux and eddy current distributions are investigated. The 2-D model shown in Fig.3 is expanded to 3-D one. The plan view is shown in Fig.7. The flux and eddy current distributions at high speed (υ_x=100m/s) obtained by using both moving and fixed coordinate systems are shown in Fig.8. The minimum value of the Peclet number is 6. A large oscillation is caused in the results obtained by using the fixed coordinate system, but the solution obtained by the proposed method using the moving coordinate system is stable.

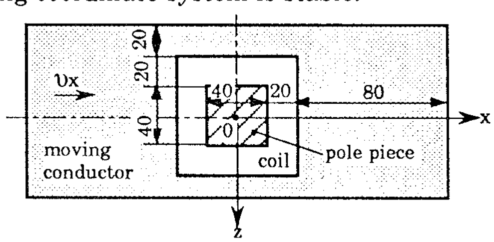

Fig. 7. 3-D analyzed model (plan view).

V. Conclusions

The method for linear analysis of the ac steady state eddy currents using the moving coordinate system, which is a combination of the phasor and the finite difference methods, is proposed. The proposed method is compared with the ordinary fixed coordinate system. The results obtained are summarized as follows :

(1) The solution obtained by the proposed method is stable, even if the speed of the moving conductor is very high. Therefore, the proposed method is effective for analysis of a high speed moving conductor.

(2) For low speed at which the Peclet number is less than unity, the fixed coordinate system should be used.

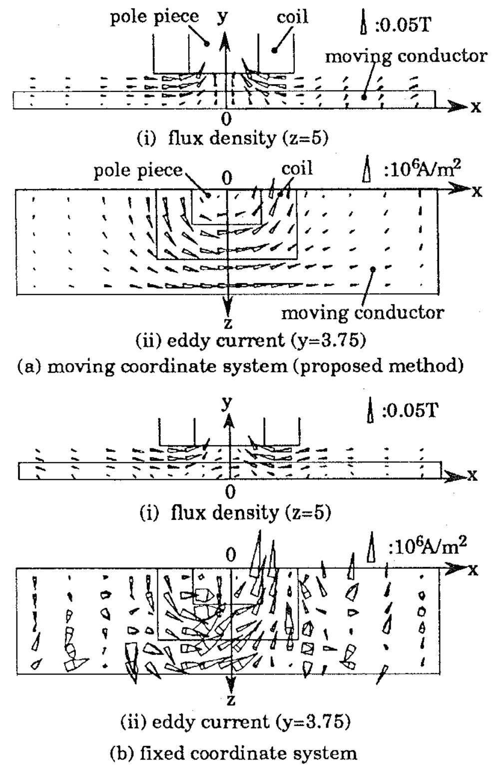

Fig.8. Comparison of flux and eddy current distributions between moving and fixed coordinate systems (υ_x=100m/s, ωt=0°).

(3) The applicable extent of the moving coordinate system is clearly shown in this paper.

The comparison of calculation and measurement of 3-D model will be reported in future.

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