Physical meaning of gradФ in eddy current analysisusing magnetic vector potential

N. Nakata
Okayama University

Norio Takahashi
Okayama University

K. Fujiwara
Okayama University

PHYSICAL MEANING OF gradϕ IN EDDY CURRENT ANALYSIS USING MAGNETIC VECTOR POTENTIALS

T. Nakata, N. Takahashi and K. Fujiwara

ABSTRACT

In the A-ϕ method, a gradϕ term (ϕ : electric scalar potential) can be neglected in some cases. In order to reduce the computing time, a physical meaning of the gradϕ term in eddy current analysis should be investigated.

The relationship between eddy current distribution and gradϕ, and the effects of boundary conditions on gradϕ are examined through several 2-D and 3-D examples. It is shown that gradϕ in 2-D analysis is a constant to modify the interlinkage flux of the conductor which is denoted by the magnetic vector potential A.

1. INTRODUCTION

In eddy current analysis using the magnetic vector potential, an electric field, namely a gradϕ term plays an important role[1,2]. This gradϕ is unnecessary under some conditions in 2-D analysis. If properties of gradϕ are clarified, a standard to judge the necessity of gradϕ can be established. Then, the computing time can be reduced in some cases. The gradϕ in 2-D analysis has been understood as a so-called mean vector potential (see Section 4) [2]. This explanation, however, is difficult to expand to 3-D analysis.

In this paper, the properties of gradϕ are examined, and the physical meaning of the gradϕ is clarified through some examples. A standard to judge the necessity of gradϕ is established from the study of gradϕ. It is shown that gradϕ does not correspond to the mean vector potential.

2. gradϕ

2.1 Introduction of gradϕ

From Faraday's law and the definition of the magnetic vector potential $\mathbb{A}$, the following equation can be obtained.

$$\mathrm{rot}(\mathbb{E}+\frac{\partial \mathbf{A}}{\partial t}) = 0 \tag{1}$$

where $\mathbb{E}$ is the electric field strength. Equation (1) implies the existence of a scalar potential ϕ, in terms of which

$$\mathbb{E} = -\frac{\partial \mathbf{A}}{\partial t} - \mathrm{grad}\,\phi \tag{2}$$

From Eq.(2), the eddy current density $\mathbb{J}e$ can be denoted as follows:

$$\mathbf{J}e = -\sigma\frac{\partial \mathbf{A}}{\partial t} - \sigma\,\mathrm{grad}\,\phi \tag{3}$$

where σ is the conductivity.

2.2 Calculation method of gradϕ

The basic equation for the 3-D eddy current analysis is written as follows:

$$\mathrm{rot}(\nu\,\mathrm{rot}\mathbf{A}) = \mathbf{J}o - \sigma\frac{\partial \mathbf{A}}{\partial t} - \sigma\,\mathrm{grad}\,\phi \tag{4}$$

where $\mathbb{J}o$ and ν are the magnetizing current density and the reluctivity respectively. From the equation of continuity of current and Eq.(3), the following equation is obtained.

$$\mathrm{div}(\mathbf{J}o - \sigma\frac{\partial \mathbf{A}}{\partial t} - \sigma\,\mathrm{grad}\,\phi) = 0 \tag{5}$$

If the vector potential $\mathbb{A}$ and the gradϕ are treated as independent unknown variables, they can be directly calculated by solving Eqs.(4) and (5) simultaneously[3].

3. PROPERTIES OF gradϕ

In this Section, the various characteristics of gradϕ are examined in order to clarify the physical meaning of gradϕ.

3.1 Analyzed model

Eddy currents in two parallel conductors placed in a transient magnetic field shown in Fig.1 are calculated. The magnetic field is uniform and perpendicular to the conductors. The applied transient field is a step function of which the flux density is 1.5(T). The conductivity σ of the conductor is 3.54×10^7(S/m). Only one-eighth of the region is analyzed because of symmetry.

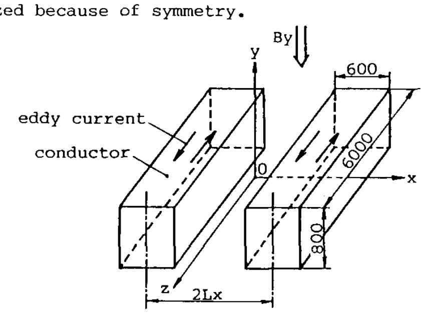


Fig.1 Analyzed model.

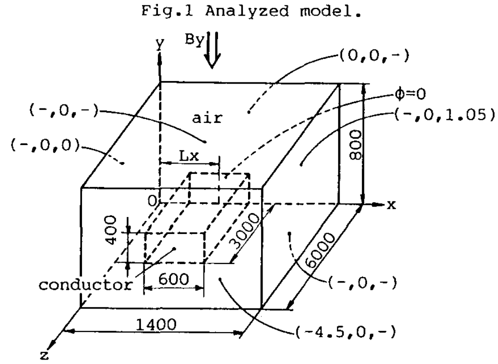


Fig.2 Boundary conditions.

Figure 2 shows the boundary conditions[4] of the analyzed region. The number in the parenthesis in the Figure denotes each component (Ax, Ay, Az). (-) means that this component is unknown. Vector potentials at the boundaries x=1400(mm) and z=6000(mm) are obtained from the following equation[4], under the assumption that the uniform magnetic field is produced by a solenoid as shown in Fig.3.

$$\oint \mathbf{A}\cdot d\,\mathbf{s} = \Phi \tag{6}$$

where Φ is the prescribed flux passing through the boundary plane a-b-c-d-a, and $\mathbf{s}$ is the unit tangential vector along the circumference of the boundary surface.

The authors are with the Department of Electrical Engineering, Okayama University, Okayama 700, Japan.

0018-9464/88/0100-0178$01.00©1988 IEEE

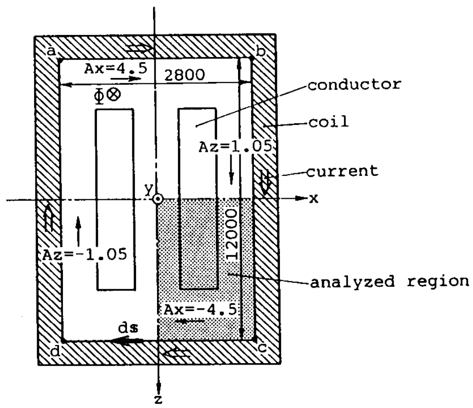


Fig.3 Relationship between excitation and boundary condition.

3.2 Relationship between the eddy current distribution and gradϕ

Figure 4 shows the distributions of Je,$-\sigma\partial A/\partial t$, $-\sigma$gradϕ and ϕ at the instant t=1(msec.). Je in Fig.4(a) is magnified 100 times as large as $-\sigma\partial A/\partial t$and $-\sigma$gardϕ in Figs.4(b) and (c).

$-\sigma\partial A/\partial t$ and $-\sigma$gradϕ are nearly uniform in the z-direction, and the equi-potential line of ϕ is parallel to the x-y plane as shown in Fig.4. This means that, in 2-D analysis, gradϕ is constant in one conductor on the analyzed x-y plane. Such a property of gradϕ in 2-D analysis can be easily proved[5].

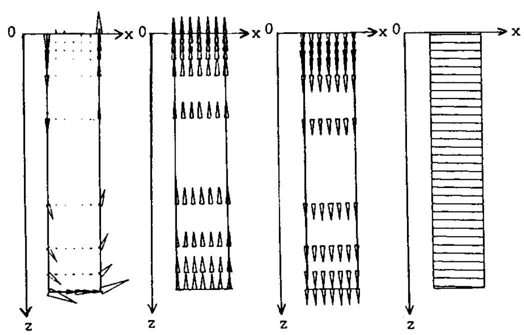


Fig.4 Je, $-\sigma\partial A/\partial t$, $-\sigma$gradϕ and ϕ at y=400(mm) (Lx=700(mm),t=1(msec.)).

3.3 Effects of the distance of the conductor from the reference plane on gradϕ

The effects of the distance Lx from the center line of the conductor to the reference plane (y-z plane in Fig.2 : on this plane, Ay=Az=0) on gradϕ are investigated.

Figure 5 shows that $-\sigma\partial A/\partial t$ and $-\sigma$gradϕ vary with the distance Lx. Je, $-\sigma\partial A/\partial t$ and $-\sigma$gradϕ in this Figure are all z-components of them. The dashed lines denote the mean values of σgradϕ along the line e-f on the surface of the conductor. Although $-\sigma\partial A/\partial t$ and $-\sigma$gradϕ vary with Lx, the sum of them, which is equal to Je in Eq.(3), is constant. Figure 6 denotes the mean value of σgradϕ along the line e-f. Figures 5 and 6 suggest that σgradϕ is increased with Lx, because Je cannot be represented by only $\sigma\partial A/\partial t$. The same results are obtained in 2-D analysis. gradϕ is a constant to adjust $\sigma\partial A/\partial t$.

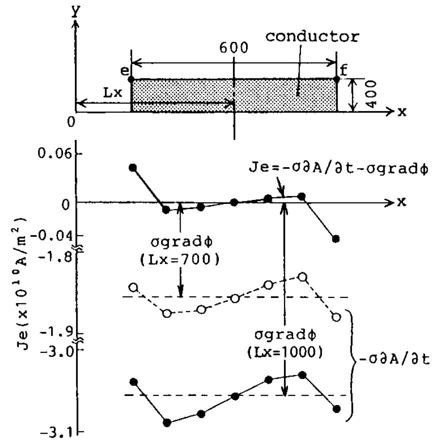


Fig.5 Effect of distance Lx on σgradϕ at y=400(mm),z=0(mm) (t=1(msec.)).

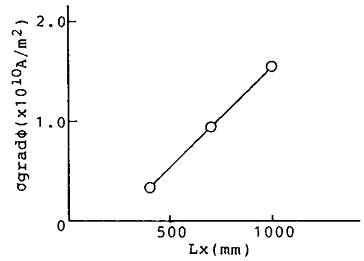


Fig.6 Relationship between σgradϕ and distance Lx at y=400(mm),z=0(mm) (t=1(msec.)).

3.4 Effects of boundary conditions

Distributions of σgradϕ is examined under two kinds of boundary conditions shown in Figs.3 and 7. The magnetic field in Fig.7 is produced by two infinitely long parallel conductors. Obtained eddy current distributions are the same for these two boundary conditions. Figure 8 shows the effects of boundary conditions on the mean values of σgradϕ along the line e-f in Fig.5. The σgradϕ is increased when the z-component of A on the boundary is increased as denoted in Fig.7.

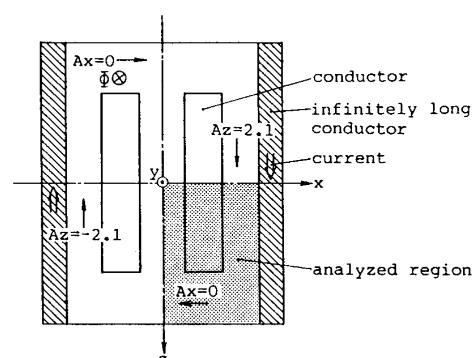


Fig.7 Excitation by infinitely long parallel two conductors.

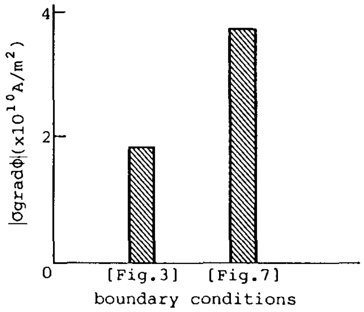
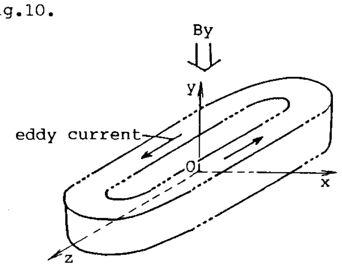


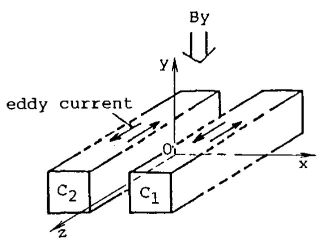
Fig.8 Effects of boundary conditions on average σgradφ along the line e-f (L_x=700(mm), t=1(msec.)).

3.5 Effects of $\partial\phi/\partial z$ on the eddy current distribution in 2-D analysis

The eddy current distributions in two infinitely long conductors placed in a uniform magnetic field, which is the same as that in Fig.1, are analyzed using 2-D finite element method. There are two kinds of arrangements of conductors as shown in Fig.9. The model in Fig.9(b) corresponds to that in Fig.1. The 2-D constructions for Figs.9(a) and (b) are the same as shown in Fig.10.

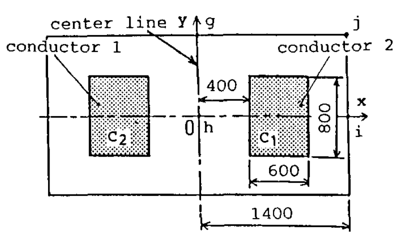


(a) two parallel conductors which are connected each other



(b) two parallel conductors which are not connected each other

Fig.9 Configurations of conductors and routes of eddy currents.

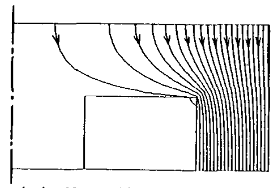


g-h: Dirichlet boundary (A=0)
i-j: Dirichlet boundary (A=2.1)
g-j,h-i: Neumann boundaries

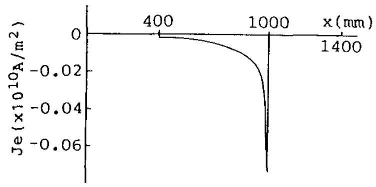
Fig.10 Sectional view of analyzed model.

The flux and eddy current distributions in Fig.9(a) are different from those of Fig.9(b) as shown in Figs.11 and 12. In the case of Fig.9(a), $\partial\phi/\partial z$ is zero. This means that $\partial\phi/\partial z$ in the ring conductor, which is symmetric with respect to the center line (y-axis), can be neglected by setting the vector potential along the center line to zero. In the case of Fig.9(b), however, $(\partial\phi/\partial z)_1$ and $(\partial\phi/\partial z)_2$ in conductors C_1 and C_2 are not equal to zero.

The flux distribution and eddy current distribution are very much changed by $\partial\phi/\partial z$, even if the cross-sectional views are the same.

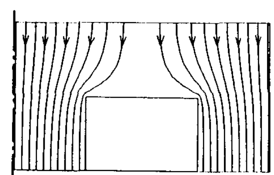


(a) flux distribution

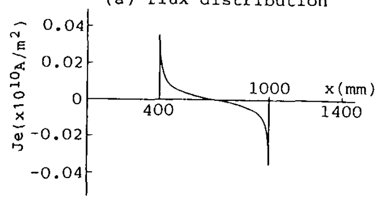


(b) eddy current distribution

Fig.11 Flux and eddy current distributions when the same two conductors placed symmetrically are connected each other ($\partial\phi/\partial z$ can be neglected, t=1(msec.)).



(a) flux distribution



(b) eddy current distribution

Fig.12 Flux and eddy current distributions when the same two conductors placed symmetrically are not connected each other ($\partial\phi/\partial z$ should be considered, t=1(msec.)).

4. PHYSICAL MEANING OF gradφ

From the properties of gradφ examined in Section 3, the physical meaning of gradφ is investigated.

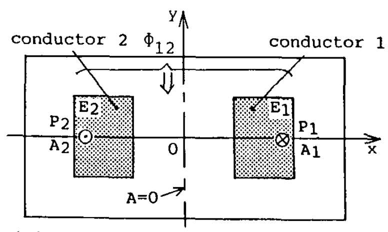
As the eddy current distributions are very much changed by gradφ($\partial\phi/\partial z$) as shown in Figs.11 and 12, the $\partial\phi/\partial z$ may have a role to adjust the distribution of $\sigma\partial A/\partial t$. Let us examine how the $\partial\phi/\partial z$ contributes to eddy current distribution in detail.

In the case of Fig.9(a), the eddy currents at the points P1 and P2, which are symmetric with respect to a center line of symmetry for the flux distribution (y-axis), are due to the flux Φ_{12} between P_1 and P_2 as shown in Fig.13(a). Where the center line of symmetry for the flux distribution is defined as the line to which the amplitudes and the directions of flux densities are the same at the two points P_1 and P_2 which are symmetric as shown in Fig.14. As the vector potential along the center line is zero as shown in Fig.13(a), the z-component of the vector potential A_1 at the point P_1 corresponds to $\Phi_{12}/2$. Electric fields E_1 and E_2 at P_1 and P_2, which produce the eddy currents, are given by

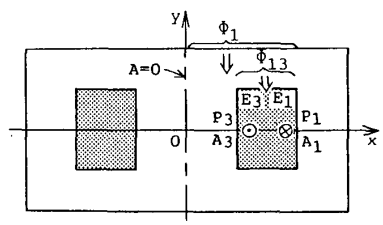
$$E_1 = -\frac{\partial}{\partial t}\left(\frac{\Phi_{12}}{2}\right) = -\frac{\partial A_1}{\partial t} \tag{7}$$

$$E_2 = \frac{\partial}{\partial t}\left(\frac{\Phi_{12}}{2}\right) = \frac{\partial A_2}{\partial t} \tag{8}$$

where A_2 is the z-component of the vector potential at the point P_2. Equations (7) and (8) denote that the eddy currents at P_1 and P_2 can be represented by A_1 and A_2 respectively. Therefore, the correction term $\partial\phi/\partial z$ is not necessary in the case of Fig.9(a).



(a) two parallel conductors which are connected each other



(b) two parallel conductors which are not connected each other

Fig.13 Relationships among fluxes, vector potentials and electric fields.

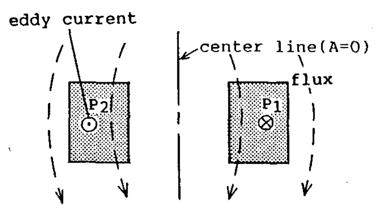


Fig.14 Explanation of a center line of symmetry for the flux distribution.

In the case of Fig.9(b), the eddy currents are due to the flux Φ_1 between P_1 and P_3 as shown in Fig.13(b), and the electric fields E_1 and E_3 are given by the following equations:

$$E_1 = -\frac{\partial}{\partial t}\left(\frac{\Phi_{13}}{2}\right) = -\frac{\partial}{\partial t}\left(\frac{A_1 - A_3}{2}\right) = -\left\{\frac{\partial A_1}{\partial t} + \frac{\partial}{\partial t}\left(-\frac{A_1 + A_3}{2}\right)\right\} \tag{9}$$

$$E_3 = \frac{\partial}{\partial t}\left(\frac{\Phi_{13}}{2}\right) = \frac{\partial}{\partial t}\left(\frac{A_1 - A_3}{2}\right) = -\left\{\frac{\partial A_3}{\partial t} + \frac{\partial}{\partial t}\left(-\frac{A_1 + A_3}{2}\right)\right\} \tag{10}$$

In this case, the eddy current at the point P_1 cannot be represented by only the vector potential A_1 (corresponding to the flux Φ_1 between the line of A=0 and the point P_1), because the center line of symmetry for the flux distribution does not coincide with the line of A=0. Therefore, the correction term $\partial\{-(A_1+A_3)/2\}/\partial t$ is introduced in Eqs.(9) and (10). This term is equal to gradϕ. Though gradϕ corresponds to the mean vector potential in the ac field, it cannot be explained by the concept of the mean vector potential in the ac-dc superimposed field.

As $(A_1+A_3)/2$ in Eqs.(9) and (10) vary with the distance Lx from the the center line (A=0), the gradϕ is changed by Lx as shown in Figs.5 and 6.

From the above-mentioned study, it can be concluded that if there is the center line of symmetry for the flux distribution, $\partial\phi/\partial z$ can be neglected by setting the vector potential along the center line to zero.

5. CONCLUSIONS

The obtained results can be summarized as follows:
(1) The eddy current density at a point cannot be calculated by only the vector potential at the point, because the vector potential at the point does not directly correspond to the interlinkage flux. Therefore, gradϕ is a correction term to modify the interlinkage flux of the conductor which is denoted by the vector potential.
(2) If a center line of symmetry for the flux distribution exists, gradϕ can be neglected by setting the vector potential along the center line to zero. Otherwise gradϕ should be considered.

By using skillfully the detailed knowledge of gradϕ, the computing time can be reduced.

Though, the physical meaning of gradϕ is discussed here mainly in 2-D analysis, further investigations of gradϕ in 3-D analysis will be reported in the other paper.

REFERENCES

[1] T.Nakata & Y.Kawase: "Finite Element Analysis of the Magnetic Characteristics in Straight Overlap Joints of Laminated Cores", Trans. of IEE, Japan, 103-B, 5, 357 (1983).
[2] T.Sato, Y.Inoue & S.Saito: "Solution of Magnetic Field, Eddy Current and Circulating Current Problems, Taking Magnetic Saturation and Effect of Eddy Current and Circulating Current Paths into Account", IEEE Winter Meeting, A 77-168 (1977).
[3] T.Nakata, N.Takahashi & K.Fujiwara: "Efficient Solving Techniques of Matrix Equations for Finite Element Analysis of Eddy Currents", Compumag Conference, Graz (1987).
[4] T.Nakata, N.Takahashi, K.Fujiwara, M.Miura & Y.Okada: "Boundary Conditions for Finite Element Analysis of 3-D Magnetic Fields", Proceedings of International Workshop for Eddy Current Code Comparison, 219, Tokyo (1986).
[5] T.Nakata, N.Takahashi & Y.Kawase: "Physical Meaning of Electric Field (gradϕ) in Eddy Current Analysis", Papers of Technical Meeting on Information Processing, IEE of Japan, IP-80-49 (1980).