Techniques for boundary element analysis of three-dimensional eddy current distribution

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TECHNIQUES FOR BOUNDARY ELEMENT ANALYSIS OF THREE-DIMENSIONAL EDDY CURRENT DISTRIBUTION

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Abstract - This paper describes an analysis of three-dimensional eddy current distribution by a boundary element method using field vector variables. In the boundary element method, a triangular element is used as a boundary element. An electric field vector and a magnetic flux density vector are defined as unknown vectors and are assumed to be constant vectors on each triangular element. For forming simultaneous equations, the computation point on the triangular element is set at the null point, where the triangular element itself doesn't induce tangential components of the electric field and the magnetic flux density.

INTRODUCTION

Boundary integral methods, which are known as boundary element method and integral equation method, are often used for the analysis of three-dimensional electric field and magnetic field because of their good applicability. Authors have proposed a boundary element method using vector variables[1]. In this paper, new techniques of the boundary element method for three-dimensional eddy current problems is presented.

The formulation of the boundary element method for the analysis of eddy current distribution is based on Maxwell's equations and is performed by the use of vector Green's theorem[2]. In the boundary element method, the boundary surfaces are divided into a number of triangular elements. Electric field vectors and magnetic flux density vectors are used as unknown vector variables in the boundary element method. And simultaneous equations are formed by evaluating boundary integrals.

Lastly, the boundary element method is applied to the conducting hollow sphere model and the conducting sphere model.

FORMULATION

Maxwell's equations for sinusoidal time dependence are given by

$$\nabla\times\vec{E} = -j\omega\vec{B} \qquad (1)$$

$$\nabla\times\vec{B} = j\omega\mu\varepsilon^*\vec{E} + \mu\vec{J} \qquad (2)$$

$$\nabla\cdot\vec{E} = 0 \qquad (3)$$

$$\nabla\cdot\vec{B} = 0 \qquad (4)$$

where $\vec{E}$ is the electric field
$\vec{B}$ is the magnetic flux density
$\vec{J}$ is the source current density
μ is the permeability
ε^* is the complex permittivity
ω is the angular frequency
j is the complex operator

ε^* in Eq. (2) is a complex number given by

$$\varepsilon^* = \varepsilon + \frac{\sigma}{j\omega} \qquad (5)$$

where ε is the permittivity
σ is the conductivity

Accordingly, the term of eddy current density, $\sigma\vec{E}$, appears on right side of Eq. (2) and the eddy current density, $\vec{J}_e$, in conductor is given by

$$\vec{J}_e = \sigma\vec{E} \qquad (6)$$

By the use of the vector Green's theorem[2], electric field, $\vec{E}_i$, and magnetic flux density, $\vec{B}_i$, in the region, V, which is enclosed by the surface, S, are obtained as follows:

$$\frac{\Omega_i}{4\pi}\vec{E}_i = -\int_V j\omega\mu\vec{J}\,\phi\,dv + \int_S\{j\omega(\vec{n}\times\vec{B})\phi - (\vec{n}\times\vec{E})\times\nabla\phi - (\vec{n}\cdot\vec{E})\nabla\phi\}\,dS \qquad (7)$$

$$\frac{\Omega_i}{4\pi}\vec{B}_i = \int_V \mu\vec{J}\times\nabla\phi\,dv - \int_S\{j\omega\mu\varepsilon^*(\vec{n}\times\vec{E})\phi + (\vec{n}\times\vec{B})\times\nabla\phi + (\vec{n}\cdot\vec{B})\nabla\phi\}dS \qquad (8)$$

where Ω_i is the solid angle at computation point, i, $\vec{n}$ is the unit normal vector at source point, and ϕ is the fundamental solution. And ϕ and $\nabla\phi$ are given by

$$\phi = \frac{\exp(-jkr)}{4\pi r} \qquad (9)$$

$$\nabla\phi = \frac{(-1-jkr)\vec{r}\exp(-jkr)}{4\pi r^3} \qquad (10)$$

where r is the distance between a source point and a computation point. And k is given by

$$k = \omega\sqrt{\mu\varepsilon^*} = \omega\sqrt{\mu(\varepsilon - j\sigma/\omega)} \qquad (11)$$

The boundary conditions for electric field and magnetic flux density on the boundary surface, S, which is the boundary between material 1 and material 2, are given by

$$\varepsilon_1\vec{E}_{i1}\cdot\vec{n} = \varepsilon_2\vec{E}_{i2}\cdot\vec{n},$$
$$\vec{E}_{i1}\cdot\vec{t}_u = \vec{E}_{i2}\cdot\vec{t}_u, \quad \vec{E}_{i1}\cdot\vec{t}_v = \vec{E}_{i2}\cdot\vec{t}_v \qquad (12)$$

$$\vec{B}_{i1}\cdot\vec{n} = \vec{B}_{i2}\cdot\vec{n},$$
$$\vec{B}_{i1}\cdot\vec{t}_u/\mu_1 = \vec{B}_{i2}\cdot\vec{t}_u/\mu_2, \quad \vec{B}_{i1}\cdot\vec{t}_v/\mu_1 = \vec{B}_{i2}\cdot\vec{t}_v/\mu_2 \qquad (13)$$

where $\vec{t}_u$ and $\vec{t}_v$ are the unit tangential vectors which intersect perpendicularly to each other.

The Boundary surface, S, is divided into a number of triangular elements, and a electric field vector and a magnetic flux density vector are assumed to be constant vectors on each triangular element.

Therefore the electric field $\vec{E}^e$ and the magnetic flux density $\vec{B}^e$ on the triangular element, e, are

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defined by constant vectors as follows:

$$\vec{E}^e = \vec{E}_i \quad (14)$$

$$\vec{B}^e = \vec{B}_i \quad (15)$$

where i is the computation point on the triangular element.

For numerical computation of the electric field vectors and the magnetic flux density vectors, the final simultaneous equations are formed by using Eqs. (7) and (8). After introducing boundary conditions for electric field and magnetic flux density, the unknown electric field vectors and unknown magnetic flux density are determined[1].

INTEGRATION OF FUNDAMENTAL SOLUTION

In Eqs. (7) and (8), the integrals to be evaluated for the triangular element, e, are given by

$$I_1 = \int_e \phi \, dS \quad (16)$$

$$\vec{I}_2 = \int_e \nabla\phi \, dS \quad (17)$$

When the computation point is on the element, e, I_1 and $\vec{I}_2$ include a singular point at r=0. In proposed method, the field point on the element, e, for forming the simultaneous equations is set at the null point, where $\vec{I}_2$ becomes zero vector. The coordinates of the null point are obtained as the solutions of the following simultaneous equations[3];

$$\left.\begin{array}{l}(I_2)_u = 0 \\ (I_2)_v = 0\end{array}\right\} \quad (18)$$

where $(I_2)_u$ and $(I_2)_v$ are the tangential components of $\vec{I}_2$ which intersect perpendicularly to each other. The solutions for Eq. (14) can be obtained easily by using Newton-Raphson method. When approximating $\exp(-jkr)\simeq 1$, I_1 at the null point, N, is obtained analytically by

$$I_1 = \frac{l_1}{2}\ln\frac{(1+\sin\theta_1)(1+\sin\theta_2)}{(1-\sin\theta_1)(1-\sin\theta_2)} + \frac{l_2}{2}\ln\frac{(1+\sin\theta_3)(1+\sin\theta_4)}{(1-\sin\theta_3)(1-\sin\theta_4)} + \frac{l_3}{2}\ln\frac{(1+\sin\theta_5)(1+\sin\theta_6)}{(1-\sin\theta_5)(1-\sin\theta_6)} \quad (19)$$

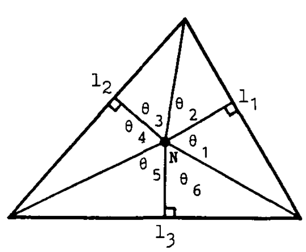

Fig. 1 A triangular element.

where l_1, l_2, l_3 are the lengths of the sides of the triangular element, and θ_1, ... , θ_5 and θ_6 are the angles shown in Fig. 1.

I_1 and $\vec{I}_2$ without singular point are computed by numerical integrations using the Gaussian quadrature formula with seven sampling points.

COMPUTATION RESULTS

In order to examine the accuracy of proposed method, a conducting hollow sphere model[4] and a conducting sphere model were chosen as three-dimensional eddy current problems.

Conducting hollow sphere model

The hollow sphere model in a uniform alternating magnetic field is shown in Fig. 2. The number of triangular elements in a twenty-fourth part of the spheres is 50. The computation results of eddy current density and magnetic flux density almost agree with analytical solutions[5] as shown in Fig. 3 and Fig. 4. And the computation results of eddy current density vectors and magnetic flux density vectors are shown in Fig. 5 and 6, respectively.

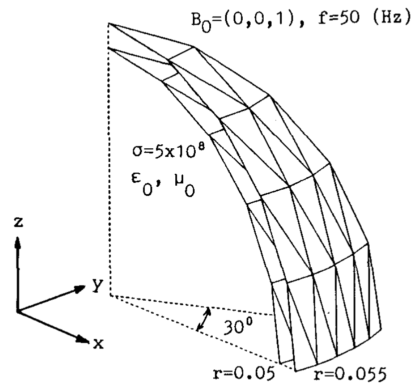

Fig. 2 A conducting hollow sphere model.

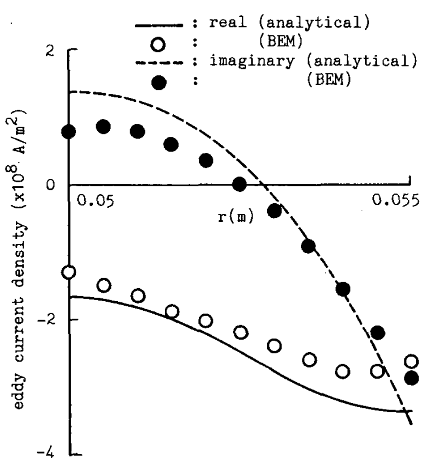

Fig. 3 Distributions of eddy current density in the hollow sphere, on x-y plane.

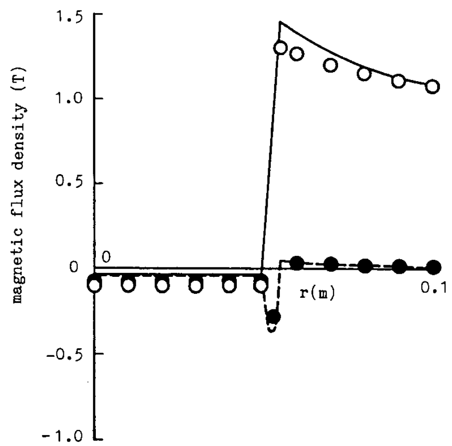

Fig. 4 Distributions of magnetic flux density in the hollow sphere, on x-y plane.

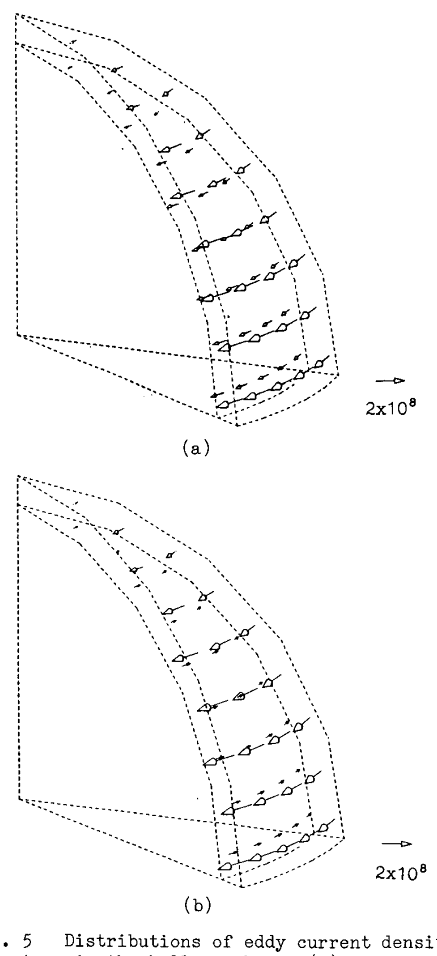

Fig. 5 Distributions of eddy current density vectors in the hollow sphere, (a) real part, (b) imaginary part.

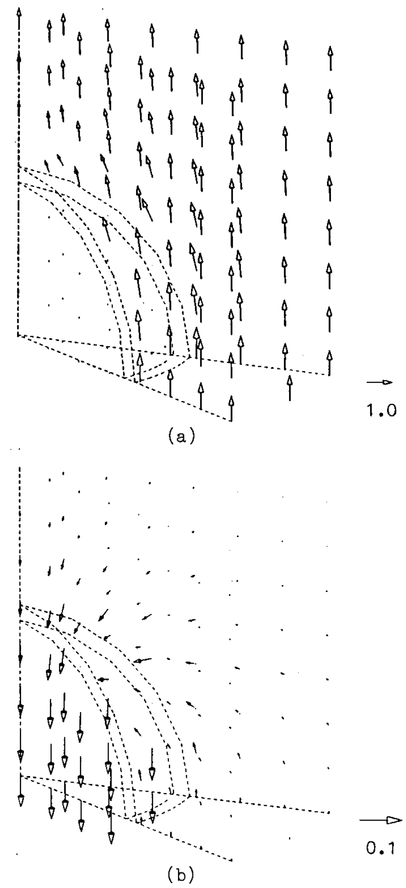

Fig. 6 Distributions of magnetic flux density vectors in the hollow sphere model, (a) real part, (b) imaginary part.

Conducting sphere model

In the conducting sphere model, the sphere has the same radius as the outer sphere of the conducting hollow sphere model. And a uniform alternating magnetic field is impressed. Figure 7 shows the conducting sphere model. The number of triangular elements in an eighth part of the sphere is 36. The computation results of eddy current distributions and magnetic flux distributions are shown in Fig. 8. The computed eddy current densities agree with analytical solutions[6].

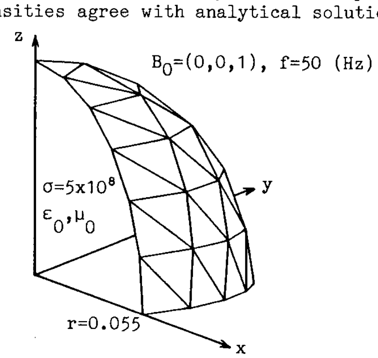

Fig. 7 A conducting sphere model.

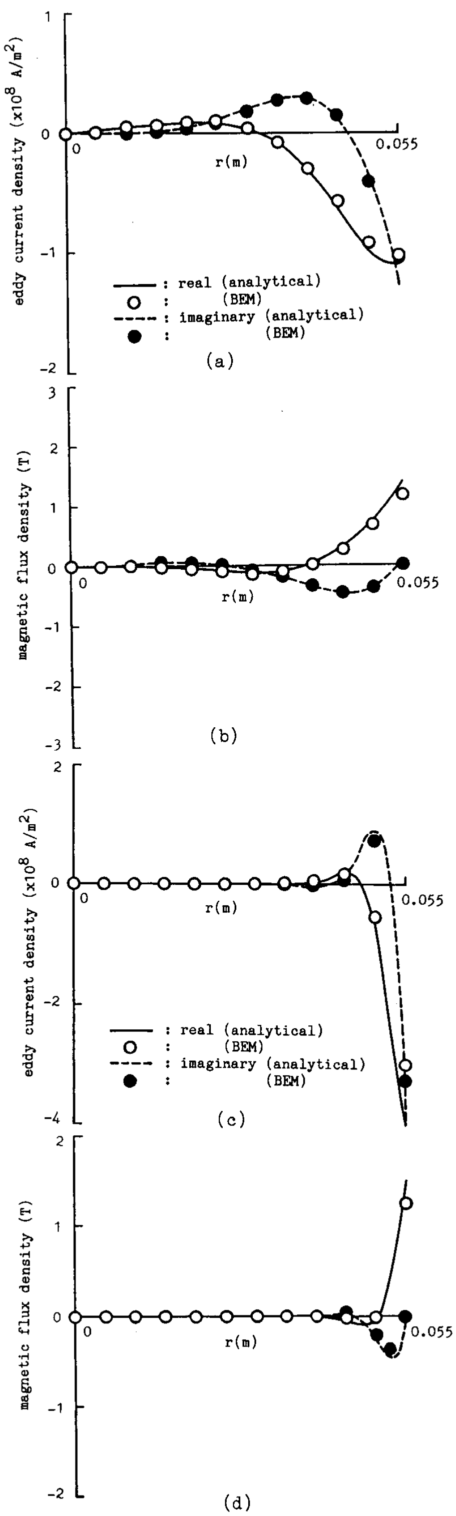

Fig. 8 Distributions of eddy current density and magnetic flux density in the sphere model on x-y plane, (a) eddy current density, f=50 Hz, (b) magnetic flux density, f=50 Hz, (c) eddy current density, f=500 Hz, (d) magnetic flux density, f=500 Hz.

CONCLUSION

In this paper, the new techniques of the boundary element method for computing three-dimensional eddy current distributions were described.

The techniques are concerned with the evaluation of the boundary integral and can be summarized as follows:

(1) In the boundary element method, the boundary surfaces are divided into a number of triangular elements on which electric field and magnetic flux density are assumed to be constant.

(2) For forming simultaneous equations, the computation point on the triangular element is set at the null point, where the triangular element itself doesn't induce tangential components of the electric field and the magnetic flux density.

(3) Boundary integrals are evaluated by analytical solutions for the element with singular point and by numerical integrations for the element without singular point.

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