A state-space based design of generalized minimum variance controller equivalent to transfer-function based design

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A State-Space Based Design of Generalized Minimum Variance Controller Equivalent to Transfer-Function Based Design

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Abstract

This paper proposes a new generalized minimum variance controller(GMVC) using state-space approach. The controller consists of a state feedback and a reduced-order observer with poles at $z = 0$. A coprime factorization of the state-space based controller is also obtained. It is shown that the GMVC designed by state-space approach is equivalent to the GMVC given by solving Diophantine equations and polynomial approach. The equivalence is proved by comparing coprime factorizations of the two controllers. From the results of this paper, it may be possible to apply advanced design schemes given by state-space control theory to the design of GMVC.

1 Introduction

Generalized minimum variance control(GMVC)[1] is widely applied in industry, particularly in process industry as many as generalized predictive control(GPC). GMVC has the special feature that a control system designer can assign the closed-loop poles of the control system by selecting coefficients in a generalized output of GMVC, whereas GPC has not the feature. As for the design of GPC, state-space based methods are already proposed by many authors and advanced design schemes given by state-space control theory can be applied to the design of GPC[2][3][4]. So far, for the design of GMVC, there exist few methods based on state-space approach and most of the design methods of GMVC are given by polynomial approach.

This paper proposes a new design method for GMVC using state-space approach. The controller in this paper consists of a state feedback and a reduced-order observer having poles at $z = 0$. The controller is equivalent to the conventional GMVC designed by polynomial approach. The equivalence is shown by obtaining a coprime factorization of the controller in state-space form and comparing a coprime factorization of the controller in this paper and the factorization of the controller of conventional GMVC.

Using the design method obtained in this paper, the following extensions will be expected.

First, it may be possible to design a controller with an observer having poles close to 1, which is robust to measurement noise. Second, authors have already proposed a strongly stable GMVC by using polynomial coprime factorization of a controller of GMVC[5][6]. However, by polynomial approach, it was difficult to obtain a strongly stable GMVC for multivariable systems having different numbers of inputs and outputs. Extending the coprime factorization in state-space form in this paper to the multivariable case, a strongly stable GMVC will be obtained for such multivariable systems. Finally by using design scheme for GMVC based on state-space approach, it will be possible to apply advanced design methods in state space control theory to the design of GMVC.

Notations: z^{-1} denotes backward shift operator: $z^{-1}y(t) = y(t-1)$. A polynomial function and a rational function are distinguished by $A[z^{-1}]$ and $A(z^{-1})$.

2 Problem Statement and GMVC Design in Polynomial Approach

Consider a single-input single-output system given by

$$A[z^{-1}]y(t) = z^{-k_m}B[z^{-1}]u(t) + \xi(t) \tag{1}$$
$$t = 0, 1, 2, \cdots$$

where $u(t)$ is the input, $y(t)$ is the output, k_m is the time delay, $\xi(t)$ is a white Gaussian noise with zero mean. $A[z^{-1}]$ and $B[z^{-1}]$ are polynomials of order n and m, and $n > m$. For notational simplicity, we use coefficients of the terms higher than m in $B[z^{-1}]$ with value 0 ($b_k = 0,\ k = m+1, \cdots, n-1$). Then polynomials $A[z^{-1}]$ and $B[z^{-1}]$ are denoted by

$$A[z^{-1}] = 1 + a_1 z^{-1} + \cdots a_n z^{-n} \tag{2}$$

$$B[z^{-1}] = b_0 + b_1 z^{-1} + \cdots b_{n-1} z^{-(n-1)} \qquad (3)$$

On the system(1), the followings are assumed. The orders and the coefficients of $A[z^{-1}]$ and $B[z^{-1}]$ are known. The time delay k_m is known and for simplicity, it is assumed that $k_m = 1$. The polynomials $A[z^{-1}]$ and $B[z^{-1}]$ are coprime.

The control objective is that the output $y(t)$ has a desirable response to the reference input $r(t)$. To this objective, the generalized minimum variance control(GMVC) given by Clarke and others[1] designs a controller to minimize the following variance of a generalized output

$$J = E[\Phi(t+1)^2] \qquad (4)$$

where $\Phi(t+1)$ is a generalized output;

$$\Phi(t+1) = P[z^{-1}]y(t+1) + Q[z^{-1}]u(t) - R[z^{-1}]r(t) \qquad (5)$$

and $P[z^{-1}], Q[z^{-1}]$ and $R[z^{-1}]$ are polynomials given by a controller designer with degrees of n_p, n_q, n_r. For notational simplicity, assuming that $n_p \leq n$, $n_q \leq n$ and that $p_k = 0$, $n_p+1 \leq k \leq n$, and $q_h = 0$, $n_q+1 \leq h \leq n$, polynomials $P[z^{-1}]$ and $Q[z^{-1}]$ are described as

$$P[z^{-1}] = 1 + p_1 z^{-1} + \cdots + p_n z^{-n} \qquad (6)$$

$$Q[z^{-1}] = q_0 + q_1 z^{-1} + \cdots + q_n z^{-n} \qquad (7)$$

Usually these polynomials are selected to obtain desirable stable closed-loop poles.

In the generalized minimum variance control law, two Diophantine equations

$$P[z^{-1}] = A[z^{-1}]E[z^{-1}] + z^{-1}F[z^{-1}] \qquad (8)$$

$$G[z^{-1}] = E[z^{-1}]B[z^{-1}] + Q[z^{-1}] \qquad (9)$$

are solved. As time delay k_m is assumed to be $k_m = 1$, then $E[z^{-1}] = 1$ and $F[z^{-1}]$ is

$$F[z^{-1}] = f_0 + f_1 z^{-1} + \cdots + f_{n-1} z^{-(n-1)} \qquad (10)$$

Then, the generalized output $\Phi(t+1)$ and its prediction $\hat{\Phi}(t+1|t)$ are given[1] by

$$\Phi(t+1) = \hat{\Phi}(t+1|t) + E[z^{-1}]\xi(t+1) \qquad (11)$$

$$\hat{\Phi}(t+1|t) = F[z^{-1}]y(t) + G[z^{-1}]u(t) - R[z^{-1}]r(t) \qquad (12)$$

Since the estimate $\hat{\Phi}(t+1|t)$ and the noise term $E[z^{-1}]\xi(t+1)$ have no correlation with each other, the control $u(t)$ to minimize the variance J is obtained by choosing $u(t)$ to make

$$\hat{\Phi}(t+1|t) = 0 \qquad (13)$$

From equation(12) and (13), the control law is

$$u(t) = \frac{R[z^{-1}]}{G[z^{-1}]}r(t) - \frac{F[z^{-1}]}{G[z^{-1}]}y(t) \qquad (14)$$

Substituting the control law(14) into the system (1), the closed-loop system is obtained as

$$y(t) = \frac{z^{-1}B[z^{-1}]R[z^{-1}]}{T[z^{-1}]}w(t) + \frac{G[z^{-1}]}{T[z^{-1}]}\xi(t) \qquad (15)$$

$$T[z^{-1}] = P[z^{-1}]B[z^{-1}] + Q[z^{-1}]A[^{-1}] \qquad (16)$$

3 Coprime Factorization of GMVC

In this section, coprime factorization of the controller (14) of GMVC is defined in order to show the equivalence of the controller (14) in polynomial form and a controller based on state-space approach proposed in the next section.

For the coprime factorization approach, consider the family of stable rational functions:

$$RH_\infty = \{G(z^{-1}) = \frac{G_n[z^{-1}]}{G_d[z^{-1}]}, \quad G_d[z^{-1}] : \text{stable polynomial}\} \qquad (17)$$

Remark For discrete-time systems, since poles given by $G_d[z^{-1}] = 0$ are stable at $z = 0$, that is, $z^{-1} = \infty$, the condition that denominator is a stable polynomial is sufficient to define RH_∞. And the properness of rational functions is not necessary. Since RH_∞ is defined only by the condition that $G_d[z^{-1}]$ is stable and $G_d[z^{-1}] = 1$ is stable, rational functions having denominator $G_d[z^{-1}] = 1$, that is, polynomials are in RH_∞.

Transfer function is expressed by a ratio of rational function in RH_∞,

$$G(z^{-1}) = N(z^{-1})/D(z^{-1}) \qquad (18)$$

where $N(z^{-1})$, $D(z^{-1})$ are rational functions in RH_∞ and are coprime in each other. Then, the stabilizing two-degree-of-freedom compensator is given in coprimely factorized form[7]:

$$u(t) = Y(z^{-1})^{-1}K(z^{-1})r(t) - Y(z^{-1})^{-1}X(z^{-1})y(t) \qquad (19)$$

where $K(z^{-1})$ are rational functions in RH_∞ and is a design parameter. $X(z^{-1})$ and $Y(z^{-1})$ are also in RH_∞ and the solutions of the following Bezout equation;

$$X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1 \qquad (20)$$

The coprime factorization of compensator (19) is defined by $X(z^{-1})$ and $Y(z^{-1})$ in RH_∞ satisfying Bezout equation (20). In Bezout equation (20), $N(z^{-1})$ and $D(z^{-1})$ are defined by

$$N(z^{-1}) = z^{-1}B[z^{-1}]/T[z^{-1}] \in RH_\infty \qquad (21)$$

$$D(z^{-1}) = A[z^{-1}]/T[z^{-1}] \in RH_\infty \qquad (22)$$

when the polynomials $P[z^{-1}]$ and $Q[z^{-1}]$ are chosen for $T[z^{-1}]$ in (16) to be stable.

4 GMVC designed by state-space approach

The state equation of the observable canonical form of the plant(1) is given by

$$x(t+1) = A_p x(t) + b_p u(t) - a_p \xi(t) \quad (23)$$
$$y(t) = c_p x(t) + \xi(t) \quad (24)$$
$$A_p = \begin{bmatrix} -a_p & \begin{matrix} I_{n-1} \\ \\ o_{1\times(n-1)} \end{matrix} \end{bmatrix}, \; a_p = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$
$$b_p = \begin{bmatrix} b_0 \\ \vdots \\ b_{n-1} \end{bmatrix}, \; c_p = [1, \; o_{1\times(n-1)}]$$

A reduced-order ($(n-1)$-order) state-observer for plant (23) and (24) is

$$w_x(t+1) = D_w w_x(t) + f_x y(t) + g_x u(t) \quad (25)$$
$$\widehat{x}(t) = P_x w_x(t) + V_x y(t) \quad (26)$$
$$T_x A_p - D_w T_x = f_x c_p, \;\; P_x T_x + V_x c_p = I_n \quad (27)$$

Since the state equation(23) is in the observable canonical form, the coefficient matrices of the observer(25) are obtained as

$$D_w = \begin{bmatrix} -d & \begin{matrix} I_{n-2} \\ \\ o_{1\times(n-2)} \end{matrix} \end{bmatrix}, \; d = \begin{bmatrix} d_1 \\ \vdots \\ d_{n-1} \end{bmatrix}$$
$$a_{p2} = \begin{bmatrix} a_2 \\ \vdots \\ a_n \end{bmatrix}, \; d_2 = \begin{bmatrix} d_2 \\ \vdots \\ d_{n-1} \\ 0 \end{bmatrix}, \; V_x = \begin{bmatrix} 1 \\ d \end{bmatrix}$$
$$P_x = \begin{bmatrix} o_{1\times(n-1)} \\ I_{n-1} \end{bmatrix}, \; T_x = [-d \; I_{n-1}]$$
$$f_x = -a_{p2} - (-a_1 + d_1)d + d_2, \; g_x = T_x b_p$$

The characteristic polynomial of the observer is

$$D_w[z^{-1}] = 1 + d_1 z^{-1} + \cdots + d_{n-1} z^{-n+1} \quad (28)$$

Then a controller given by state feedback $u(t) = -L\widehat{x}(t)$ and observer (25) is coprimely factorized as in the next Lemma.

<u>**Lemma 1**</u>[7] The coprime factorization of a controller given by a state feedback $u(t) = -L\widehat{x}(t)$ and observer (25) is given by

$$u(t) = -\frac{X(z^{-1})}{Y(z^{-1})} y(t) \quad (29)$$

where

$$X(z^{-1}) = LV_x + LP_x(zI - D_w)^{-1} f_x \quad (30)$$
$$Y(z^{-1}) = 1 + LP_x(zI - D_w)^{-1} g_x \quad (31)$$

Consider the case that the generalized output $\Phi(t+1)$ is a simple one

$$\Phi(t+1) = y(t+1) - R[z^{-1}]r(t) \quad (32)$$

Then using this Lemma, a controller to minimize J is obtained in state-space form.

<u>**Lemma 2**</u> Given the polynomial $D_0[z^{-1}] = 1$ of the form (28), and let $E[z^{-1}]$, $F_0[z^{-1}]$ and $G_0[z^{-1}]$ be the solutions of the following Diophantine equations,

$$D_0[z^{-1}] = A[z^{-1}]E[z^{-1}] + z^{-1}F_0[z^{-1}] \quad (33)$$
$$G_0[z^{-1}] = g_0 + z^{-1}G_1[z^{-1}] = E[z^{-1}]B[z^{-1}] \quad (34)$$

Then the controller to make

$$\hat{\Phi}(t+1|t) = \hat{y}(t+1|t) - R[z^{-1}]r(t) = 0 \quad (35)$$

is given by

$$u(t) = \frac{R[z^{-1}]}{G_0[z^{-1}]} r(t) - \frac{F_0[z^{-1}]}{G_0[z^{-1}]} y(t) \quad (36)$$

and one of coprime factorization $Y(z^{-1})^{-1}X(z^{-1})$ of the compensator $F_0[z^{-1}]/G_0[z^{-1}]$ is

$$X(z^{-1}) = \frac{1}{g_0} F_0[z^{-1}] \quad (37)$$
$$Y(z^{-1}) = \frac{1}{g_0} G_0[z^{-1}] \quad (38)$$

In state space approach, an observer to give $\hat{y}(t+1|t)$ is observer (25) with the following coefficients:

$$D_0 = \begin{bmatrix} o_{(n-1)\times 1} & \begin{matrix} I_{n-2} \\ o_{1\times(n-2)} \end{matrix} \end{bmatrix}, \; d = o_{(n-1)\times 1} \quad (39)$$
$$f_x = -a_{p2}, \; g_0 L P_x = P_L = [1 \;\; o_{1\times(n-2)}] \quad (40)$$
$$g_0 L V_x = V_L = [-a_1], \; L = \frac{1}{g_0} c_p A_p \quad (41)$$

The controller (36) is given in state-space form as state feedback

$$u(t) = -L\widehat{x}(t) + \frac{1}{g_0} R[z^{-1}]r(t) \quad (42)$$

and observer (25). The controller is coprimely factored as

$$X(z^{-1}) = LV_x + LP_x(zI - D_0)^{-1} f_x \quad (43)$$
$$Y(z^{-1}) = 1 + LP_x(zI - D_0)^{-1} g_x \quad (44)$$

Then the following equations are obtained.

$$F_0[z^{-1}] = V_L + P_L(zI - D_0)^{-1} f_x \quad (45)$$
$$G_0[z^{-1}] = g_0 + P_L(zI - D_0)^{-1} g_x \quad (46)$$

<u>**Proof**</u> Using equations (33) and (34)

$$y(t+1) = F_0[z^{-1}]y(t) + G_0[z^{-1}]u(t) + E[z^{-1}]\xi(t+1) \quad (47)$$

Then an estimate of $y(t+1)$ is

$$\hat{y}(t+1|t) = F_0[z^{-1}]y(t) + G_0[z^{-1}]u(t) \quad (48)$$

From $\hat{y}(t+1|t) - R[z^{-1}]r(t) = 0$, the control law (36) is obtained.

In state equation (23), $\hat{y}(t+1|t)$ is given as

$$\hat{y}(t+1|t) = \boldsymbol{c}_p\hat{\boldsymbol{x}}(t+1|t) = \boldsymbol{c}_pA_p\hat{\boldsymbol{x}}(t) + g_0u(t) \quad (49)$$

$$\boldsymbol{c}_p\boldsymbol{b}_p = b_0 = g_0 \quad (50)$$

From $\hat{y}(t+1|t) - R[z^{-1}]r(t) = 0$, controller (36) for generalized output (32) is equivalent to

$$u(t) = -\frac{1}{g_0}\boldsymbol{c}_pA_p\hat{\boldsymbol{x}}(t) + \frac{1}{g_0}R[z^{-1}]r(t) \quad (51)$$

From Lemma 1, the control law which consists of the state feedback (42) and observer (25) with coefficients (39), (40) and (41) to estimate $\hat{x}(t)$ is factored as equations (43) and (44). Sets of equations (37), (38) and (43), (44) are the coprime factorizations of the same controller (36) or (51). A coprime factorization is unique except for multiplying by a unimodular function[7] and the factorizations (37), (38) and (43), (44) have the same denominator polynomial $D_0[z^{-1}] = z^{-n+1}\det(zI - D_0) = 1$. These facts imply the equivalence of equations, (37), (38) and (43), (44). Since

$$(zI - D_0)^{-1} = \begin{bmatrix} z^{-1} & -z^{-2} & \cdots & (-1)^{n+1}z^{-n} \\ & z^{-1} & \cdots & (-1)^{n}z^{-n+1} \\ & & \ddots & \vdots \\ & & & z^{-1} \end{bmatrix} \quad (52)$$

$X(z^{-1})$ and $Y(z^{-1})$ of (43) and (44) are polynomials. Then from (37) and (38), equations (45) and (46) hold.

To obtain a compensator for the generalized output (5), we split the output $\Phi(t+1)$ into three parts and the term with reference input $r(t)$,

$$\begin{aligned}\Phi(t+1) &= (1 + z^{-1}P_1[z^{-1}])y(t+1) + Q[z^{-1}]u(t) \\ &\quad -R[z^{-1}]r(t) \\ &= y(t+1) + y_1(t) + u_1(t) - R[z^{-1}]r(t) \quad (53)\end{aligned}$$

where

$$1 + z^{-1}P_1[z^{-1}] = P[z^{-1}] \quad (54)$$

$$P_1[z^{-1}] = p_1 + p_2z^{-1} + \cdots + p_nz^{-n} \quad (55)$$

$$y_1(t) = P_1[z^{-1}]y(t) \quad (56)$$

$$u_1(t) = Q[z^{-1}]u(t) \quad (57)$$

The estimate of $\Phi(t+1)$ is given by

$$\hat{\Phi}(t+1|t) = \hat{y}(t+1|t) + \hat{y}_1(t) + \hat{u}_1(t) - R[z^{-1}]r(t) \quad (58)$$

and we will obtain three observers to estimate $\hat{y}(t+1|t)$, $\hat{y}_1(t)$ and $\hat{u}_1(t)$.

Using observer (25) with (39), (40) and (41), the estimate of $\hat{y}(t+1|t)$ is

$$\hat{y}(t+1|t) = P_L\boldsymbol{w}_x(t) + V_Ly(t) + g_0u(t) \quad (59)$$

Observers with poles at $z = 0$ to estimate $y_1(t)$ and $u_1(t)$ of (56) and (57) are

$$\begin{aligned}\boldsymbol{w}_y(t+1) &= D_0\boldsymbol{w}_y(t) + \boldsymbol{f}_yy(t) \quad (60)\\ \hat{y}_1(t) &= P_L\boldsymbol{w}_y(t) + p_1y(t) \\ \boldsymbol{f}_y &= [p_2,\ p_3,\ \cdots,\ p_n]^T \\ \boldsymbol{w}_u(t+1) &= D_0\boldsymbol{w}_u(t) + \boldsymbol{q}_1u(t) \quad (61)\\ \hat{u}_1(t) &= P_L\boldsymbol{w}_u(t) + q_0u(t) \\ \boldsymbol{q}_1 &= [q_1,\ \cdots,\ q_{n-1}]^T\end{aligned}$$

These three observers are made into single observer

$$\boldsymbol{w}(t+1) = D_0\boldsymbol{w}(t) + (\boldsymbol{f}_x + \boldsymbol{f}_y)y(t) + (\boldsymbol{g}_x + \boldsymbol{q}_1)u(t) \quad (62)$$

$$\boldsymbol{w}(t) = \boldsymbol{w}_x(t) + \boldsymbol{w}_y(t) + \boldsymbol{w}_u(t) \quad (63)$$

and an estimate of $\Phi(t+1)$ is

$$\begin{aligned}\hat{\Phi}(t+1|t) &= P_L\boldsymbol{w}(t) + (V_L + p_1)y(t) + (g_0 + q_0)u(t) \\ &\quad -R[z^{-1}]r(t) \quad (64)\end{aligned}$$

Then a controller in state-space form to give (13) is obtained in the next theorem.

<u>**Theorem**</u> A minimum variance controller in state-space form to give (13) consists of observer (62) and controller

$$u(t) = \frac{1}{g_0 + q_0}\{R[z^{-1}]r(t) - P_L\boldsymbol{w}(t) - (V_L + p_1)y(t)\} \quad (65)$$

Coprime factorization of the controller is

$$X(z^{-1}) = \frac{1}{g_0 + q_0}\{V_L + p_1 + P_L(zI - D_0)^{-1}(\boldsymbol{f}_x + \boldsymbol{f}_y)\} \quad (66)$$

$$Y(z^{-1}) = 1 + \frac{1}{g_0 + q_0}P_L(zI - D_0)^{-1}(\boldsymbol{g}_x + \boldsymbol{q}_1) \quad (67)$$

The controller is equivalent to polynomial form (14).

<u>**Proof**</u> The equivalence of these controllers is shown as follows. Using solutions $F_0[z^{-1}]$ and $G_0[z^{-1}]$ of equations (33) and (34) and $P_1[z^{-1}]$ in equation (55), solutions $F[z^{-1}]$ and $G[z^{-1}]$ of Diophantine equations (8) and (9) are given as

$$F[z^{-1}] = F_0[z^{-1}] + P_1[z^{-1}] \quad (68)$$

$$G[z^{-1}] = G_0[z^{-1}] + Q[z^{-1}] \quad (69)$$

Then coprime factorization of compensator $F[z^{-1}]/G[z^{-1}]$ (19) is

$$X(z^{-1}) = \frac{1}{g_0 + q_0}(F_0[z^{-1}] + P_1[z^{-1}]) \quad (70)$$

$$Y(z^{-1}) = \frac{1}{g_0 + q_0}(G_0[z^{-1}] + Q[z^{-1}]) \quad (71)$$

In these equations, polynomials $F_0[z^{-1}]$ and $G_0[z^{-1}]$ are given by (45) and (46). From (56) and (61), polynomials $P_1[z^{-1}]$ and $Q[z^{-1}]$ are given by

$$P_1[z^{-1}] = p_1 + P_L(zI - D_0)^{-1} f_y \quad (72)$$

$$Q[z^{-1}] = q_0 + P_L(zI - D_0)^{-1} q_1 \quad (73)$$

Substituting these polynomials into (70) and (71), the state-space forms of $X(z^{-1})$ and $Y(z^{-1})$ are obtained by

$$X(z^{-1}) = \frac{1}{g_0 + q_0}[\{V_L + P_L(zI - D_0)^{-1} f_x\} + \{p_1 + P_L(zI - D_0)^{-1} f_y\}] \quad (74)$$

$$Y(z^{-1}) = \frac{1}{g_0 + q_0}[\{g_0 + P_L(zI - D_0)^{-1} g_x\} + \{q_0 + P_L(zI - D_0)^{-1} q_1\}] \quad (75)$$

which are equal to (66) and (71). Since (70) and (71) are coprime, (66) and (67) are also coprime. Using relations (68), (69), (45), (46), (72) and (73), the two controllers (65) in state-space form and (14) in polynomial form are same.

5 Example

In this section, an example is given to show the equivalence of a polynomial form and a state-space form of GMVC. Consider a plant (1) with $k_m = 1$ and

$$A[z^{-1}] = 1 + 0.7z^{-1} + 0.3z^{-2} + 0.1z^{-3} \quad (76)$$

$$B[z^{-1}] = 0.8 + 0.5z^{-1} + 0.1z^{-2} \quad (77)$$

A generalized output (4) is given by

$$P[z^{-1}] = 1 + 1.8z^{-1} + 0.1z^{-2} \quad (78)$$

$$Q[z^{-1}] = 0.2 - 2.2z^{-1} + 0.56z^{-2} \quad (79)$$

$$R[z^{-1}] = 1 - 0.26z^{-1} \quad (80)$$

Solving Diophantine equations (8) and (9), polynomials to define the controller (14) in polynomial form are

$$E[z^{-1}] = 1 \quad (81)$$

$$F[z^{-1}] = 1.1 - 0.2z^{-1} - 0.1z^{-2} \quad (82)$$

$$G[z^{-1}] = 1 - 1.7z^{-1} + 0.66z^{-2} \quad (83)$$

A simulation is conducted in the case where reference input $r(t)$ is the rectangular wave with period of 100 steps and amplitude +1 and −1 and Gaussian white noise $\xi(t)$ with mean 0 and variance 0.03^2. Fig.1 shows output $y(t)$ and input $u(t)$ generated by polynomial form controller (13) having polynomials (81), (82) and (83) in solid lines and reference $r(t)$ in dotted line.

An observable canonical form of the plant with $A[z^{-1}]$ and $B[z^{-1}]$ of (77) is

$$x(t+1) = \begin{bmatrix} -0.7 & 1 & 0 \\ -0.3 & 0 & 1 \\ -0.1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0.8 \\ 0.5 \\ 0.1 \end{bmatrix} u(t) - \begin{bmatrix} 0.7 \\ 0.3 \\ 0.1 \end{bmatrix} \xi(t) \quad (84)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(t) + \xi(t)$$

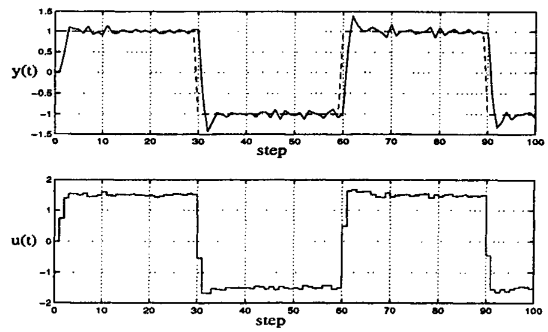

Figure 1: output $y(t)$(upper) and input $u(t)$(lower) by polynomial approach

An observer to estimate the generalized output with polynomials (78), (79) and (80) is

$$w(t+1) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} w(t) + \begin{bmatrix} -0.2 \\ -0.1 \end{bmatrix} y(t) + \begin{bmatrix} -1.7 \\ 0.66 \end{bmatrix} u(t) \quad (85)$$

$$\hat{\Phi}(t+1|t) = \begin{bmatrix} 1 & 0 \end{bmatrix} w(t) + 1.1y(t) + u(t) \quad (86)$$

A controller (65) in state-space form is

$$u(t) = -\begin{bmatrix} 1 & 0 \end{bmatrix} w(t) + 1.1y(t) - r(t) + 0.26r(t-1) \quad (87)$$

Simulated output $y(t)$ and input $u(t)$ from state-space form controller (87) are shown in Fig.2 and are same to output and input in Fig.1 by polynomial form controller.

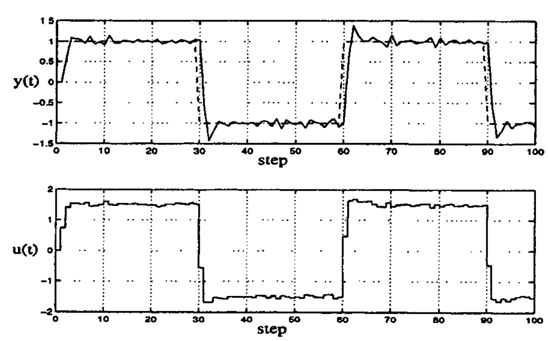

Figure 2: output $y(t)$(upper) and input $u(t)$(lower) by state-space approach

6 Conclusion

This paper gives a design method for generalized minimum variance controller (GMVC) in state-space form. The controller consists of a state feedback and an observer with poles at $z = 0$. It is shown that the state-space form controller is equivalent to a polynomial form controller. Also obtained is a coprime factorization in state-space form of GMVC.

Using results of this paper, the following extensions will be expected. First, a controller of GMVC having observers with poles close to $z = 1$, which is robust to measurement noise. Second, comparing Youla parametrization in state-space form to the coprime factorization obtained in this paper, a new design parameter will be introduced into GMVC in state-space form from Youla parametrization. Finally, advanced design methods in state-space control theory can be applied to GMVC using the state-space form of GMVC.

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