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A Design of a Strongly Stable Generalized Predictive Control Using Coprime Factorization Approach

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Abstract

This paper proposes a new generalized predictive control (GPC) having new design parameters. In selecting the design parameters, the controller becomes a strongly stable GPC, that is, not only the closed-loop system is stable, but also the controller itself is stable. The parameters are introduced by applying the coprime factorization approach and comparing Youla parametrization of stabilizing compensators to the controller by the standard GPC.

1 Introduction

Generalized predictive control (GPC) technique is first proposed by Clarke and others in 1987 [1]. The control technique has been accepted by many of practical engineers and applied widely in industry [2]. In applying to industry, safty is the most important. For safety, it is desirable to design a controller itself to be stable in addition to the closed-loop stability, that is, to design a strongly stable controller. The strongly stable controller ensures the boundedness of control input even when the feedback loop is cut by an accident. So far many papers concerned with the stability of GPC have been published, but most of the papers are devoted to obtain design methods ensuring the stability of closed-loop system [4]. And less attention is paid to the design of strongly stable GPC.

In this paper a method to design a strongly stable GPC is proposed by extending a standard GPC. The method consists of three steps: The first step is to design a GPC with the closed-loop to be stable by standard GPC technique. In the second step, the GPC obtained in step 1 is extended by adding new design parameters. The parameters are introduced by comparing Youla parametrization[3] of coprimely factored stabilizing compensators to the controller by the standard GPC. At the last step, by selecting the newly introduced parameters in the extended GPC, the controller is made to be stable without changing the closed-loop stability.

It will be shown that for any design parameters the extended GPC has the same closed-loop poles to the ones of the original standard GPC and also shown that, although the control input will change according to the selection of parameters, the gradient of the objective function with respect to control input is equal to 0. Hence for any values of the design parameters, the newly extended GPC keeps the closed-loop system to be stable and the objective function of the original standard GPC to be minimal, the parameters are searched only in considering to make the controller itself stable.

Coprime factorization approach is used by Wams et al [5] in designing predictive control but their design method is to obtain closed-loop stability.

Notations: z^{-1}denotes backward shift operator: z^{-1} $y(t) = y(t-1)$ and $\Delta = 1 - z^{-1}$. Polynomial and rational functions are distinguished by $A[z^{-1}]$ and $A(z^{-1})$.

2 Problem Statement and Standard GPC

Consider a single-input single-output system given by CARIMA model,

$$A[z^{-1}]y(t) \;=\; z^{-k_m}B[z^{-1}]u(t) + \frac{C[z^{-1}]}{\Delta}\xi(t) \quad (1)$$

where $u(t)$ is input, $y(t)$ is output, k_m is time delay, $\xi(t)$ is a white Gaussian noise with zero mean. $A[z^{-1}]$, $B[z^{-1}]$, $C[z^{-1}]$ are polynomials with degrees n, m, l and $A[z^{-1}]$, $C[z^{-1}]$ are monic. The followings are assumed;

[A.1] The degrees n, m, l of $A[z^{-1}], B[z^{-1}], C[z^{-1}]$ and the time delay k_m are known. For simplicity k_m is assumed to be $k_m = 1$.
[A.2] In adaptive case, the coefficients of $A[z^{-1}], B[z^{-1}]$ and $C[z^{-1}]$ are unknown, but the nominal values of the coefficients are known.
[A.3] The polynomials $A[z^{-1}]$ and $B[z^{-1}]$, $A[z^{-1}]$ and $C[z^{-1}]$ are coprime. And $C[z^{-1}]$ is stable.

In standard GPC technique [1], the control objective is to minimize the following objective function J;

$$E[\sum_{j=N_1}^{N_2} \{y(t+j) - w(t+j)\}^2 + \sum_{j=0}^{N_u-1} \lambda_j \{\Delta u(t+j)\}^2] \quad (2)$$

where the average is taken over the noise $\xi(t)$, $w(j)$ is reference input, N_1 and N_2 are prediction horizons, N_u is control horizon and λ_j's are weighting factors on control inputs. For simplicity it is assumed that $N_1 = 1, N_u = N_2$.

In this section and the next section, non-adaptive case is considered. Hence the coefficients of $A[z^{-1}], B[z^{-1}]$ and $C[z^{-1}]$ are assumed to be known.

In order to compare the controller of the standard GPC to the most general compensator in Youla parametrization in the next section, the controller structure of standard GPC [1] is given.

First, consider the following Diophantine equation for $j = 1, 2, \cdots, N_2$;

$$C[z^{-1}] = \Delta A[z^{-1}]E_j[z^{-1}] + z^{-j}F_j[z^{-1}] \quad (3)$$

where $E_j[z^{-1}], F_j[z^{-1}]$ are polynomials with degree $j-1$ and n, and $E_j[z^{-1}]$ is monic.

Using equations (1) and (3), the optimal estimate is obtained by

$$\begin{aligned} \hat{y}(t+j|t) &= E[y(t+j)] \\ &= [F_j[z^{-1}]y(t) + E_j[z^{-1}]B[z^{-1}] \\ &\quad \cdot \Delta u(t+j-1)]/C[z^{-1}] \end{aligned} \quad (4)$$

To separate the future values and past values of $u(t)$ in (4), $E_j[z^{-1}]B[z^{-1}]$ is separated as

$$E_j[z^{-1}]B[z^{-1}] = C[z^{-1}]R_j[z^{-1}] + z^{-j}S_j[z^{-1}] \quad (5)$$

where $R_j[z^{-1}]$ and $S_j[z^{-1}]$ are polynomials with degree of $j-1$ and $n_3 = \max\{m, l\} - 1$ and $r_0, r_1, \cdots, r_{j-1}$ are the coefficients of $R_j[z^{-1}]$.

Define the following signal $h_j(t)$, vectors $\hat{\boldsymbol{y}}$, $\boldsymbol{u}$, $\boldsymbol{h}$, $\boldsymbol{\epsilon}$, $\boldsymbol{w}$ and matrices $\boldsymbol{R}$, $\boldsymbol{\Lambda}$ as

$$C[z^{-1}]h_j(t) = F_j[z^{-1}]y(t) - S_j[z^{-1}]\Delta u(t-1) \quad (6)$$

$$\hat{\boldsymbol{y}} = [\hat{y}(t+1|t), \hat{y}(t+2|t), \cdots, \hat{y}(t+N_2|t),]^T \quad (7)$$
$$\boldsymbol{u} = [\Delta u(t), \Delta u(t+1), \cdots, \Delta u(t+N_2-1)]^T \quad (8)$$
$$\boldsymbol{h} = [h_1(t), h_2(t), \cdots, h_{N_2}(t)]^T \quad (9)$$
$$\boldsymbol{\epsilon} = [\epsilon(t+1), \epsilon(t+2), \cdots, \epsilon(t+N_2)]^T \quad (10)$$
$$\boldsymbol{w} = [w(t+1), w(t+2), \cdots, w(t+N_2)]^T \quad (11)$$
$$\boldsymbol{R} = \begin{bmatrix} r_0 & & & 0 \\ r_1 & r_0 & & \\ \vdots & & \ddots & \\ r_{N_2-1} & r_{N_2-2} & \cdots & r_0 \end{bmatrix} \quad (12)$$
$$\boldsymbol{\Lambda} = \text{diagonal matrix of } \{\lambda_1, \lambda_2, \cdots, \lambda_{N_2}\} \quad (13)$$

Then the estimate $\hat{\boldsymbol{y}}$ and the objective function J of (2) are given in vector form:

$$\hat{\boldsymbol{y}} = \boldsymbol{R}\boldsymbol{u} + \boldsymbol{h} \quad (14)$$

$$\begin{aligned} J &= E[\ (\boldsymbol{R}\boldsymbol{u} + \boldsymbol{h} - \boldsymbol{w})^T (\boldsymbol{R}\boldsymbol{u} + \boldsymbol{h} - \boldsymbol{w}) \\ &\quad + \boldsymbol{u}^T \boldsymbol{\Lambda} \boldsymbol{u}\ ] + E[\boldsymbol{\epsilon}^T \boldsymbol{\epsilon}] \end{aligned} \quad (15)$$

Solving the linear equation $\partial E[J]/\partial \boldsymbol{u} = 0$ of $\boldsymbol{u}$ and extracting the first element of $\boldsymbol{u}$, the optimal control input $\Delta u(t)$ to minimize J is obtained as

$$\begin{aligned} \Delta u(t) &= P[z^{-1}]w(t+N_2) - (F_p[z^{-1}]y(t) \\ &\quad + S_p[z^{-1}]\Delta u(t-1))/C[z^{-1}] \end{aligned} \quad (16)$$

$$\begin{aligned} \boldsymbol{p} &= [p_1, p_2, \cdots, p_{N_2}] \\ &= [1\ 0\ \cdots\ 0] \left(\boldsymbol{R}^T\boldsymbol{R} + \boldsymbol{\Lambda}\right)^{-1} \boldsymbol{R}^T \end{aligned} \quad (17)$$

$$P[z^{-1}] = p_{N_2} + p_{N_2-1}z^{-1} + \cdots + p_1 z^{-(N_2-1)} \quad (18)$$
$$\begin{aligned} F_p[z^{-1}] &= p_1 F_1[z^{-1}] + p_2 F_2[z^{-1}] + \\ &\quad \cdots + p_{N_2} F_{N_2}[z^{-1}] \end{aligned} \quad (19)$$
$$\begin{aligned} S_p[z^{-1}] &= p_1 S_1[z^{-1}] + p_2 S_2[z^{-1}] + \\ &\quad \cdots + p_{N_2} S_{N_2}[z^{-1}] \end{aligned} \quad (20)$$

Substituting the control input (16) into the system (1), and defining polynomial $T[z^{-1}]$ as

$$C[z^{-1}]T[z^{-1}] = \Delta A[z^{-1}]C[z^{-1}] + z^{-1}D_p[z^{-1}] \quad (21)$$
$$D_j[z^{-1}] = \Delta A[z^{-1}]S_j[z^{-1}] + B[z^{-1}]F_j[z^{-1}]$$
$$\begin{aligned} D_p[z^{-1}] &= p_1 D_1[z^{-1}] + p_2 D_2[z^{-1}] + \\ &\quad \cdots + p_{N_2} D_{N_2}[z^{-1}] \end{aligned} \quad (22)$$

we get the closed-loop system

$$\begin{aligned} y(t) &= \frac{z^{-1}B[z^{-1}]P[z^{-1}]}{T[z^{-1}]} w(t+N_2) \\ &\quad + \frac{C[z^{-1}] + z^{-1}S_p[z^{-1}]}{T[z^{-1}]} \xi(t) \end{aligned} \quad (23)$$

The transfer function of the controller (16) from $w(t+N_2)$ to $u(t)$ is

$$C[z^{-1}]P[z^{-1}] / \left(C[z^{-1}]\Delta + S_p[z^{-1}]z^{-1}\Delta\right)$$

and it has unstable pole $z^{-1} = 1$. If the feedback loop is cut by an accident, then the control input will be divergent, even when the closed-loop is stable.

To solve this problem, we will introduce new design polynomials in the controller (16) by comparing the most general stabilizing compensator, Youla parametrization form, to the controller (16).

3 Extended GPC with design parameters

For coprime factorization approach, the family of stable rational functions are considered;

$$\begin{aligned} RH_\infty &= \{G(z^{-1}) = \frac{G_n[z^{-1}]}{G_d[z^{-1}]}, \\ &\quad G_d[z^{-1}] : \text{stable polynomial}\} \end{aligned} \quad (24)$$

The transfer function is given in the form of a ratio of rational functions in RH_∞,

$$G(z^{-1}) = N(z^{-1})/D(z^{-1}) \quad (25)$$

where $N(z^{-1})$, $D(z^{-1})$ are rational functions in RH_∞ and are coprime in each other. Then, all of the stabilizing two-degree-of-freedom compensator is given in Youla parametrization [3];

$$u(t) = C_1(z^{-1})w(t+N_2) - C_2(z^{-1})y(t) \quad (26)$$
$$C_1(z^{-1}) = (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}K(z^{-1}) \quad (27)$$
$$C_2(z^{-1}) = (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}(X(z^{-1}) + U(z^{-1})D(z^{-1})) \quad (28)$$

where $U(z^{-1})$ and $K(z^{-1})$ are rational functions in RH_∞ and are design parameters. $X(z^{-1})$ and $Y(z^{-1})$ are also in RH_∞ and the solutions of the following Bezout equations;

$$X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1 \quad (29)$$

We assume that GPC controller (16) for the given plant (1) is already designed and the weighting factor λ_j's are chosen so that the closed-loop characteristic $T[z^{-1}]$ is stable. Comparing the plant (1) to (25), we can choose $N(z^{-1})$ and $D(z^{-1})$ in RH_∞ as

$$N(z^{-1}) = z^{-1}B[z^{-1}]/T[z^{-1}] \quad (30)$$
$$D(z^{-1}) = A[z^{-1}]/T[z^{-1}] \quad (31)$$

Substituting (30) and (31) into Youla parametrization (26)-(28) and comparing the form (26)-(28) having $U(z^{-1}) = 0$ to the controller (16), we get relations;

$$X(z^{-1}) = F_p[z^{-1}]/C[z^{-1}] \quad (32)$$
$$Y(z^{-1}) = (C[z^{-1}] + z^{-1}S_p[z^{-1}])\,\Delta/C[z^{-1}] \quad (33)$$
$$K(z^{-1}) = P[z^{-1}] \quad (34)$$

Since $C[z^{-1}]$ is assumed to be stable, $X(z^{-1})$, $Y(z^{-1})$ and $K(z^{-1})$ are in RH_∞ By using Diophantine equation (3), it is confirmed that $N(z^{-1})$, $D(z^{-1})$ of (30), (31) and $X(z^{-1})$, $Y(z^{-1})$ of (32), (33) satisfy Bezout equation (29).

In order to extend the controller (16), instead of choosing $U(z^{-1})$ as $U(z^{-1}) = 0$, we use $U(z^{-1})$ as a newly introduced design parameter for the controller (16). To simplify the description of the controller, using new two design polynomials $U_n[z^{-1}]$ and $U_d[z^{-1}]$, we use

$$U(z^{-1}) = \frac{U_n[z^{-1}]}{U_d[z^{-1}]}T[z^{-1}] \quad (35)$$

Then substituting (30)-(35) into (26)-(28), we get a newly extended GPC;

$$\{U_d[z^{-1}]\,(C[z^{-1}] + z^{-1}S_p[z^{-1}])\,\Delta - U_n[z^{-1}]C[z^{-1}]z^{-1}B[z^{-1}]\}\,u(t) = U_d[z^{-1}]C[z^{-1}]P[z^{-1}]w(t+N_2) - (U_d[z^{-1}]F_p[z^{-1}] + U_n[z^{-1}]C[z^{-1}]A[z^{-1}])y(t) \quad (36)$$

To calculate this controller, we separate the leading term and the remaining terms in the polynomial multiplied by $u(t)$ in the left-hand side of (36) as

$$U_d[z^{-1}]\,(C[z^{-1}] + z^{-1}S_p[z^{-1}])\,\Delta - U_n[z^{-1}]C[z^{-1}]z^{-1}B[z^{-1}] = g_0 + z^{-1}G'[z^{-1}] \quad (37)$$

Then the controller (36) is calculated by

$$u(t) = -\frac{1}{g_0}\,(U_d[z^{-1}]F_p[z^{-1}] + U_n[z^{-1}]C[z^{-1}] \cdot A[z^{-1}])\,y(t) - \frac{1}{g_0}G'[z^{-1}]u(t-1) + \frac{1}{g_0}U_d[z^{-1}]C[z^{-1}]P[z^{-1}]w(t+N_2) \quad (38)$$

When $U_n[z^{-1}]$ and $U_d[z^{-1}]$ are chosen as $U_n[z^{-1}] = 0$, $U_d[z^{-1}] = 1$, the controller (36) or (38) coincides with the controller (16) designed by the standard GPC.

Theorem 1: Using the controller (36) or (38), the followings hold;

(i) The closed-loop system is given by

$$y(t) = \frac{z^{-1}B[z^{-1}]P[z^{-1}]}{T[z^{-1}]}w(t+N_2) + \left(\frac{C[z^{-1}] + z^{-1}S_p[z^{-1}]}{T[z^{-1}]} - \frac{z^{-1}B[z^{-1}]C[z^{-1}]}{T[z^{-1}]\Delta} \cdot \frac{U_n[z^{-1}]}{U_d[z^{-1}]}\right)\xi(t) \quad (39)$$

(ii) For any $U_n[z^{-1}]$ and $U_d[z^{-1}]$, the control input $u(t)$ by (36) or (38) satisfies

$$\frac{\partial E\,[J]}{\partial \boldsymbol{u}} = 0 \quad (40)$$

(iii) From (36), the poles of the controller itself are the roots of equation,

$$U_d[z^{-1}]\,(C[z^{-1}] + z^{-1}S_p[z^{-1}])\,\Delta - U_n[z^{-1}]z^{-1}C[z^{-1}]B[z^{-1}] = 0 \quad (41)$$

Remark (i) From (39), it is shown that the transfer function from $w(t+N_2)$ to $y(t)$ is equal to the one by the standard GPC [1], and is independent of the choice of the design parameters $U_d[z^{-1}]$ and $U_n[z^{-1}]$.

Remark (ii) The poles of the controller can be designed by selecting polynomials $U_d[z^{-1}]$ and $U_n[z^{-1}]$ in (41), without changing the poles of the closed-loop system. We may design a strongly stable controller, by sequentially designing first λ_j's for the closed-loop poles (the roots of $T[z^{-1}] = 0$) to be stable, second $U_n[z^{-1}]$ and $U_d[z^{-1}]$ for the controller poles (the roots of (41)) to be stable.

Proof. Equations (39) is derived from (3) and (30)-(36). The first element of $\boldsymbol{u}$ satisfying (40) is given by

$$\Delta u(t) = \boldsymbol{p}^T(-\boldsymbol{h}+\boldsymbol{w}) \quad (42)$$

$$\boldsymbol{p}^T\boldsymbol{h} = \frac{F_p[z^{-1}]}{C[z^{-1}]}y(t) - \frac{S_p[z^{-1}]}{C[z^{-1}]}\Delta z^{-1}u(t) \quad (43)$$

$$\boldsymbol{p}^T\boldsymbol{w} = P[z^{-1}]w(t+N_2) \quad (44)$$

Using (43), (44), (36) and the plant equation (1), equation (42) holds. Q.E.D.

4 Adaptive Extended GPC

In this section, under the assumption that only the nominal values of the coefficients of the system (1) are known and the true values are unknown, we will propose an adaptive GPC by adding a parameter identification law to the extended controller (38). First choose the design parameter $U_d[z^{-1}]$ and $U_n[z^{-1}]$ so as that the poles of controller are stable using the nominal values of $A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$. Then the adaptive GPC consists of the following identification law and the controller (38) calculated using identified values $\hat{a}_1(t),\cdots,\hat{c}_l(t)$ in each sampling time.

The identification law is;

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\Gamma(t-1)\psi(t-1)}{1+\psi^T(t-1)\Gamma(t-1)\psi(t-1)}\varepsilon(t) \quad (45)$$

$$\Gamma(t) = \Gamma(t-1) - \frac{\lambda\Gamma(t-1)\psi(t-1)\psi^T(t-1)\Gamma(t-1)}{1+\lambda\psi^T(t-1)\Gamma(t-1)\psi(t-1)}$$
$$\Gamma(0) = \alpha I,\ \alpha > 0 \quad (46)$$

$$\varepsilon(t) = y(t) - \hat{\theta}^T(t-1)\psi(t-1) \quad (47)$$

$$\eta(t) = y(t) - \hat{\theta}^T(t)\psi(t-1) \quad (48)$$

$$\hat{\theta}(t) = [\hat{a}_1(t),\cdots,\hat{a}_n(t),\hat{b}_0(t),\cdots,\hat{b}_m(t), \hat{c}_1(t),\cdots,\hat{c}_l(t)]^T \quad (49)$$

$$\psi(t) = [-y(t-1),\cdots,-y(t-n),u(t-k_m),\cdots, u(t-k_m-m),\eta(t-1),\cdots,\eta(t-l)]^T \quad (50)$$

where $\hat{a}_1(t),\cdots,\hat{c}_l(t)$ are the identified value, λ $(0 < \lambda \le 1)$ is a forgetting factor.

Theorem 2: If the assumptions [A.1]~[A.3] and the following assumptions [A.4]~[A.6] hold,

[A.4] The nominal values of the coefficients are equal to the true values and λ_j's are selected such that $T[z^{-1}]$ is stable.

[A.5] Transfer function $C[z^{-1}] - \dfrac{\lambda}{2}$ is strongly positive real.

[A.6] The signal $\psi(t)$ satisfies PE(Persistently Spanning) condition, that is, the matrix

$$R = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\psi(t-1)\psi^T(t-1) \quad (51)$$

is positive definite,

then the tracking error $e(t) = y(t) - w(t)$ converges to zero $a.s.$(almost surely, converge except on a set having probability zero).

Proof. Using similar way by [6], it is proved that the error of parameter identification $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ and the error of output estimation, $z(t) = \eta(t) - \xi(t)$ converge to zero $a.s.$. Then the tracking error can be proved to converge from the fact that the error is expressed by $\tilde{\theta}(t)$ and $z(t)$. Q.E.D.

5 Example

Consider a system and an objective function

$$y(t) = \frac{z^{-1}(0.5-1.5z^{-1})}{(1+0.6z^{-1}+0.7z^{-2})}u(t) + \frac{\xi(t)}{\Delta(1+0.6z^{-1}+0.7z^{-2})} \quad (52)$$

$$J = \sum_{j=1}^{5}(y(t+j)-w(t+j))^2 + \sum_{j=1}^{5}(\Delta u(t+j-1))^2$$
$$N_2 = N_u = 5,\ \lambda_j = 1 \quad (53)$$

and the reference input $w(t)$ of a rectangular wave with amplitude 1.0 and period of 50 steps.

The control law (16) by standard GPC [1] is

$$u(t) = \frac{-0.0802-0.0074z^{-1}-0.156z^{-2}-0.3767z^{-3}+0.1452z^{-4}}{1-0.9042z^{-1}-0.0958z^{-2}}w(t+N_2) - \frac{-0.0908-0.3397z^{-1}-0.0447z^{-2}}{1-0.9042z^{-1}-0.0958z^{-2}}y(t) \quad (54)$$

and the closed-loop system is

$$y(t) = \frac{-0.0401z^{-1}+0.1166z^{-2}-0.0669z^{-3}+0.0457z^{-4}+0.6376z^{-5}-0.2177z^{-6}}{1-0.3496z^{-1}+0.028z^{-2}-0.2032z^{-3}}w(t+N_2) + \frac{1+0.0958z^{-1}}{1-0.3496z^{-1}+0.028z^{-2}-0.2032z^{-3}}\xi(t) \quad (55)$$

The poles of the closed-loop system are $z^{-1} = 1/0.712$, $1/(-0.181 \pm 0.503i)$ and stable. But the pole of the controller is $z^{-1} = 1$, $1/(-0.0958)$ and includes unstable one, so this controller is not strongly stable.

Selecting the design parameters in the extended GPC controller (38) as $U_d = 1.0$, $U_n = 0.4$, the controller is

$$u(t) = \frac{-0.0802 - 0.0074z^{-1} - 0.156z^{-2} - 0.3767z^{-3}}{1 - 1.1042z^{-1} + 0.5042z^{-2}} \frac{+ 0.1452z^{-4}}{} w(t + N_2) - \frac{0.3092 - 0.0997z^{-1} + 0.2353z^{-2}}{1 - 1.1042z^{-1} + 0.5042z^{-2}} y(t) \quad (56)$$

and the closed-loop system is

$$y(t) = \frac{-0.0401z^{-1} + 0.1166z^{-2} - 0.0669z^{-3}}{1 - 0.3496z^{-1} + 0.028z^{-2} - 0.2032z^{-3} + 0.0457z^{-4} + 0.6376z^{-5} - 0.2177z^{-6}} w(t + N_2) + \frac{1 - 1.1042z^{-1} + 0.5042z^{-2}}{1 - 1.3496z^{-1} + 0.3776z^{-2} - 0.2312z^{-3} + 0.2032z^{-4}} \xi(t) \quad (57)$$

The poles of the closed-loop system from $w(t + N_2)$ to $y(t)$ by this controller are not changed from the poles by (55). The poles of the controller are $z^{-1} = 1/(0.552 \pm 0.447i)$, $|z^{-1}| = 1/0.71$ and are improved to be stable, that is, the controller is strongly stable.

Assuming the coefficients of the plant (52) are unknown and the variance of noise is $\sigma = 0.025$, computer simulations are conducted. In the simulations, the forgetting factor in (45)~(50) is $\lambda = 0.99$, the initial value of $\Gamma(0)$ is $1.0I$ and the initial values of identified coefficients are set to be equal to 0.8 of the true values.

Simulation results are shown in Fig.1 using the controller (16) by standard GPC [1], and Fig.2 with the controller (38) proposed in this paper. In Fig.2 the output without noise shown by dotted lines is same to the one in Fig.1. This fact shows that the output response to reference input is not changed by introducing the design parameters $U_n[z^{-1}]$ and $U_d[z^{-1}]$.

In the simulations, the solid lines give the output responses with noise and feedback cut at step $t = 100$. Fig.1 shows that the output by the controller (16) is divergent, whereas, the output in Fig.2 by the controller (38) stays bounded.

6 Conclusion

In this paper, the controller designed by GPC [1] is extended to a GPC including new design polynomials by using coprime factorization approach and comparing the most general two-degree-of-freedom compensator in Youla parametrization form. Without changing the response of output to reference input, the poles of the compensator can be changed by selecting the newly introduced design polynomials in the proposed controller.

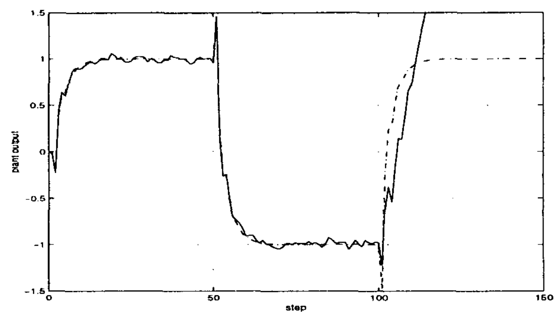

Figure 1: Control result by standard GPC

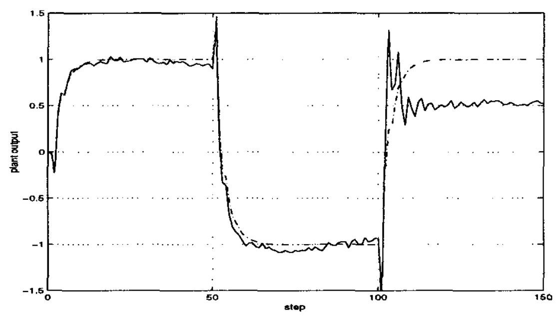

Figure 2: Control result by extended GPC proposed in this paper

To find parameter values giving the controller stable poles requires try-and-error method and to obtain a method finding the parameter values straightforwardly remains as a future work.

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