Journal of the Faculty of Environmental Science and Technology, Okayama University
Vol.4, No.1, pp.119-130, February 1999

Water Table Recession in Subsurface Drained Soils

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(Received October 19, 1998)

Theoretical drainage equations are intensively tested in many parts of humid and arid regions and are commonly used in drainage design. However, this is still a great concern in Japan as the drainage design is exclusively based on local experiences and empirical basis. There is a need therefore to evaluate the theoretical drainage equations under Japanese field conditions to recommend equations for design of subsurface drainage systems. This was the main motivation for this study. While drainage requirements of different crops are difficult to define quantitatively, adequate drainage can probably be provided by designing for a given water table recession rate. The method used was to measure the rate of water table recession and drain outflow after cessation of a considerable amount of rainfall. Based on these measurements, soil parameters were determined and nine drainage equations were tested in simulating water table recession against actual field data of two drained experimental sites, located in Soja city west of Okayama prefecture, Japan, having drain spacing of 10 and 8.5 m for a period of two years. An experimental drainage equation was also derived and theoretical equations were recommended for design of subsurface drainage systems.

Keywords: Simulation, Soil parameters, Subsurface drainage, Water table

1. INTRODUCTION

In Japan, paddy fields represent more than 50% of the total agricultural lands with approximately 2.7 million hectares of high productivity. Rice production which had continued to increase rapidly developed a surplus and caused a social problem due to the decline of farmers zeal for rice farming and shortage of rural labors. Therefore, the greatest task facing Japanese agriculture now is the "paddy reorganization work" (Tsutsui, 1996) of introducing dry-foot crops into the paddy fields. Under such new cropping system, some of paddy fields will be used permanently for the other crops and some alternately use as paddy and dry crops fields. In either case and in particularly the latter case, it is vitally important that the subsurface drainage should be as effective as possible for lowering the water table after a rainfall (average annual rainfall is 1,788 mm).

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The predicted performance of subsurface drainage systems is exclusively based on local experience and designed on an empirical basis, mainly due to lack of some soil properties such as hydraulic conductivity, drainable porosity and depth of impermeable layer below soil surface which are costly and cumbersome to measure under field conditions. These properties are extremely important and needed by various drainage equations for designing the drainage system. Although some testing and evaluation of these equations have already been reported (Johnston et al., 1965; French and O'Callaghan, 1966; Nwa and Twocock, 1969; Skaggs et al., 1973; El-Mowelhi and Hermsmeier, 1982; Buckland et al., 1987), they have never been tested under Japanese field conditions. Moreover, literature on the relative capabilities and the results' credibility of the newly developed drain spacing equations by Singh et al. (1992) is scarce and is still an important concern.

In this study, a methodology was given for determining different soil parameters needed for drainage equations using observations of water table height above drains and subsurface drain outflows. Nine drainage equations, including the newly developed equations, were then evaluated in simulating water table recession midway between drains of the drained soils for a period of two years, 1997 and 1998.

2. DRAINAGE EQUATIONS

One of the first drainage equations based on the rate of water table recession was derived by Glover (Dumm, 1954) for an initial flat water table. Tapp and Moody (Dumm, 1964) modified Glover equation to consider a fourth degree parabola as the initial water table shape. Their equation may be written as:

$$S^2 = \frac{\pi^2 KDt}{f \ln\left(\frac{3.7}{\pi}\frac{m_o}{m_t}\right)} \tag{1}$$

where S is the drain spacing; t is the time; m_o and m_t are the heights of the water table at the midpoint between drains for t = 0 and t = t, respectively; D is the average initial depth of water bearing stratum, = $d_e + m_o/2$, d_e is Hooghoudt equivalent depth; K is the hydraulic conductivity; and f is the drainable porosity.

Hammad (1962) used potential theory and the assumption that the receding water table between drains is nearly flat to derive equations for water table recession in both shallow and deep soils. Because the field observations in this study were made on a soil with a shallow impermeable layer, Hammad equation for thin layers (d/S< 0.25) was only used where d is the depth of impermeable layer below the drains. This equation may be written as:

$$S = \frac{2\pi K t}{f \ln\left(\frac{m_o}{m_t}\right) \ln\left(\frac{S^2}{2\pi^2 rd}\right)} \tag{2}$$

where r is the radius of the drain pipe.

Van Schilfgaarde (1963) derived an equation based on Dupuit-Forchheimer theory taking into account the correction for effect of convergence of flow toward the drains by making use of Hooghoudt equivalent depth. His solution avoids the assumption of a constant water bearing stratum, D. This equation may be written as:

$$S^2 = \frac{9A^2K(d_e + m_t)(d_e + m_o)t}{2f(m_o - m_t)} \tag{3}$$

where A is a constant defined as $\{1-[d_e/(d_e+m_o)]^2\}^{1/2}$.

Van Schilfgaarde (1964) also derived an equation for an initially parabolic water table and modified Glover equation to correct for convergence of flow toward the drains. The modified equation is expressed as:

$$S^2 = \frac{9Kd_e t}{f \ln\left[\dfrac{m_o(m_t + 2d_e)}{m_t(m_o + 2d_e)}\right]} \tag{4}$$

Bouwer and van Schilfgaarde (1963) developed an equation (Integrated Hooghoudt equation) based on the steady state theory for predicting the rate of fall of water table. They assumed that the instantaneous drainage rate midway between drains is equal to the steady state drainage rate and used it to predict the rate of fall of the water table midway between the drains by introducing a correction factor c for the shape of the water table. This equation may be written as:

$$S^2 = \frac{8Kd_e t}{f\,c \ln\left[\dfrac{m_o(m_t + 2d_e)}{m_t(m_o + 2d_e)}\right]} \tag{5}$$

where c is a correction factor defined as the ratio of the average flux between drains to the flux midway between drains. For $0.02<(m_o/S)<0.08$, c = 0.8 and for $(m_o/S)>0.15$, c = 1.0. However, higher values of c can be expected for the early initial stages of water table recession (Bouwer and van Schilfgaarde, 1963; Moustafa and Yomota, 1997). c-values of 1.0 and 0.8 were used with this equation and the results were compared.

Youngs (1985) developed an equation based on Hooghoudt equivalent depth steady-state drainage equation and the hodograph analysis for drains of optimum size to predict the water table recession. It may be written as:

$$m_t = m_o \Big/ \left[1 + \frac{(a-1)\,K\,m_o^{a-1}\,t}{f\,(S/2)^a}\right]^{1/(a-1)} \tag{6}$$

where

$$a = 2\left(\frac{2d}{S}\right)^{\frac{2d}{S}} \quad , \quad 0 \le \frac{2d}{S} \le 0.35$$

$$a = 1.36 \quad , \quad \frac{2d}{S} \rightarrow \infty$$

Singh et al. (1992) have extended the same technique of Bouwer and van Schilfgaarde (1963) using the steady-state equations of Ernst (1962), Dagan (1964) and van Beers (1965) to develop unsteady-state solutions.

The Integrated Ernst equation may be written as:

$$m_t = m_o \exp\left[\frac{-Kt}{f\,c\left(\dfrac{S^2}{8d} + \dfrac{S}{\pi}\ln\dfrac{d}{\pi r}\right)}\right] \tag{7}$$

The Integrated Dagan equation may be written as:

$$m_t = m_o \exp\left(\frac{-Kt}{S\,f\,c\,F_D}\right) \tag{8}$$

where $F_D = \frac{1}{4}\left[\frac{S}{2d} - \frac{2}{\pi}\ln\left(2\cosh\frac{\pi r}{d} - 2\right)\right]$

The Integrated van Beers equation may be written as:

$$m_t = \frac{2\,d}{\left[\frac{m_o + 2d}{m_o}\exp\left\{\frac{8Kdt}{(S+B)^2\;f\,c}\right\} - 1\right]} \tag{9}$$

where $B = d\ln\left(\frac{d}{\pi r}\right)$

The foregoing drainage equations were chosen as they represent a variety of assumptions regarding the initial shape of the water table, boundary conditions, and techniques and some of them (Eqns. 7, 8 and 9) are newly developed and are still in need for a validation scheme to be undertaken before adopting them for drainage design.

3. EXPERIMENTS

Two experimental sites of approximately 0.2 ha each used as paddy fields of Maekawa district project, which covers a total area of 80.5 ha and is located in Soja city west of Okayama prefecture, Japan, were selected for this study. Subsurface polyethylene (PE) drain pipes of 75 and 80 mm in diameter and 100 m in length were installed in 1994 at 10 and 8.5 m spacing for site 1 and site 2, respectively, at 0.8 m depth. The soils are silty clay and silty sand in both sites, respectively. A set of observation wells of 55 mm in diameter was installed in the two sites to about 1.0 m depth in a single line across 1/3 the pipe length from the outlet and spaced as: just outside the drain trench, 1/8 spacing, 1/4 spacing, and at midway between drains. The sites were chosen where the adjacent lands are subsurface drained to prevent lateral movement of water and with different drain spacing to allow derivation of an experimental drainage equation and comparison of drainage equations under different boundary conditions.

Water table heights midway between drains in the two sites were measured continuously using water level recorders, while the heights at the remaining locations were measured using a measuring tape and recorded by hand in the field. In order to obtain site specific rainfall data, a rain-gage recorder was installed in the field so numerous records were obtained for the response of the water table to rainfall and the resulting recession rate. Subsurface drain outflows were measured by means of a measuring cylinder and a stopwatch each time the position of the water table was measured. The outflow through the drains right and left of the tested drain was also measured at each time to check the uniformity of performance by all drains. Experiments were carried out in 1997 and 1998 after rice harvesting period where the soils were fallow to guarantee the sites to be under soil-water conditions suitable for other dry crops. Measurements were done after a considerable amount of rainfall was ceased and the water table was near the soil surface.

4. RESULTS AND DISCUSSION

4.1 Water table response to rainfall

The response of the water table height to rainfall as measured at the midway between drains is shown in Fig. 1. As shown, there is a sharp rise of the water table due to the rainfall. The magnitude of the rise depends on the rate and the amount of rainfall, initial water table height and the antecedent soil moisture. Figure 1a indicates that the highest water table resulted from a rainfall in site 1 during the first period of observations occurred 16 to 20 hours afterwards, whereas in the second period (Fig. 1c) it occurs 5 to 13 hours afterwards (7 hours in average). This is probably attributed to the difference in field soil consolidation conditions during the two periods of observations. The highest water table in site 2 (Fig. 1d) occurred 3 to 7 hours afterwards (5 hours in average) and the water table height was generally lower than that of site 1, indicating the distinction between soil characteristics of the two sites and the effect of drain spacing on the water table height.

During the rain-free periods, it can be seen that there is a clear difference in the recession rate of water table in the two sites. The average rates for site 1 are 0.52 and 0.97 cm/hr during the two periods of observations (Figs. 1a, 1c), respectively. The rate of site 2 is higher as compared with site 1 to be 1.85 cm/hr in average (Fig. 1d). However, under the same soil consolidation conditions, the variation and recession rates were higher in winter than in late spring in the two sites. The variation in recession rates of the two sites was reaffirmed by the variation in drainage outflow rate which was observed higher in site 2 comparing to site 1.

According to Japanese criteria (MAFF, 1979), the water table depth should be maintained at an average depth of 50 cm below soil surface for the initial few days after the rainfall having ceased for most of the crops (Table 1). However, based on the observed water table depths and recession rates in the experimental sites, this criterion cannot be assured in site 1 chiefly in case of perennial crops whereas it will be valid in site 2 for all crops (Table 1).

Table 1. Observed water table depth comparing to applied criteria after cessation of the rainfall, cm

Crops	Site 1				Site 2		Applied criteria	
	first period (a)		second period (c)					
	2-3 days	7 days	2-3 days	7 days	2-3 days	7 days	2-3 days	7 days
paddy fields							30-40	40-50
dry-foot crops	28-40	90	28-45	113	90-134	312	40-50	50-60
perennial crops							50-60	60-100

These results reveal that there are distinct differences between the two sites of observations regarding the recession rate of water table. These variations could be attributed to many factors. Among them are the depth and spacing of drain pipes, soil texture, cropping pattern, amount and uniformity distribution of the applied water to the field. However, the drain pipes were installed in both sites at the same depth of approximately 0.8 m and the sites were fallow during the time of observations which reduce the effect of evapotranspiration on the recession of water table (Moustafa, 1998) in the two sites. Moreover, the amount of rainfall was the same and uniformly distributed over the two sites. Furthermore, the texture of top soil of about 30 cm was almost the same in both sites. Because the

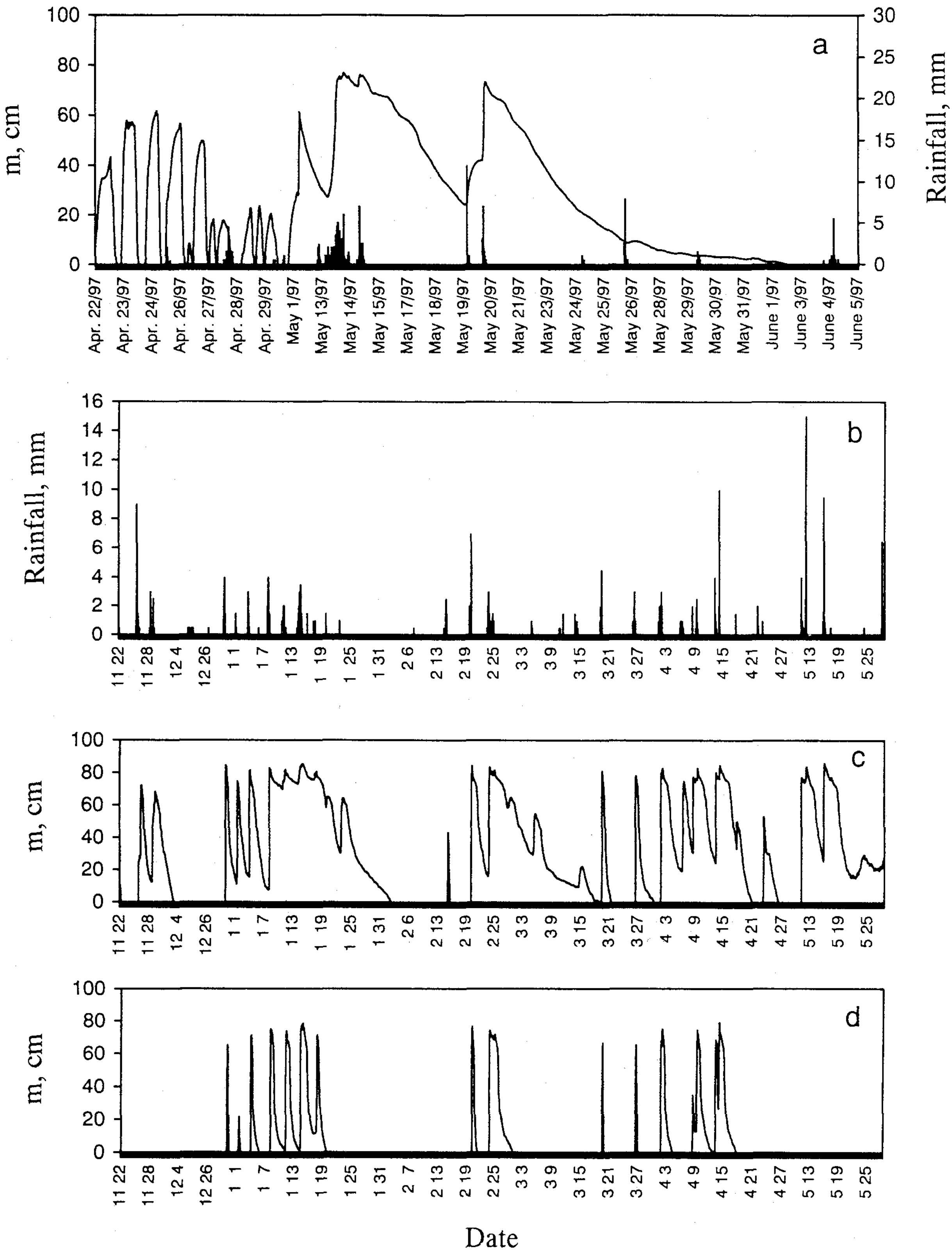

Fig. 1. Observed water table height midway between drains (m) response to rainfall and subsequent drainage for site 1 (a, c) and site 2 (d).

uniformity of water application and drainage rate from the top soil or the root zone represents significant effects on the final recession rate and drainage outflow (Moustafa, 1998), we concluded that these factors might not affect the recession rates obtained from our experimental sites. This may be taken as an indication that a major part of the difference in water table recession of the two sites might be attributed to the difference in drain spacing of both sites. Similar results to ours were previously reported by Skaggs et al. (1973) and Buckland et al. (1987) for different soils.

The usable periods of observations in the experimental sites after the cessation of rainfall were determined as given by Dieleman and Trafford (1984) and were found to be between 24 and 27 hours. This result was used in selecting the time periods for simulation of water table recession in the experimental sites.

4.2 Determination of soil hydrological parameters

Field measurements on actually operating systems may offer the only effective way to verify the initial assumptions concerning the hydrological properties on which the design of a subsurface drainage has to be based and to extend the design to adjacent areas. Therefore, drainable porosity (f) was evaluated based on the water table height and drain outflow of two consecutive time periods. Impermeable layer depth (d) was determined using the well known Hooghoudt equation (1940) which was solved for varying equivalent depth (d_e) at different drainage rates and water table heights midway between drains. Then the relationship between d_e and hydraulic conductivity (K) was established to find an approximate value of the impermeable layer depth. The relationship indicated that K converges asymptotically with increasing d_e. The points at the asymptotic inflection were used to compute the average effective d_e (Moustafa and Yomota, 1997). Using this value with the radius and spacing of drains, the depth of impermeable layer below the drains was determined using the tables prepared by Hooghoudt (1940).

To consider the transient flow conditions in evaluation of d_e, equation (3) was also solved for the relationship between K and d_e using the observed water table heights at different time intervals. The results showed that the impermeable layer depth calculated with transient conditions is reaffirmed the previous calculations with steady-state conditions using Hooghoudt equation. The calculated d_e was then used to determine the effective hydraulic conductivity value. These calculated soil parameters are presented in Table 2 for both experimental sites.

Table 2. Calculated soil parameters

Soil parameter	Site 1	Site 2
K, (m/d)	0.14	0.27
f	0.05	0.06
d_e, (m)	0.75±0.06	0.72±0.03
d, (m)	1.50	1.25

K-values as computed that way take into account the perforation restriction of the drain pipes, stratified and anisotropic soil conditions and the inherent spatial variability of hydraulic conductivity encountered in the fields. The calculated K therefore may be of a valuable to be used in the drainage design of similar areas. The use of water table height-drain outflow method presented in this study to calculate f, d_e, and K has moreover great advantages to save time and money which are normally encountered in the field and laboratory works for direct measurements of these soil parameters as required for drainage design. Similar procedures can be repeated for only good representative areas rather to do expensive tests at every site for design purposes.

4.3 Experimental drain spacing equation

To derive an experimental drainage equation, the water table height above drains should be related to different soil properties as any other theoretical transient drainage equation. Thus, using field observations of water table recession, such relation was derived by equating the relative water table recession (m_t/m_o) to the dimensionless variable KDt/fS^2. The most appropriate relationship between these two dimensionless parameters was computed in an exponential form as:

$$y = a e^{bx} \tag{10}$$

in which; $y = m_t/m_o$, $x = KDt/fS^2$

Four drainage events (I,...,IV) were considered for site 1 and two events (V, VI) for site 2. Results are shown in Fig. 2 for drainage events I and V and the coefficients of least squares exponential regression equations for all events are presented in Table 3.

Table 3. Least squares regression coefficients for the experimental drainage equations

Coefficient	Site 1				Site 2	
	I	II	III	IV	V	VI
a	1.09	1.01	1.01	1.11	1.04	1.05
b	-13.002	-14.988	-14.032	-14.048	-14.043	-14.061
R^2	0.990	0.949	0.960	0.936	0.906	0.923

I at m_o=62.35 cm (21/5/97 13:00), IV at m_o=74.95 cm (18/5/98 12:15)
II at m_o=84.35 cm (20/2/98 13:17), V at m_o=75.80 cm (15/1/98 13:05)
III at m_o=74.85 cm (2/4/98 18:53), VI at m_o=71.80 cm (25/2/98 10:05)

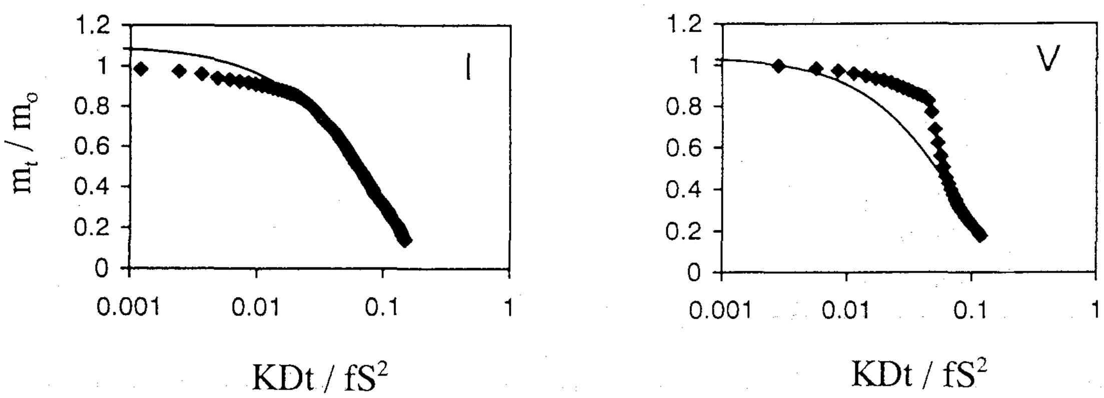

Fig. 2. Relationship between observed relative water table recession (m_t/m_o) and soil hydrological parameters and flow geometry (KDt/fS^2).

These results indicate that average values of 1.05 and -14.029 for a and b, respectively could be used with an experimental drainage equation representing an overall solution for drain depth and spacing combinations in the area of Maekawa district project. This equation might be written as:

$$S^2 = \frac{14.029\ KDt}{f \ln\left(1.05 \frac{m_o}{m_t}\right)} \tag{11}$$

which is identical in form to equation (1). The only difference being the value of the coefficients. However, equation (11) does not assume a constant thickness of the water bearing stratum (D) as has been done with equation (1), but it was derived with variable D since the assumption of constant D is not strictly valid (van Schilfgaarde, 1963). Furthermore, the value of m_t/m_o at $t = 0$ should equal 1.0 and the derived empirical equation (11) satisfactorily verifies this condition at reasonably acceptable accuracy. This equation could be therefore used in drainage design in areas of soil and climatic conditions similar to the experimental sites.

4.4 Simulation of water table recession by different drainage equations

Graphical comparisons in prediction of relative water table recession (m_t/m_o) of different theoretical drainage equations (from Eqns. 1 through 9) to actual field data of drainage events I and II for site 1 and V for site 2 are shown in Fig. 3. The other drainage events within each site were similar to those presented in Fig. 3. Hammad equation [Eqn. (2)] more closely approximates field observations, particularly for last periods of t. Integrated Ernst and Dagan equations [Eqns. (7) and (8)] yield almost same results which are the most inaccurate results when compared with the other equations. They have a tendency to underestimate m_t/m_o for initial values of t, whereas they yield an overestimate for the other periods of t. A summary of deviations in m_t/m_o shown in Fig. 3 is given in Table 4 as the average absolute deviations for all values of m_t/m_o.

Table 4. Average deviations in m_t/m_o predicted by different drainage equations, %

Equation	Site 1				Site 2	
	I	II	III	IV	V	VI
(1)	37	53	41	27	43	42
(2)	10	22	12	12	24	21
(3)	16	54	36	25	46	43
(4)	29	37	27	19	35	34
(5), c=1.0	37	42	31	22	39	39
(5), c=0.8	22	33	23	17	31	30
(6)	45	53	39	28	53	52
(7), c=0.8	45	62	45	31	59	58
(8), c=0.8	45	62	45	31	59	58
(9), c=0.8	31	48	34	24	46	45

Depending upon the actual field observations and the initial water table height (m_o), predictions in m_t/m_o from equation (1) through (9) varied. Equations (2), (5) with c = 0.8, (4), and (5) with c = 1.0 provided the best predictions when compared with the other equations under a variety of field conditions. Equations (4) and (5) with c

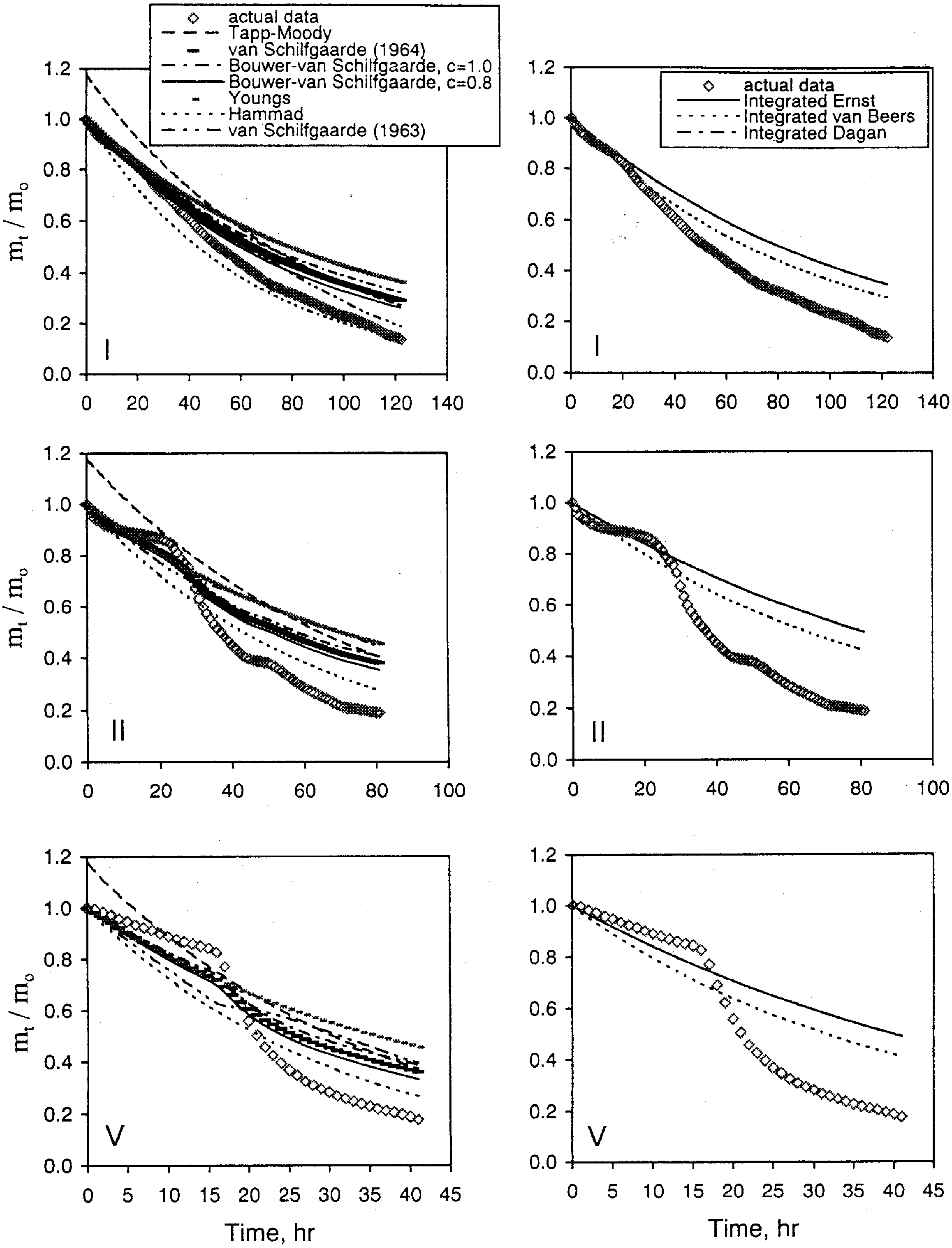

Fig. 3. Graphical comparison of water table recession predictions of different theoretical drainage equations against field data.

= 0.8 produced almost similar results, as expected, since they differ only in minor assumptions regarding the initial shape and subsequent change in shape of the water table during recession. The Hammad equation gives the best results in all drainage events of the two drain spacing of the experimental sites. Deviations in the predictions associated with Integrated van Beers equation [Eqn. (9)] are less than those associated with Integrated Ernst and Dagan equations [Eqns. (7) and (8)]. This result is in agreement with the results obtained by Singh et al. (1992). Generally, all equations gave better agreement but still gave high deviations in m_t/m_0 for site 1 (S = 10 m), except case II, comparing with site 2 (S = 8.5 m). The high deviations in simulating m_t/m_0 may be attributed to the use of a constant value of drainable porosity (f).

For all six drainage events of the two sites, the most reliable predictions of m_t/m_0 with equation (5), were obtained using c = 0.8. Same results were obtained with equations (7), (8), and (9). Indeed, this was expected because of low values of m_0/S for all the examined drainage events. This may be taken as a verification that the recession of water table is faster midway between drains than near the drains. Same result was obtained by Nwa and Twocock (1969) in a sandy loam soil, whereas Buckland et al. (1987) showed that the recession of water table profile was uniform between the drains in a clay loam to clay soil and a c-value of 1.0 yielded more reliable predictions in drain spacing as compared with c = 0.8 using equation (5).

5. CONCLUSIONS

Water table recessions and drainage outflows following rainfall events were measured for drain spacings of 10 and 8.5 m in two experimental sites for two years. This gives an excellent opportunity to determine the most important soil parameters needed for drainage design and to compare different predictions of water table recession using theoretical drainage equations with the actual field data under different boundary conditions.

Results indicate that the order of preference for the most reliable four equations in simulating water table recession in the experimental sites is Hammad, Bouwer and van Schilfgaarde with c = 0.8, van Schilfgaarde (1964), and Bouwer and van Schilfgaarde with c = 1.0. A c-factor of 0.8 was found preferable to 1.0 with the newly developed integrated equations of Ernst, Dagan, and van Beers. Integrated van Beers equation gives more reliable predictions for all boundary conditions examined comparing to Integrated Ernst and Dagan equations that gave the most worst results for all drainage events tested.

ACKNOLEDGEMENTS

The authors would like to express their thanks and appreciation to the Associate Professor Takeshi Miura for his help during the field work and manipulating the water level recorder's field data. Appreciation is also expressed to all students of Irrigation and Drainage Laboratory for their support during the course of this study. Special thanks to the owners of the experimental sites in Maekawa district area for providing their lands for conducting this study. The assistance of Soja City Administration Office staff is also acknowledged.

REFERENCES

Bouwer, H. and van Schilfgaarde, J., 1963. Simplified method of predicting fall of water table in drained land. Trans. ASAE 6(4): 288-291, 296.

Buckland, G.D., Sommerfeldt, T.G. and Harker, D.B., 1987. A field comparison of transient drain spacing equations in a southern Alberta lacustrine soil. Trans. ASAE 30(1): 137-142.

Dagan, G., 1964. Spacing of drains by an approximate method. J. Irrig. and Drain. Div. ASCE 90(1): 41-66.

Dieleman, P.J. and Trafford, B.D., 1984. Drainage testing. FAO Irrigation and Drainage Paper 28, 172 pp.

Dumm, L.D., 1954. New formula for determining depth and spacing of subsurface drains in irrigated lands. Agric. Engng. 35(10): 726-730, October.

Dumm, L.D., 1964. Transient flow concept in subsurface drainage: its validity and use. Trans. ASAE 7(2): 142-146, 151.

El-Mowelhi, N.M. and Hermsmeier, L.F., 1982. Tile drainage performance compared to theory. Trans. ASAE 25: 981-983.

Ernst, L.F., 1962. Grondwaterstromingen in de verzadigde zone en hun berekening bij aanwezigheid van horizontale evenwijdige open leidingen. Versl. Landbouwk. Onderz. 67-15, 189 pp.

French, B.E. and O'Callaghan, J.R., 1966. A field test of drain spacing equations for agricultural land. J. Agric. Engng. Res. 11(4): 282-295.

Hammad, H.Y., 1962. Depth and spacing of tile drain systems. J. Irrig. and Drain. Div. ASCE 88(1): 15-34.

Hooghoudt, S.B., 1940. Algemeene beschouwing van het probleem van de detailontwatering en de infiltratie door middel van parallel loopende drains, greppels, slooten en kanalen. Versl. Landbouwk. Onderz. 46-14: 515-707.

Johnston, W.R., Letey, J. and Pillsbury, A.F., 1965. Tile drainage performance compared to theoretical predictions. Trans. ASAE 8(4): 548-549, 552.

Ministry of Agriculture, Forestry and Fisheries (MAFF), Japan, 1979. Design criteria of subsurface drainage. (in Japanese)

Moustafa, M.M., 1998. Time-dependent drainage from root zone and drainage coefficient under different irrigation management levels for subsurface drainage design in Egypt. Irrig. and Drain. Systems 12(2): 141-159.

Moustafa, M.M. and Yomota, A., 1997. Performance of subsurface drainage compared to applied criteria in Japan. Proc., Chugoku-Shikoku Conference No. 52., JSIDRE, PP. 224-226, Takamatsu, Japan.

Nwa, E.U. and Twocock, J.G., 1969. Drainage design theory and practice. J. Hydrology 9: 259-276.

Singh, K.M., Singh, O.P., Sewa Ram and Chauhan, H.S., 1992. Modified steady state drainage equations for transient conditions in subsurface drainage. Agric. Water Mgmt. 20: 329-339.

Skaggs, R.W., Kriz, G.J. and Bernal, R., 1973. Field evaluation of transient drain spacing equations. Trans. ASAE 16(3): 590-595.

Tsutsui, H., 1996. World irrigation VI: Agriculture and irrigation in Japan. J. Irrig. Engng. and Rural Planning 30(2): 7-32.

van Beers, W.F.J., 1965. Some nomographs for the calculation of drain spacings. Bulletin No. 8, ILRI, Wageningen, The Netherlands, 48 pp.

van Schilfgaarde, J., 1963. Design of tile drainage for falling water tables. J. Irrig. and Drain. Div. ASCE 89: 1-11.

van Schilfgaarde, J., 1964. Closure-design of tile drainage for falling water tables. J. Irrig. and Drain. Div. ASCE 90: 71-73.

Youngs, E.G., 1985. A simple drainage equation for predicting water-table drawdowns. J. Agric. Engng Res. 31: 321-328.