

# ***Plant Layout Method Considering Material Handling Cost and Maintainability***

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## SYNOPSIS

This paper presents a method using simulated annealing(SA) and genetic algorithm(GA) to solve the plant layout problem in which the layout is evaluated by material handling cost and maintainability. In the former study about facility layout problem, it was either the minimization of the objective function consisting of transport cost or the maximization of the objective function consisting of closeness rating. In this paper, both transport cost and maintainability were included in the objective function to be minimized. The plant layout problem can be basically classified as the combinatorial optimization problem. For this problem, this paper proposes the heuristic procedures to obtain a suboptimal layout solution by combining SA with GA. From the simulation by computer, it concluded that the method which SA is combined with GA is more efficient than the method which utilizes SA and GA independently.

## 1. INTRODUCTION

Recently, by variability of consumer and short life of products and etc, production system is changed from mass production to job shop type production. So we must construct new production system or reform existing production system<sup>(1)</sup>. Plant layout influences the productivity of whole production system and become the important problem from the point view of the maintenance of facilities. The layout problem in which facilities are to be assigned to locations and in which there are interactions between facilities related with their location belongs to the NP complete problem and only suboptimal solution can be obtained by feasible computational time<sup>(2)</sup>.

In this paper, we propose the layout method for the sub-optimal solution to minimize the transportation cost and to make easy the facility maintainability by using genetic algorithm(GA)<sup>(3)</sup> and simulated annealing(SA). GA is adopted to search large scale solution space and SA is adopted to search small scale solution space.

## 2. PROPOSED FACILITY LAYOUT PROCEDURE

### 2.1 Formulation of the layout problem

#### 2.1.1 The transportation cost

In the facility layout problem in which  $N$  facilities are assigned to  $N$  available locations,  $f(i)$  denotes

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facility which is assigned to location  $P_i$  and also  $f = \{f(1), f(2), \dots, f(N)\}$  denotes the assignment vector of the facility layout. The transportation cost can be expressed in the following equation (1), where the distance between locations  $P_i$  and  $P_j$  is represented as  $d_{ij}$  and the volume of the transportation between facilities  $f(i)$  and  $f(j)$  is represented as  $v_{f(i)f(j)}$ .

$$HC(f) = (1/2) \sum_{i=1}^N \sum_{j=1}^N d_{ij} v_{f(i)f(j)} \quad (1)$$

### 2.1.2 Evaluation of maintainability

After the determination of the facility layout, the facility maintainability influences the running cost of the production system. Accordingly, it is important to take the maintainability of the facility as well as the material handling into consideration at the phase of the planning of the facility allocation.

In this paper, the relationship between the maintainability and the facility layout is expressed by the sum of the product, the closeness rating for maintenance and the reciprocal number of the distance between facilities. The closeness rating for maintenance  $m$  is numerized from mutual relationship between facilities, if absolutely necessary,  $m=6$ , if very important,  $m=5$ , if important,  $m=4$ , if ordinary important,  $m=3$ , if unimportant,  $m=2$ , if undesirable,  $m=1$ , if absolutely undesirable,  $m=0$ . And the closeness rating of maintenance is the element of the matrix of the closeness rating of the maintenance  $M = \{m_{ij}\}$ , where  $i, j = 1, 2, \dots, N$ . And the estimating expression of the maintainability is defined by the following equation and the facility layout which maximizes this equation as much as possible is sought.

$$FM(f) = (1/2) \sum_{i=1}^N \sum_{j=1}^N m_{f(i)f(j)} / d_{ij} \quad (2)$$

But in this paper, to transfer the above mentioned problem to the minimization problem as well as the transportation cost, the estimating expression of the unti-maintainability which can be obtained by taking the reciprocal number of the  $FM(f)$  is adopted in this paper.

$$UFM(f) = (1/2) \sum_{i=1}^N \sum_{j=1}^N d_{ij} / m_{f(i)f(j)} \quad (3)$$

### 2.1.3 The objective function of facility layout

The objective function of the facility layout, where the layout is expressed by the assignment vector  $f$ , consists of the transportation cost and the estimate expression of the unti-maintainability, and is made as small as possible. In this case, to be equally evaluated mutually, the transportation cost and the unti-maintainability are normalized and those are expressed by  $SHC(f)$  and  $SUFM(f)$  respectively and defined as follows.

$$\begin{aligned} SHC(f) &= (1/2) \sum_{i=1}^N \sum_{j=1}^N \frac{d_{ij}}{d_{\max}} \cdot \frac{v_{f(i)f(j)}}{v_{\max}} \\ &= (1/2 \cdot d_{\max} \cdot v_{\max}) \sum_{i=1}^N \sum_{j=1}^N d_{ij} v_{f(i)f(j)} \end{aligned} \quad (4)$$

$$\begin{aligned}
SUFM(f) &= (1/2) \sum_{i=1}^N \sum_{j=1}^N \frac{d_{ij} / d_{\max}}{m_{f(i)f(j)} / m_{\max}} \\
&= (m_{\max} / 2 \cdot d_{\max}) \sum_{i=1}^N \sum_{j=1}^N d_{ij} / m_{f(i)f(j)}
\end{aligned} \tag{5}$$

,where

$$\begin{aligned}
d_{\max} &= \max_i \max_j d_{ij} \\
v_{\max} &= \max_i \max_j v_{ij} \\
m_{\max} &= 6
\end{aligned} \tag{6}$$

Consequently the objective function of the facility layout in this paper is defined by the following equation and the layout which has the evaluation value as small as possible is sought.

$$EV(f) = SHC(f) + SUFM(f) \tag{7}$$

## 2.2 The Application of SA to the Facility Layout Problem

### 2.2.1 Outline of SA

Neural network is classified as the layered network and the bidirectional associative network. The former consists of the several layers which are connected from the input layer toward the output layer and are called the perceptron network too. The latter is called as Hopfield network and in this network synapse between neurons is symmetrically interconnected. Furthermore, neurons change their state synchronously and finally reach the equilibrium state<sup>(4)</sup>. In Hopfield network, the energy function in which the lowest energy state corresponds to the best solution of the combinatorial problem was introduced from the analogy between the combinatorial optimization and the statistical thermal mechanism<sup>(5)</sup>. Furthermore, Hinton et al.<sup>(6)</sup> introduced Boltzmann machine into Hopfield network to give the probabilistic motion and prevent the convergence at the local minima.

About the neuron  $i$ , the probability of  $u_i=1$  is obtained by the next equation.

$$\Pr(u_i = 1) = 1 / \{1 + \exp(-\text{sum}_i / T)\} \tag{8}$$

,where  $\text{sum}_i$  denotes the sum of input to the neuron  $i$  and  $T$  expresses the temperature of the network. At the limitation when  $t=0$ , Boltzmann machine acts as same as Hopfield network. The time for reaching the optimal solution can be reduced by simulated annealing in which the cooling is done sufficiently slow.

### 2.2.2 Facility layout procedure introduced SA

In order to apply SA to the seeking process of the minimum facility layout solution, the change of the evaluation value of the facility layout is made equivalent to the change of energy of the network when the layout vector is altered from  $f_k$  to  $f_l$  by the transition of the network state. And the change of the energy is defined by the next equation.

$$\Delta E = EV(f_k) - EV(f_l) \tag{9}$$

If the transition of the state of neurons is permitted only to the direction of decreasing  $\Delta E$ , there is a possibility for the network to converge at the local minima. To prevent this convergence and to accept the transition to the worse state, the acceptance probability  $p$  of the transition from  $f_k$  to  $f_l$  is defined by

$$p = \begin{cases} 1 & , \text{ if } EV(f_i) \leq EV(f_k) \\ \exp(-\Delta E/T) & , \text{ if } EV(f_i) > EV(f_k) \end{cases} \quad (10)$$

, where if  $p=1$ ,  $f_i$  is chosen. If  $p \neq 1$ ,  $p_0$  ( $0 \leq p_0 < 1$ ) as the value of selection is given. Then if  $p > p_0$ ,  $f_i$  is selected as the assignment vector. On the other hand, if  $p \leq p_0$ ,  $f_k$  is selected.

$T$  denotes the temperature of the network and  $T$  is started at a high temperature and is gradually reduced. This control way of temperature corresponds to the annealing of physical system. In this paper, is adopted the cooling schedule<sup>(7)</sup> which is expressed by the next equation (11).

$$T_r = 10.0 * (0.9)^{r-1} \quad (11)$$

, where  $r$  denotes the number of repetition.

### 2.3 Application of GA to the Facility Layout Problem

GA is one of the random search method based on the mechanics of natural genetics and natural selection by reproduction, crossover and mutation, and in this algorithm the change of generation is repeated and the individuals which have the higher fitness are increased and finally the suboptimal solution is obtained among these individuals<sup>(8)~(11)</sup>.

When GA is applied simply to the facility layout problem of this paper, the probability for the infeasible solution to be derived may be high because of the occurrence of individuals which have the lethal gene. For example, when the facility is selected from the facilities group ( $F_a, F_b, F_c, F_d, F_e$ ) to be arranged to one of the locations ( $P_1, P_2, P_3, P_4, P_5$ ), we assume that the chromosomes of parent are as follows.

$$\begin{aligned} PA_1 &= (F_a, F_b, F_d, F_e, F_c) \\ PA_2 &= (F_d, F_c, F_a, F_b, F_e) \end{aligned}$$

If the crossover genetic operation in which cut-point is set between the second and the third terms of each chromosomes is adopted, the following chromosomes of the resulting offsprings can be obtained.

$$\begin{aligned} CH_1 &= (F_a, F_b, F_a, F_b, F_e) \\ CH_2 &= (F_d, F_c, F_d, F_e, F_c) \end{aligned}$$

From the result of  $CH_1$ , the facility  $F_a$  is located into two location  $P_1$  and  $P_3$ , and  $F_b$  is also arranged to two locations  $P_2$  and  $P_4$ . This result shows the conflict to the condition of one facility to one location correspondence, and  $CH_2$  has the same conflict with the condition. Therefore  $CH_1$  and  $CH_2$  have the lethal gene and both are the infeasible solutions of the facility layout problem. So, the coding method to convert to the string of chromosome is used to make the application of GA to the facility layout problem possible.

- (1) Make the facility standard list in which suffixes of the notations showing the facilities are arranged in alphabetical order.
- (2) Convert the ordinary number from the beginning of the facility standard list to the string of the chromosome, where the element of the facility standard list is eliminated if it has been already coded. For example, we assume that the strings of parents  $PA_1$  and  $PA_2$  above mentioned are as follows.

$$PA_{1c} = (1, 1, 2, 2, 1), \quad PA_{2c} = (4, 3, 1, 1, 1).$$

(3) Make the genetic operation such as crossover and mutation to these strings. And we assume that the offsprings of crossover in which the cutpoint is set between second and third term are as follows.

$$CH_{1c} = (1, 1, 1, 1, 1), \quad CH_{2c} = (4, 3, 2, 2, 1)$$

(4) After the genetic operation, transform strings to the allocation vectors inversely.

For example,  $CH_{1c}$  and  $CH_{2c}$  are transformed inversely and the allocation vectors of offsprings can be obtained as follows.

$$CH_1 = (Fa, Fb, Fc, Fd, Fe), \quad CH_2 = (Fd, Fc, Fb, Fe, Fa)$$

#### 2.4 Facility Layout Technique by GA & SA

A search from the wide range is difficult only by SA<sup>(12)</sup>, because SA works on only one solution at a time. On the other hand, GA searches the solution among the population of solutions and explores the search space by generating new generation. But in GA, there is the possibility of the premature convergence.

Therefore, for the facility layout problem, the approach in which SA and GA are combined is proposed in this paper. SA is applied to the population of the solutions and for the transition of the state GA is adopted in this approach. This approach will hereinafter be abbreviated SAGA method.

(step 1) Give the initial population of the allocation vectors  $S_0 = \{f_1, f_2, \dots, f_m\}$  and the maximum repetition number  $NN$  and set  $i=0$ .

(step 2) Search an arrangement vector  $f^*$  which can satisfy the equation (12).

$$EV(f^*) = \min_i EV(f_i) \quad (12)$$

(step 3) Apply GA and choose two arrangement vectors (parents vector) at random from  $S_0$ , and generate two new vectors by the genetic operation of the crossover and the mutation. Form the  $m$  allocation vectors expressed as  $S_1$  in total in this way. And obtain the sum of sets by the equation (13).

$$U = S_0 \cup S_1 \quad (13)$$

(step 4) Arrange  $EV(f)$  in descending order, where  $f \in U$ .

$$EV(f_1) \geq EV(f_2) \geq \dots \geq EV(f_m) \geq \dots \geq EV(f_{2m}) \quad (14)$$

Select  $m$  allocation vectors from this ordered  $EV(f)$  and they are expressed as  $S_0$  newly.

(step 5) If

$$EV(f_i) \leq EV(f^*), \quad (15)$$

then make  $f^* = f_i$ , and go to (step 6).

If

$$EV(f_i) > EV(f^*), \quad (16)$$

then

$$\Delta E = EV(f_i) - EV(f^*) \quad (17)$$

And calculate the acceptance probability according to the equation (18).

$$p = \exp(-\Delta E/T_i) \quad (18)$$

,where

$$\Delta E = EV(f_i) - EV(f^*). \quad (19)$$

And, give  $p_0$  by the unique random number of range  $[0,1]$ . If  $p_0 < p$ , then  $f^* = f_1$ . If not, leave  $f^*$  as it is .

(step 6) Make  $i$  increment as  $i = i + 1$ .

(step 7) If  $i < NN$ , return to (step 3). If  $i \geq NN$ , finish the calculation process and get that suboptimal solution  $f^*$ .

### 3. RESULT OF SIMULATION AND CONSIDERATION

The distance matrix used in this experiment was made from the random number of the normal distribution  $N(10.0,5.0)$ . On the other hand, ten volume matrices made from the normal distributed random numbers of constant mean,  $m=10.0$  and coefficient of variations : $CV=0.05,0.10,0.15,\dots,0.50$  were used for the simulation experiment. These data were corresponding to the change in the flow for the machine equipment by the change of the product model. And, as for the matrix of the closeness rating for maintenance, the element consisted of the uniform distributed random number of the range  $[0,6]$ . Furthermore, the number of the facility and the location was varied from 5 to 50 in steps of 5<sup>(13)</sup>.

The initial solutions to each problems were introduced by ATSP<sup>(14)</sup>, HURWICZ<sup>(15)</sup>, MAT<sup>(16)</sup> which were classified into the construction procedures, and heuristic search procedure. And the sub-optimal solution was obtained by SAGA belonging to a improvement procedure and for the sake of confirmation of effectiveness of this layout method, the solution of SAGA was compared with the one of SA and GA which were the layout method consisting of simulated annealing and GA respectively.

#### 3.1 Evaluation Value of Facility Layout

The evaluation value of the equipment arrangement was influenced significantly by the number of the facility which is the character of the input data, and by the coefficient of variation. In Fig.1, we indicated the averaged evaluation values of SA,GA and SAGA in which the initial layout solutions were introduced by 4 construction procedures above mentioned.

A difference for the evaluation value in each technique increased together with the number of the facility. When the evaluation value of the SA technique was made 100%,the evaluation value of GA was 97.9 % and that of SAGA was 86. 0% on the average. And, as for the result of two way classification of analysis of variance in which two factors were the number of facility and the facility layout method, the significant differences of each two factors were confirmed at the 5% significant level. And the relationship between the coefficient of variation of volume matrix and the evaluation value of the improved solution by each facility layout methods was indicated in Fig.2,where the evaluation value of the SA technique was made 100%.

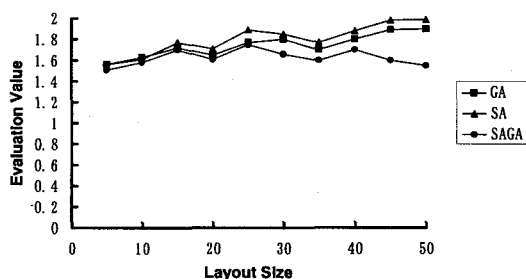


Fig 1. Evaluation Value

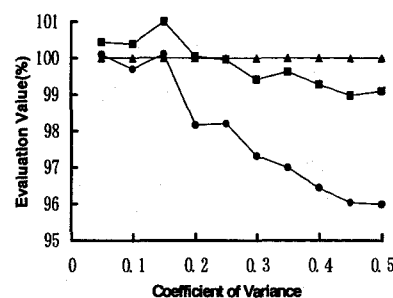


Fig 2. Evaluation Value and Coefficient variation

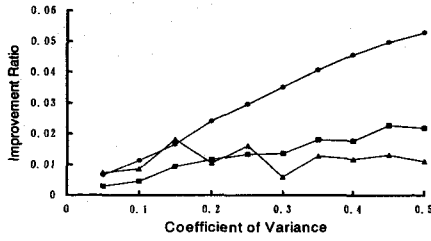


Fig 3. Coefficient Variation and Improvement Rate

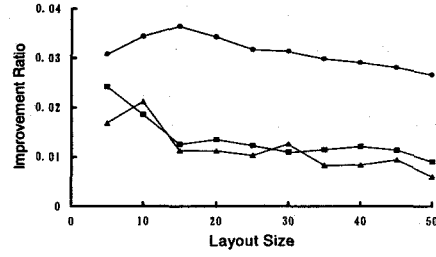


Fig 4. The Number of Facility and Improvement Rate

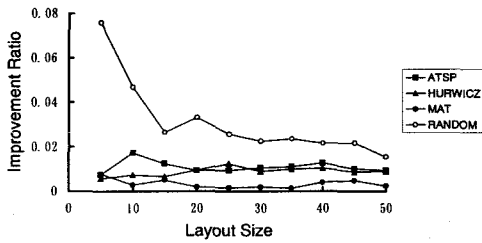


Fig 5. Effect of Layout Size of Initial Solution

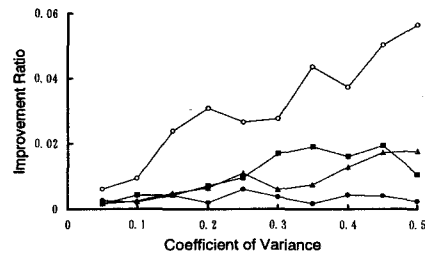


Fig 6. Effect of Coefficient of Variation of Initial Solution

Therefore, it was clarified that the efficient facility layout could be obtained by SAGA according to the increasing of the coefficient of variation and the variance of the transportation value.

### 3.2 The Improvement Rate

GA, SA and SAGA which are classified into the improvement procedure look for the most suitable solution by improving the initial facility layout plan. On that occasion, to examine how much the facility layout of the initial plan was improved, the improvement rate *imp* was defined in the equation (20) and was calculated, where  $EV_i$ ,  $EV_{imp}$  were the evaluation values of the initial layout and the improved layout respectively.

$$imp = (EV_i - EV_{imp}) / EV_i \tag{20}$$

An improvement rate of SAGA was significantly larger than that of GA and SA from the result of Fig. 3 and Fig.4.

The improvement rate of each technique became large in accordance with the change of the coefficient of variation of the volume matrix. Especially in SAGA, the difference was remarkable in comparison with other improvement techniques. The coefficient of variation of the volume matrix becomes large in the job shop type production and SAGA becomes an effective facility layout technique in such a production system.

Furthermore, there are few influences of the number of the facility on the improvement ration of each

method. But, the improvement rate of SAGA became stably high value. To examine the influences of the construction procedures, the relationship of the number of facility and the improvement rate of SAGA were shown in Fig. 5 and that of the coefficient of variation of the volume matrix and the improvement rate of SAGA were expressed in Fig. 6.

From these results, it was clarified that the improvement rate became large when the layout plan of random search method was used as the initial solution. The difference could be evident when the number of the equipment was small and the coefficient of variation of the volume matrix was large.

Therefore, the significant influences of the initial layout plan on the improvement procedure was clarified and the effectiveness of the construction procedures except the random search method could be confirmed.

#### 4. CONCLUSIONS

In this paper, we proposed the facility layout method in which simulated annealing of Boltzmann machine and genetic algorithm were used together. And the evaluation values of facility layout and the improvement rate of SAGA were compared with those of SA and GA in computational experiment and the following results were obtained.

- 1) The evaluation value of facility layout plan by SAGA became smallest among SAGA, SA and GA. Therefore it was clarified that SAGA could present the efficient facility layout plan.
- 2) The improvement rate of SAGA became largest from the comparison with those of SA and GA, so the sub-optimal layout solution could be efficiently obtained by SAGA.
- 3) Because the initial layout solution influenced the improvement rate of improvement procedure, the time for the convergence to the sub-optimal solution became shorter by using the initial solution by construction procedures such as ATSP, HURWICZ and MAT.

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