Memoirs of the Faculty of Engineering, Okayama University, Vol. 43, pp. 49-54, January 2009

Position Control of 2-Link SCARA Robot by using Internal Model Control

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(Received November 22, 2008)

In this paper, the controlled target is the SCARA robot with two links, and the object is fine control of the arm head position of the robot. To attain the object, Internal Model Control (IMC) is introduced. A nonlinear equations are for robot dynamics formulated by solving Lagrange equation, and is linearized to design control system by IMC. The controller of IMC is designed or synthesisted as the inverse system of the linearized model, and IMC filter model is selected. Also, reference filter is introduced to make the improvement of performance. The result of control performance by IMC is compared with that of PID numerically, accuracy and incoherency are confirmed.

1 INTRODUCTION

In general, SCARA(Selective Compliance Assembly Robot Arm) robot, is a type of horizontal drive, and is used as the arrangement of parts to a printed wiring board and a product assembly, and is usually controlled by PID compensator to attain an exact moving [1]. In controlling the SCARA robot, the arm head is moved by motors and the system is a MIMO (Multiple-Input Multiple-Output) system, so each link is required its accuracy and incoherency. In general, gains of PID controller are determined through trial and error processes by skilled experts, even though the adjustment of gains following to alterations of modeling errors and frictions are difficult.

In this study, the controlled target is the SCARA robot with two links, and the arm head of the robot is tried to be controlled by using IMC (Internal Model Control) [2],[6]. In general, controller of IMC model is made by the inverse of the target system model, and by using the model in control system performance is developed. The difference the output of the target and the model is feedbacked and the output of the target is good performance by reflecting the difference to the controller. A nonlinear equation is formulated solving Lagrange equation and then it is linearized [3],[4],[5]. In this reaserch, the controller is constructed by inverse system without solving equations for controller [6]. IMC filter is a selected model, but performance is poor only by strutting IMC filter. To develop the performance, a reference filter is constructed. Through these improvement of IMC system, the adjustment gain becomes to be easily determined. Designing is confirmed the effects of the reference filter for direct drive motor by the experiment [7]. The control performance by IMC is compared with that of PID numerically, the accuracy and the incoherency are confirmed.

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2 SCARA ROBOT MODEL

In this section, a nonlinear equation about the SCARA robot with two links is formulated by solving Lagrange equation, and is linearized to design control system by IMC.

2.1 Lagrange Equation of Motion for SCARA Robot

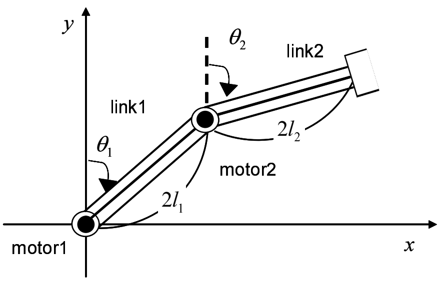

Fig. 1 In SCARA robot model

Fig. 1 shows a model of SCARA robot with two links. In this model, parameters are defined as shown in Table 1, where the center of gravity is given at the center of a link to simplify the calculation, l_i is a distance between link edge to the gravity point. θ_i is an angle between y axis and link.

Table 1 Parameters

m_i	mass of link i [kg]
l_i	length of link i [m]
I_i	moment of inertia of link i [kgm^2]
f_i	viscous frictional coefficient of shaft of the motor i [kgm^2/s]
J_i	moment of inertia of motor i [kgm^2]

From Fig. 1, coordinate of the center of gravity for links are expressed as follows.

$$x_1 = l_1 \sin\theta_1 \tag{1}$$
$$y_1 = l_1 \cos\theta_1 \tag{2}$$
$$x_2 = 2l_1 \sin\theta_1 + l_2 \sin\theta_2 \tag{3}$$
$$y_2 = 2l_1 \cos\theta_1 + l_2 \cos\theta_2 \tag{4}$$

Also, coordinate of potision of arm are as follows.

$$x = 2l_1 \sin\theta_1 + 2l_2 \sin\theta_2 \tag{5}$$
$$y = 2l_1 \cos\theta_1 + 2l_2 \cos\theta_2 \tag{6}$$

If it assumes that the arm is homogeneous, then $dm = \rho dx$ and the density is $\rho = m_1/2l_1$. The moment of inertia around vertical axis to link 1, I_{g1}, and moment of inertia around vertical axis to link 1,I_1, through that link edge are gained as follows.

$$I_{g1} = \int_{-l_1}^{l_1} r^2 dm = \rho \int_{-l_1}^{l_1} x^2 dx = \frac{2}{3}\rho l_1^3 \tag{7}$$
$$I_1 = I_{g1} + m_1 l_1^2 = \frac{1}{3} m_1 l_1^2 + m_1 l_1^2 = \frac{4}{3} m_1 l_1^2 \tag{8}$$

Similarly, I_2 is calculated by

$$I_2 = \frac{4}{3} m_2 l_2^2 \ . \tag{9}$$

Considering moment of inertia of each motor, then kinetic energy function $\mathcal{T}$ is given by

$$\mathcal{T} = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} I_2 \dot{\theta}_2^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 \{4 l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 4 l_1 l_2 \cos(\theta_2 - \theta_1) \dot{\theta}_1 \dot{\theta}_2\} \ . \tag{10}$$

The potential energy function $\mathcal{U}$ is neglected because SCARA robot moves only on $x - y$ plane. So, the potential energy function is

$$\mathcal{U} = 0 \ . \tag{11}$$

Besides, the dissipative energy function $\mathcal{D}$ is expressed by considering friction f as follows.

$$\mathcal{D} = \frac{1}{2} f_1 \dot{\theta}_1^2 + \frac{1}{2} f_2 (\dot{\theta}_2 - \dot{\theta}_1)^2 \tag{12}$$

By assigning these energy functions to Lagrange equation (13), generalized coordinate θ is by equation (14) and generalized force T is given by equation (15), the nonlinear equation is formulated as equation (16)

$$\frac{d}{dt}\frac{\partial \mathcal{T}}{\partial \dot{\theta}_i} - \frac{d}{dt}\frac{\partial \mathcal{U}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{T}}{\partial \theta_i} + \frac{\partial \mathcal{U}}{\partial \theta_i} + \frac{\partial \mathcal{D}}{\partial \dot{\theta}_i} = \tau_i \tag{13}$$

$$\theta = [\theta_1, \theta_2], \tag{14}$$
$$T = [\tau_1, \tau_2] \tag{15}$$

$$J_0(\theta)\ddot{\theta} + D_0(\theta)\dot{\theta}^2 + F_0\dot{\theta} = T \tag{16}$$

where,

$$J_0(\theta) = \begin{bmatrix} J_{11} & J_{12}\cos(\theta_2 - \theta_1) \\ J_{21}\cos(\theta_2 - \theta_1) & J_{22} \end{bmatrix}$$
$$D_0(\theta) = \begin{bmatrix} 0 & -J_{12}\sin(\theta_2 - \theta_1) \\ J_{21}\sin(\theta_2 - \theta_1) & 0 \end{bmatrix}$$
$$F_0 = \begin{bmatrix} f_1 + f_2 & -f_2 \\ -f_2 & f_2 \end{bmatrix} \tag{17}$$

In equation (17), J_{**} are terms about mass, length and moment of inertia except angle in matrix J_0, D_0.

$$J_{11} = (m_1 + 4m_2)l_1^2 + I_1 + h_1 \quad (18)$$
$$J_{12} = 2m_2 l_1 l_2 \quad (19)$$
$$J_{21} = J_{12} \quad (20)$$
$$J_{22} = m_2 l_2^2 + I_2 + h_2 \quad (21)$$

2.2 Derivation of the Linearized Equation

The gained equation is a nonlinear equation, so it isn't able to derive transfer function. Therefore, the nonlinear equation is linearized and using it the controller is designed for IMC system. Also, in IMC system, linearized transfer function is used to design control system as its model $\tilde{P}$.

At first, torque is considered. A potential energy is zero in SCARA robot, so the torque $\bar{T}$ to maintain thw gravity at equilibrium is not necessary.

$$\bar{T} = 0 \quad (22)$$

Next, each equilibrium position is considered. Here, in the equation (6) note similarities between matrix J_0 and D_0, all θ is consists of $\theta_2 - \theta_1$. If angle of two links is same as follows,

$$\bar{\theta}_1 = \bar{\theta}_2 \quad (23)$$

then inside of trigonometrical function $\theta_2 - \theta_1$ becomes 0 then trigonometrical function is always 1 or 0. That is, when two links are parallel, it is able to linearize without considering linearized angles.

By putting the parameter as follow

$$X = [\theta, \dot{\theta}]^T, \quad (24)$$

the linear equation with 4 order is gained.

$$\dot{X} = AX + BU \quad (25)$$
$$Y = CX \quad (26)$$

where,

$$A = \begin{bmatrix} 0 & E \\ 0 & -J^{-1}F \end{bmatrix} : (4 \times 4),$$
$$B = \begin{bmatrix} 0 \\ -J^{-1} \end{bmatrix} : (4 \times 2),$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} : (2 \times 4) \quad (27)$$

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, F = F_0, E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (28)$$

The transfer function matrix $\tilde{P}$ is gained by following equation.

$$\tilde{P}(s) = \frac{1}{s(s+\alpha)(s+\beta)} \begin{bmatrix} k_{11}(s+a), k_{12}(s-b) \\ k_{21}(s-b), k_{22}(s+c) \end{bmatrix} \quad (29)$$

where, all poles and zeros are

$$\alpha, \beta, a, b, c > 0 \quad (30)$$

3 DESIGNING CONTROL SYSTEM

In this section, control system is designed in 3-steps. The first is designing IMC controller, second is IMC filter in IMC system, and the last is the reference filter.

3.1 IMC System

3.1.1 IMC Controller

The first step is to design IMC controller $\tilde{Q}(s)$. Fig.2 shows construction ofIMC system. In IMC system, the

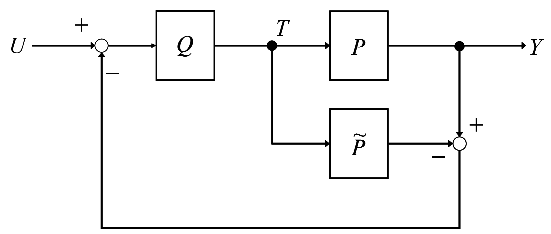

Fig. 2 IMC system

contoller is determined by input type U and target system $\tilde{P}$ as follows

$$\tilde{Q} = \tilde{P}_M^{-1} W^{-1} \{W \tilde{P}_A^{-1} U_M\}_* U_M^{-1} . \quad (31)$$

Here, $\{*\}_M$ describes minimum phase function and $\{*\}_A$ describes allpass function. A symbol $\{\}_*$ describes omitting fraction that include poles after partial fraction expansion. However, in general IMC controller is used inverse system of model $\tilde{P}$. Inverce matrix of model is

$$\tilde{P}^{-1}(s) = -\begin{bmatrix} J_{11}s(s+c) & J_{12}s(s-b) \\ J_{21}s(s-b) & J_{22}s(s+a) \end{bmatrix} . \quad (32)$$

According to IMC system, condition for $\tilde{Q}$ has not unstable poles in transfer function. The equation (32) has only zeros, but it has not pole. So, equation (32) satisfies the condition for controller. Therefore, IMC controller is expressed as follow.

$$\tilde{Q}(s) = \tilde{P}^{-1}(s) \tag{33}$$

3.1.2 IMC Filter

The second step is the design of IMC filter $F(s)$. For robustness $\tilde{Q}$ has to be augmented by low-pass filter $F(s)$. IMC filter is formed to removed refluence of noise and refluence of modeling error. In principle both the structure and the parameters of $f(s)$ should be determined such that between performance and robustness become an optimal compromise. To simplify the design we fix the filter structure and search over a small number of filter parameters, usually just one, to obtain desired robustness characteristics. $F(s)$ is expressed as follows.

$$F(s) = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix} \tag{34}$$

and transfer function $f(s)$ is selected as follow.

$$f_i(s) = \frac{n_i \lambda_i s + 1}{(\lambda_i s + 1)^{n_i}},\ i = 1, 2 \tag{35}$$

where, n_i, is the order of IMC filter, is selected large enough to make IMC controller $\tilde{Q}$ proper, and λ_i is the adjustable parameter.

$$Q(s) = \tilde{Q}F \tag{36}$$

If λ is a large number, control system is reached more robustness, but if λ is small number, control system becomes poor performance. Because in the equation (32) all elements are consist of two order, then IMC controller becomes proper.

$$n_1 = n_2 = 3 \tag{37}$$

In IMC design of the controller for MIMO system is determined by inner-outer factorization, so the controller is uniqueness. However, in this paper the controller is determined by inverse of model without inner-outer factorization, because of invertible model.

3.2 Reference Filter

By above two steps, the design of control system is completed. In third step, reference filter $F_r(s)$ is added to improve performance in Fig. 3. The reference filter is low-pass filter and is formed as follows

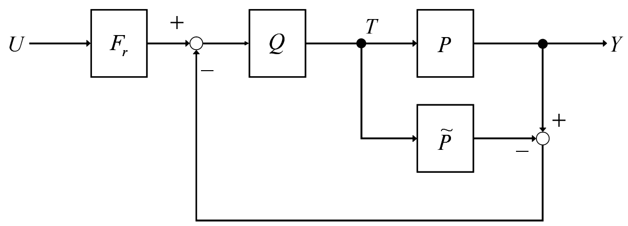

Fig. 3 IMC system with reference filter

$$F_r(s) = \begin{bmatrix} f_{r1}(s) & 0 \\ 0 & f_{r2}(s) \end{bmatrix} \tag{38}$$

here, transfer function $f_{rl}(s)$ is

$$f_{ri}(s) = \frac{1}{T_{ri}s + 1} \tag{39}$$

where, T_{rl} is the adjustable parameter and is determined as follws

$$f_{ri}(s) = \frac{1}{T_{ri}s + 1} = \frac{1}{n_i \lambda_i s + 1} \tag{40}$$

in this form denominator is the same as numerator of equation (35), and by this form performance is expected. In Fig. 3, by introduction F_R transfer function $G(s)$ is expessed as follows .

$$G(s) = \frac{\tilde{Q}PFF_R}{1 + \tilde{Q}F(P - \tilde{P})} \tag{41}$$

In equation (42), if it assumes that the model is perfect about target system, then equation (42) become

$$\begin{aligned} G(s) &= FF_R \\ &= \begin{bmatrix} f_1 f_{r1}(s) & 0 \\ 0 & f_2 f_{r2}(s) \end{bmatrix} \end{aligned} \tag{42}$$

this assumption is not realistic, but it performed an approximation to constatation reference advantage of filter by this assumption. $G(s)$ is consists of IMC filter and reference filter in equation (42), then it is gained as follow.

$$G(s) = \begin{bmatrix} \frac{1}{(\lambda_1 s + 1)_1^n} & 0 \\ 0 & \frac{1}{(\lambda_2 s + 1)_2^n} \end{bmatrix} \tag{43}$$

In equation (43), this system finally depends on only λ_i, because n_i is determined by model. Therefore the structure is simplicity and be able to adjust easily by using IMC system with reference filter.

4 NUMERICAL SIMULATION

Numerical simulation is performed by using the values of the parameters in Table 2.

Table 2 values of parameters in simulation

parameter	value
m_1	0.080 [kg]
m_2	0.064 [kg]
l_1	0.070 [m]
l_2	0.056 [m]
f_1	0.002 [kgm^2/s]
f_2	0.002 [kgm^2/s]
h_1	0.0025 [kgm^2]
h_2	0.0025 [kgm^2]

4.1 Effect of IMC filter

The simulation result is shown in Figures 4 and 5. Fig. 4 shows the result that of IMC system with IMC filter and without IMC filter. The result of IMC system without IMC filter overshoots, but the result of IMC system with IMC filter for performance was improved in Fig. 4. As a result performance becomes good by addition IMC filter in Fig. 4 ($n_1 = 3, n_2, \lambda_1 = 0.01, \lambda_2 = 0.01$).

Fig. 5 shows the result when time constant of IMC filter was changed. From above lin, $T_R < n_i\lambda_i = 0.01$, $T_R = n_i\lambda_i = 0.03$, $T_R > n_i\lambda_i = 0.06$, In Fig. 5, good performance is confirmed by $T_{Ri} = n_i\lambda_i$.

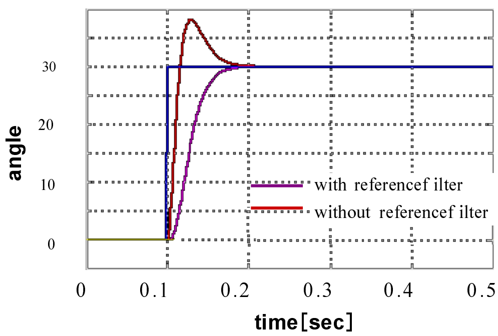

Fig. 4 Effect of refference filter

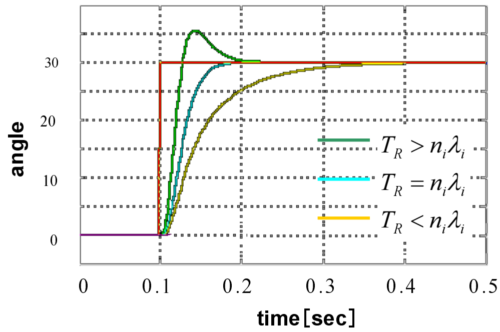

Fig. 5 Effects of refference filter

Table 3 values of parameters in controller

	IMC		PID		
	λ	n	K_p	K_i	K_d
u_1	0.01	3	0.5	0.5	0.5
u_2	0.01	3	1.0	1.0	1.0

4.2 Comparison of IMC with PID

Gain parameters of IMC and PID controller are used the values in Table 3. In Table 3, u_i expressed the controller of ith motor. As step input reference angle input in 0.1 [sec], $\theta_1 = 30$ [deg],$\theta_2 = 0$ [deg]

The result of comparison to PID control is shown in Fig. 6∼9. Fig. 6,7 show the result for reference angle and Fig. 8,9 show the arm position. In Fig. 6, the result of θ_2 by using PID controller is vibrating about 1[deg], but the result of θ_2 by IMC remains 0 [deg], and performance of quick settling time in IMC is gained.

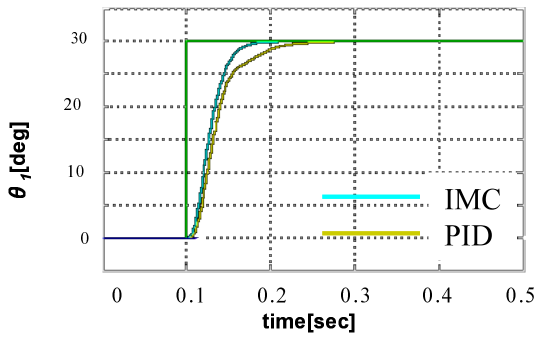

Fig. 6 Comparison for link angel θ_1

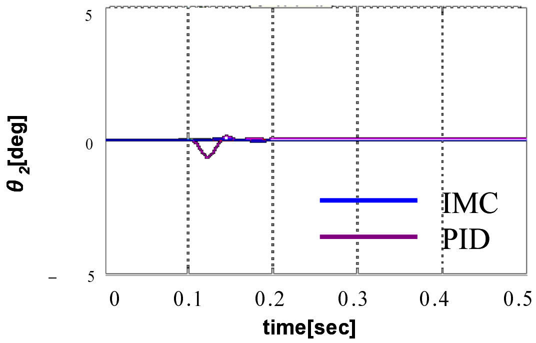

Fig. 7 Comparison for link angel θ_2

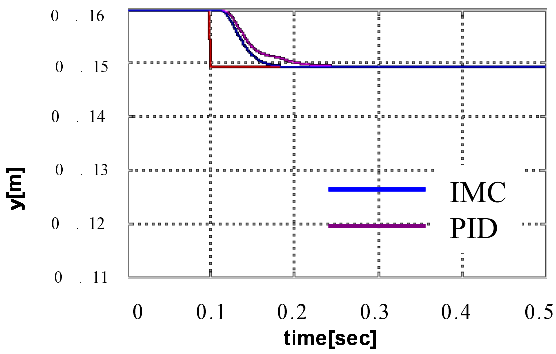

Fig. 9 Comparison for arm position y

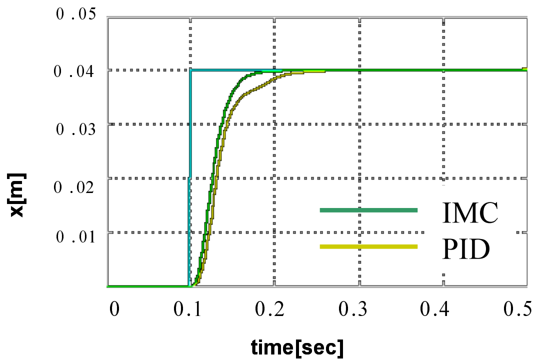

Fig. 8 Comparison for arm position x

5 CONCLUSIONS

IMC system with reference filter is used to attain good performance in designing the controller of the SCARA robot with two links. And then the control is determined easily by simple manner. The control performance by IMC is compared with that of PID numerically, and the superiority of IMC is confirmed.

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