

Tracking Control of Combination Vehicles with Preview Feedforward Compensation

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A tractor-trailer vehicle in the factory might move on the route determined beforehand. However, automation of a tractor-trailer vehicle is difficult so that it is necessary to consider the nonlinearity of a vehicle and a trailer. In this research, the effective tracking control method of a tractor-trailer vehicle is proposed. The control method using time-state control form for a preview control is presented, and be applied a tractor-trailer vehicle with nonholonomic characteristics. Results of numerical experiments are presented to check effectiveness of the proposed control method.

1 INTRODUCTION

A tractor-trailer vehicle is widely used to carry materials from the supplied stock part to the conveyance part in the factory. Because there is an advantage that the large amount of supplied materials can be transported by a tractor-trailer with low-cost. The operation requires a lot of skill for driver when a tractor-trailer vehicle moves along a curved law and parks into a garage. Because of such a difficult operation, the drive support of four-wheeled vehicle for the parking support system is appeared in advance. However, automation is not seen in the tractor-trailer vehicle. The tractor-trailer vehicle cannot be stabilized by a static feedback that is called nonholonomic system. To deal with this problem, using a time varying controller method[1] and a time-state control form method[2], a new control system is proposed. The proposed control method is applied to the tracking control of the vehicle. At the same time, a method using a geometric transformations[3] is proposed. In many cases, the tractor-trailer carrier in the factory moves on the route

decided beforehand. Therefore, it may lead large contribution in the automation of the driving a tractor-trailer vehicle if the new control method by which the vehicle follows the route automatically. However, it is limited to a two-wheeled or a three-wheeled vehicle that is examined beforehand, and the tractor-trailer vehicle is not treated. In this research, the purpose is to cultivate tracking control method of the tractor-trailer vehicle. It assures a good follow up capability using preview control for the problem where the target route is known beforehand. Moreover, because the vehicle is nonholonomic system, time-state control form is used in the application of the proposed method. The effectiveness of the proposal method is examined through numerical experiments and the laboratory experiments.

2 TRACTOR-TRAILER VEHICLE MODEL

The tractor-trailer vehicle model that treats by this research is shown in Fig.1.

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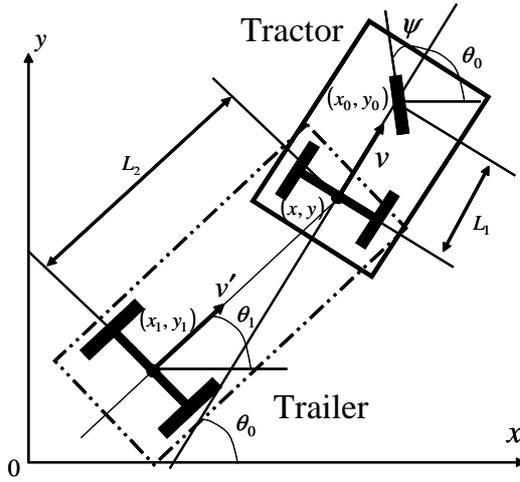


Fig. 1 Tractor-trailer vehicle model

L_1 and L_2 are length of the tractor and trailer of the vehicle, (x, y) is coordinate of midpoint of the rear wheels of a tractor, v and v' are the speeds of the vehicle. Assuming that the wheels don't skid, velocity constraints are given the Eq. (1).

$$\begin{aligned} \dot{x}_0 \sin(\theta_0 + \psi) - \dot{y}_0 \cos(\theta_0 + \psi) &= 0 \\ \dot{x} \sin \theta_0 - \dot{y} \cos \theta_0 &= 0 \\ \dot{x}_1 \sin \theta_1 - \dot{y}_1 \cos \theta_1 &= 0 \end{aligned} \quad (1)$$

Moreover, the next expressions are obtained as angular velocity restraints.

$$\begin{aligned} \dot{\theta}_0 &= \frac{v}{L_1} \tan \psi \\ \dot{\theta}_1 &= \frac{v}{L_2} \sin(\theta_0 - \theta_1) \end{aligned} \quad (2)$$

Let inputs are $u_1 = v$, $u_2 = \dot{\psi}$. Eq. (1) and Eq. (2) give state equation as shown in Eq. (3) which is non-holonomic system with velocity and position constraint.

$$\frac{d}{dt} \begin{bmatrix} x \\ y \\ \theta_0 \\ \theta_1 \\ \psi \end{bmatrix} = \begin{bmatrix} \cos \theta_0 & 0 \\ \sin \theta_0 & 0 \\ \frac{\tan \psi}{L_1} & 0 \\ \frac{\sin(\theta_0 - \theta_1)}{L_2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (3)$$

3 TRACKING CONTROL OF TRACTOR-TRAILER VEHICLE

This chapter describes the method when the vehicle follows the route. In the following, elements and its functions of the proposed method will be presented.

3.1 Tracking Control Method

It is assumed that the objective route is given as shown in Fig. 2 by discrete sequence of points when the vehicle runs.

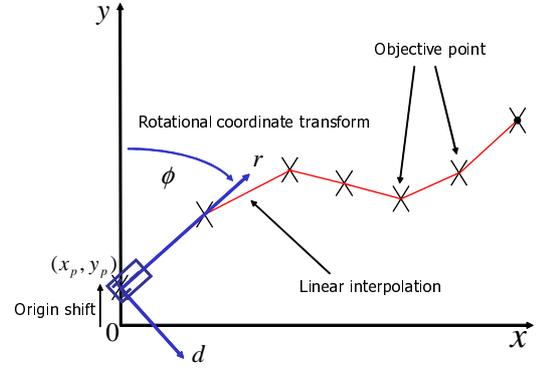


Fig. 2 Objective path for tracking control method

First, it interpolates between the point and the point by linear interpolation and it is assumed a new objective route. Next, y axis is matched to the traveling direction axis by the rotational coordinate transform and origin shift. As a result, the error from the route can be treated with d axis. Moreover, the purpose of it is to evade that the input diverges at the singular point if angle of the vehicle becomes 90 degrees when time-state control form is used. After that, the vehicle is stabilized to r axis by state feedback control from present objective point to next objective point. Finally, it is possible to follow to the target route by repeating from the rotational coordinate transform to the state feedback control.

3.2 Rotational Coordinate Transform and Origin Shift

The $x - y$ coordinate system is converted into $d - r$ coordinate system by the rotational coordinate transform and origin shift. The relation between the $x - y$ coordinate system and the $d - r$ coordinate system is as follows.

$$\begin{bmatrix} d \\ r \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} x - x_p \\ y - y_p \end{bmatrix} \quad (4)$$

(x_p, y_p) is coordinates of the objective point and ϕ is a rotation angle of the coordinate system. As for ϕ , y axis is decided to correspond with objective route of linear interpolation. When the angle of the vehicle in

$d-r$ coordinate system is assumed to be φ_0 and φ_1 , the next expressions are obtained.

$$\begin{aligned}\varphi_0 &= \frac{\pi}{2} - \theta_0 + \phi \\ \varphi_1 &= \frac{\pi}{2} - \theta_1 + \phi\end{aligned}\quad (5)$$

Rewriting Eq. (3) by Eqs. (4) and (5), Eq. (6) can be derived.

$$\begin{aligned}\dot{d} &= \dot{\phi}P_r + \sin \varphi_0 u_1 \\ \dot{r} &= -\dot{\phi}P_d + \cos \varphi_0 u_1 \\ \dot{\varphi}_0 &= \dot{\phi} - \frac{v}{L_1} \tan \psi \\ \dot{\varphi}_1 &= \dot{\phi} - \frac{v}{L_2} \sin(\phi_1 - \phi_0)\end{aligned}\quad (6)$$

Eq. (6) is the state equation concerning r , d , φ_0 , φ_1 , ψ assuming ϕ is constant because the objective route is straight.

$$\frac{d}{dt} \begin{bmatrix} r \\ d \\ \varphi_0 \\ \varphi_1 \\ \psi \end{bmatrix} = \begin{bmatrix} \cos \varphi_0 & 0 \\ \sin \varphi_0 & 0 \\ -\frac{\tan \psi}{L_1} & 0 \\ -\frac{\sin(\varphi_1 - \varphi_0)}{L_2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\quad (7)$$

3.3 Conversion to Time-State Control Form

The constraint like Eq. (1) is called nonholonomic constraint which can't denote position and angle, and the constraint that can't be integrated. The system including such constraints are called nonholonomic system, and it is observed as the system which it is difficult to control generally. There is a time-state control form method to nonholonomic system. This method is suitable for the control of the vehicle. Because it is easy for driving the vehicle forward and back to be easy, and to apply the linear control theory. This chapter explains the control method that uses a time-state control form. Eq. (7) is converted into a time-state control form. Let transformed state variable and input are $[z_1, z_2, z_3, z_4, z_5]^T$ and $[\mu_1, \mu_2]^T$, respectively. As a coordinate transform, consider the following equation.

$$\begin{aligned}z_1 &= r \\ z_2 &= -\frac{\sec^3 \varphi_0 \sec \varphi_1 \tan \psi}{L_1 L_2} \\ &\quad + \frac{\sec^2 \varphi_0 \sec^3 \varphi_1 \sin(\varphi_1 - \varphi_0)}{L_2^2} \\ &\quad \times \{\cos \varphi_0 + \sin \varphi_1 \sin(\varphi_1 - \varphi_0)\} \\ z_3 &= -\frac{\sec \varphi_0 \sec^2 \varphi_1 \sin(\varphi_1 - \varphi_0)}{L_2} \\ z_4 &= \tan \varphi_0 + \sec \varphi_0 \sec \varphi_1 \sin(\varphi_1 - \varphi_0) \\ z_5 &= d - \frac{L_2}{2} \ln \left(\frac{1 + \sin \varphi_1}{1 - \sin \varphi_1} \right)\end{aligned}\quad (8)$$

As a input transform, consider the following equation.

$$\begin{aligned}\mu_1 &= \cos \varphi_0 u_1 \\ \mu_2 &= \frac{3 \tan^2 \psi \sin \varphi_0}{L_1^2 L_2 \cos^5 \varphi_0 \cos \varphi_1} - \frac{\cos^2 \varphi_0 \sin(\varphi_1 - \varphi_0)}{L_2^3 \cos^3 \varphi_0 \cos^4 \varphi_1} \\ &\quad + \frac{\tan \psi \cos \varphi_0 \cos \varphi_1}{L_1 L_2^2 \cos^4 \varphi_0 \cos^3 \varphi_1} \\ &\quad + \frac{3 \tan \psi \cos \varphi_1 \sin \varphi_1 \sin(\varphi_1 - \varphi_0)}{L_1 L_2^2 \cos^4 \varphi_0 \cos^3 \varphi_1} \\ &\quad - \frac{4 \cos \varphi_0 \sin \varphi_1 \sin^2(\varphi_1 - \varphi_0) + \sin^3(\varphi_1 - \varphi_0)}{L_2^3 \cos^3 \varphi_0 \cos^4 \varphi_1} \\ &\quad - \frac{\sec^2 \psi}{L_1 L_2 \cos^4 \varphi_0 \cos \varphi_1} u_2\end{aligned}\quad (9)$$

Eq. (8) and Eq. (9) give a time-state control form.

$$\begin{aligned}\Sigma_1 : \\ \dot{z}_1 &= \mu_1 \\ \Sigma_2 : \\ \frac{d}{dz_1} \begin{bmatrix} z_5 \\ z_4 \\ z_3 \\ z_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_5 \\ z_4 \\ z_3 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mu_2\end{aligned}\quad (10)$$

The expression above is composed of subsystem Σ_1 and Σ_2 . The singular point is generated by conversion when the angle of the vehicle and the steering angle at 90 degrees. This problem is solved by the rotational coordinate transform and origin shift introduced by subsection 3.1. It is the feature of this system that subsystem Σ_2 is differentiated by z_1 , usually temporal differentiation. Therefore, the linear control theory can be applied by monotonously increasing (or decreasing) z_1 just like time, subsystem Σ_2 comes to linear system.

3.4 Preview Control

The tractor-trailer vehicle is able to track the objective route by rotational coordinate transform and origin shift, and evasion of singular point. The error increases very much at the rotational coordinate transform in the next objective point, because it is the settling control to the traveling direction axis between the objective point and next objective point. The vehicular swept path becomes not smooth because of this problem. Moreover, even if the controller parameter is adjusted to prevent this problem, the error from the objective route increases. Therefore, the operation beforehand of cutting the steering wheel before the curve is added to the vehicle. To achieve such an operation, a preview control[4] is taken. When the vehicle understands the reference value in the future, the preview

feedforward compensation that adds the input corresponding to it is designed as shown in Fig. 3.

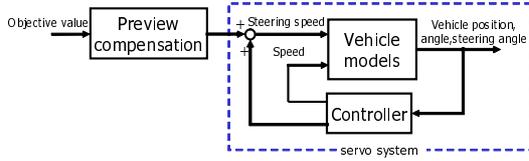


Fig. 3 Block diagram of preview control system

Application to the real system is foreseen and it designs in the discrete-time system. Because it is constant that the input of subsystem Σ_1 is controlled, the preview feedforward compensation like Fig. 3 is designed for subsystem Σ_2 . The expanded system that adds deviation to the state variable is defined, and the preview feedforward compensation is designed to that system.

3.4.1 Servo System

The system that discretizes the subsystem Σ_2 by k is defined as follows.

$$\begin{aligned} Z(k+1) &= A_d Z(k) + B_d \mu_2(k) \\ y(k) &= C_d Z(k) \end{aligned} \quad (11)$$

Here, it is $Z = [z_5, z_4, z_3, z_2]^T$. The deviation e is defined as follows.

$$\begin{aligned} e(k) &= R(k) - y(k) \\ R(k) &: \text{vector of objective value} \end{aligned} \quad (12)$$

The expanded system is given by the first-order deviation e and difference of y .

$$\begin{aligned} \begin{bmatrix} e(k+1) \\ \Delta Z(k+1) \end{bmatrix} &= \begin{bmatrix} I & -C_d A_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} e(k) \\ \Delta Z(k) \end{bmatrix} \\ &+ \begin{bmatrix} -C_d B_d \\ B_d \end{bmatrix} \Delta \mu_2(k) + \begin{bmatrix} I \\ 0 \end{bmatrix} \Delta R(k+1) \end{aligned} \quad (13)$$

Δ is the first-order deviation symbol. Define $Z_0 = [e(k), Z(k)]^T$, the expanded system is shown as follows.

$$Z_0(k+1) = \Phi_0 Z_0(k) + G \Delta \mu_2(k) + G_R \Delta R(k+1) \quad (14)$$

The feedback controller designs the optimal servo system that settles z_5 in the reference value. The reason of not considering the angle of vehicle as reference value is that the angle of vehicle tracks to it naturally. Because the angle of vehicle is adjusted by turning a handle in order to regulate the deviation on d axis to zero.

3.4.2 Design of Preview Control Feedforward Compensation

It is assumed that the target value from M_R step ahead is already-known as of present. The evaluation function including the error and the input is defined.

$$J = \sum_{k=-M_r+1}^{\infty} [Z_0^T(k) Q Z_0(k) + \Delta \mu_2^T(k) H \Delta \mu_2(k)] \quad (15)$$

The reason of summing from $k = -M_r + 1$ is that the input is changed M_R step ago. The reference value is changed as shown in Fig. 4. The evaluation function is minimized by control input as follows.

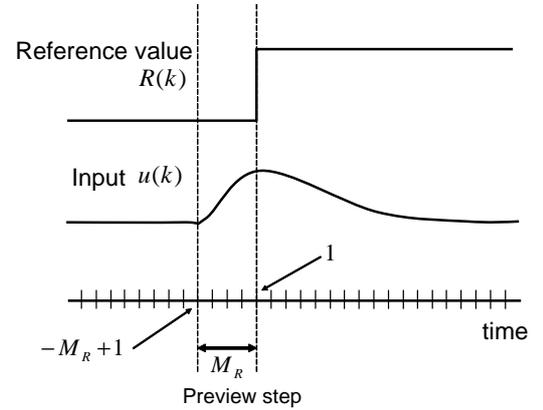


Fig. 4 Preview control

$$\Delta \mu_2(k) = F_0 Z_0(k) + \sum_{j=0}^{M_R} F_R(j) \Delta R(k+j) \quad (16)$$

Here, $F_R(j)$ is term which is added by prevision. It is the control system that the feedforward compensation is added to the optimal servo system. The preview feedforward coefficient $F_R(j)$ is obtained to minimize the evaluation function of Eq. (15).

$$\begin{aligned} F_R(0) &= 0 \\ F_R(j) &= -[H + G^T P G]^{-1} G^T ([\Phi_0 + G F_0]^T)^{j-1} P G_R \quad (j \geq 1) \end{aligned} \quad (17)$$

Eq. (17) is the preview feedforward compensation which adds the input of M_R step ahead in Fig. 3. P is positive definite solution which given by the riccati equation as follows.

$$P = Q + \Phi_0^T P \Phi_0 - \Phi_0^T P G [H + G^T P G]^{-1} G^T P \Phi_0 \quad (18)$$

3.4.3 To Decide Reference Value

Fig. 5 shows how to take the preview reference value. The setup preview reference value is obtained from

present vehicle position in transformed system. In this research, It is used to a preview control that the reference value is obtained by z_5 of Eq. (8), because the value of d axis is regarded only.

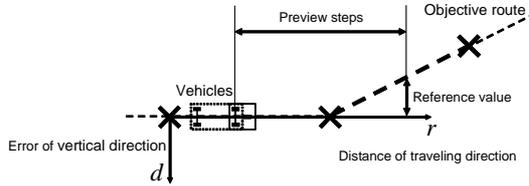


Fig. 5 Prospective reference

4 VERIFICATION BY EXPERIMENT

4.1 Numerical Experimental Result

The effect of preview control is confirmed by the reference route as shown in Fig. 6. The parameter of the vehicle is $L_1 = 0.2[m]$ and $L_2 = 0.19[m]$ in simulation. The preview feedforward compensation was designed by $M_r = 0.2[m]$. The controller of Fig. 3 is designed by section 3.4.1. The results of the control system without the preview feedforward compensation is shown in Fig. 6 and 7, and the results of control system with the preview feedforward compensation is shown in Fig. 8 and 9. The error area and peak input are shown in Table. 1.

Table. 1 Effect of preview control

	without preview	with preview
error area [cm ²]	187.5	174.4
peak input[rad/s]	3.0	-0.8

As against the control system without preview, the error area is decreased at 7[%], and the peak input is decreased at 73[%] in the control system with preview. The effectiveness of preview control follows from these results.

4.2 Experiment of Real System

The experimental system is shown in Fig. 10. The vehicle of the same size as simulation is configured by LEGO MINDSTORMS NXT. The control system at the velocity and the angular velocity of the vehicle used the PI control. The position and the angle of the vehicle are measured by pasting seals with different

color to the body, and processing the image of the USB camera. The control signal generated with PC by the Bluetooth communication is transmitted to the vehicle.

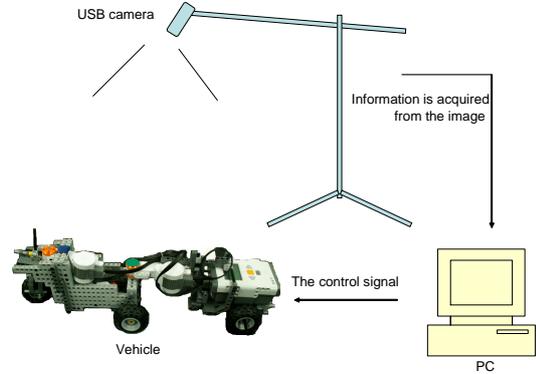


Fig. 10 Experimental system

The result of control system without preview is shown in Fig. 11 and the result of control system with preview is shown in Fig. 12.

In Fig. 11, the vehicle turns in order to correct error, and it slide off the route. Because a large input enters the vehicle as shown in Fig. 7, after that, it cannot return the steering wheel. The vehicle using a preview control follows without sliding off the route. Both Fig. 11 and Fig. 12 show deviations from simulation results, because the velocity control system of the vehicle did not show ideal response.

5 CONCLUSION

In this research, tracking control of the tractor-trailer vehicle for the case of a predetermined reference route was studied. The control design technique was proposed to combine a time-state control form and a preview control. The effectiveness of the proposed method was confirmed by experiments. In the future tasks, considering the quality of the vehicle's velocity control system is to be considered and the control system is to be designed to check the performance with constraints.

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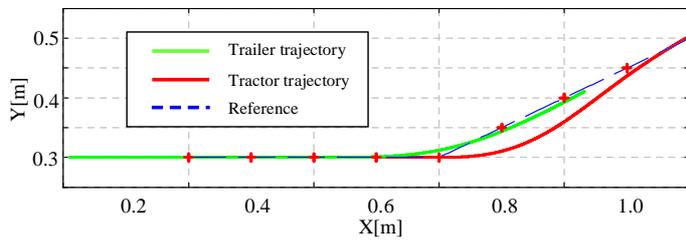


Fig. 6 Swept path of vehicle without preview control

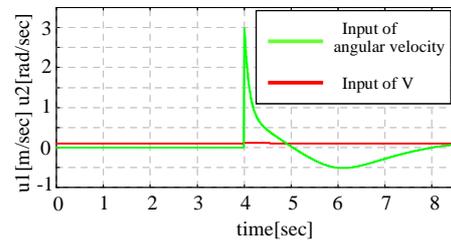


Fig. 7 Input without preview control

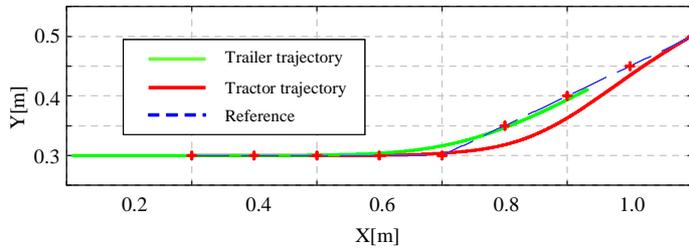


Fig. 8 Swept path of vehicle with preview control

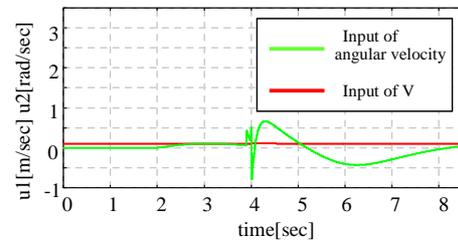


Fig. 9 Input with preview control

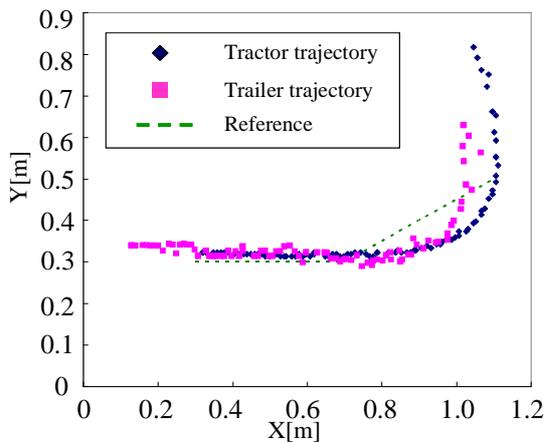


Fig. 11 Swept path of vehicle without preview control

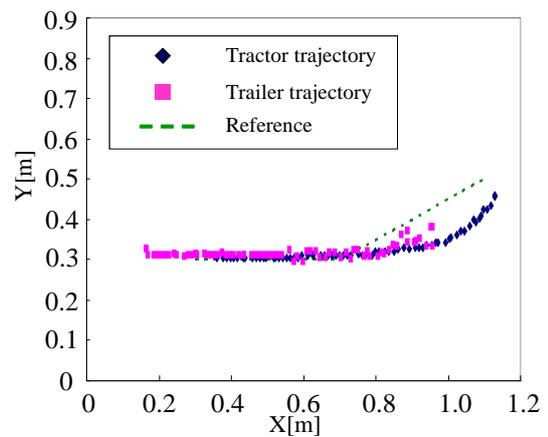


Fig. 12 Swept path of vehicle with preview control

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