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Analysis of Brushless-dc Motor Drive System Taking Account of a Load

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Synopsis

In the power converter of ac drive system the switching behavior of semiconductors causes the distorted voltage and current waveforms, thus the drive characteristics are affected by them. It is, especially, an essential problem that the distorted current waveforms induce the torque ripple. It is assumed in the previous analysis that the input voltage of inverter is a ripple-free dc one. In industry, however, the input voltage of inverter is gained by rectifying the ac voltage. The torque ripple is caused by not only the behavior of inverter but also the behavior of converter. It is required to develop the analysis taking account of the both behaviors.

In this paper, the analysis of brushless-dc motor drive system is proposed taking account of the ac supply, power converter, motor and load. This analytical method is the most suitable one for the analysis of practical system. The effects of factors, e.g. filter constants, inverter frequency, phase relation between ac supply and inverter cycle, load and inertia on the speed variation are revealed by this analysis. Further, the decision of filter constants is discussed from a view of speed variation.

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1. Introduction

There is an increasing tendency to make use of inverter-fed ac machines in the control of variable-speed motors. From a viewpoint of energy saving, the variable-speed control of ac motors using an inverter or a cycloconverter is investigated and adapted to the industry. The adaptation of an inverter-fed permanent magnet synchronous motor (PMSM) is promising in the field of smaller power because of the high efficiency of the motor [1]. The analysis of voltage source inverter (VSI)- or current source inverter (CSI)-fed permanent magnet synchronous motor is described [2-5].

The torque pulsation produced in such drive systems is found to be an important problem, and its influence in brushless-dc drives has been reported in the literature [6]. In industrial applications, the dc source is achieved by rectifying the ac supply and is the input voltage of the inverter. Thus, not only the behavior of inverter but also the behavior of rectifier influences the torque pulsation. Most of papers, however, have only dealt with the torque pulsation of the inverter-fed ac motor which arises due to the behavior of inverter; that is, the input voltage of the inverter is assumed to be a ripple-free dc voltage.

On the other hand, the harmonics of input current and the power factor in the transistor inverter-fed induction motor are discussed [7]. The discussion on the whole performance of motor drive system including the converter, however, is not presented. The authors have proposed the analysis of a brushless-dc drive system taking account of a behavior of converter [8], and discussed the estimation of torque pulsation [9]. In these analyses, however, the motor speed is assumed to be constant, and then it is impossible to discuss the effect of torque pulsation on the speed variation. In practice the effect of torque pulsation is on the speed variation. The speed variation depends on the characteristics of load. Thus, it is necessary to analyze all over the drive system taking account of a load for the synthetic estimation of drive system.

In this paper, we propose the analysis of a brushless-dc drive system taking account of a load to clarify the effect of torque pulsation on the speed variation [10]. In this analysis, all the behaviors of system from a source to a load are taken into account. Thus, it is a powerful tool for the analysis of practical system. The effects of factors, e.g. filter constants, an inverter frequency, a

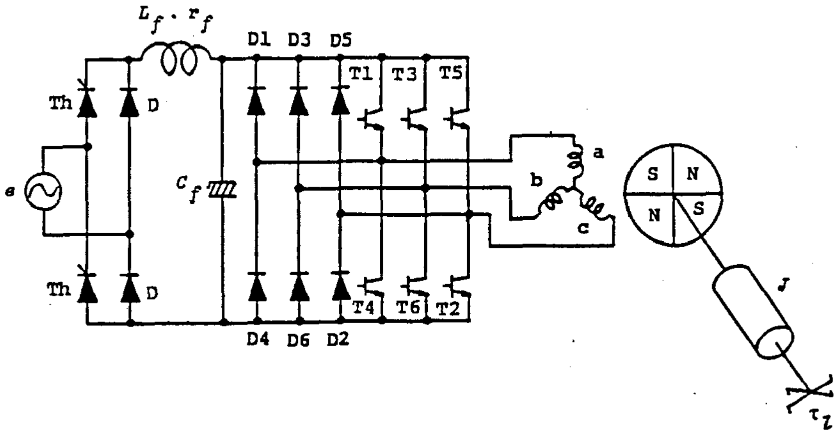


Fig.1 Brushless-dc motor drive system

phase relation between the source and the inverter, a load and an inertia of system on the speed variation are analytically clarified by using this analysis. Furthermore, the decision of filter constants is discussed from a view of the speed variation.

2. Drive System and Speed Variation

Fig.1 shows a brushless-dc drive system. This system is composed of a half-controlled bridge converter (HCBC), an inverter (INV), a permanent magnet synchronous motor (PMSM) and a load. The load has a square of speed-torque characteristic and the energy saving is expected by applying the variable speed control of the motor on this load.

Fig.2 shows the waveforms and the speed variation. The dc voltage is controlled by thyristors, and the voltage has a component of $2n$ multiple of source frequency. The developed torque has harmonics of $6n$ multiple of inverter

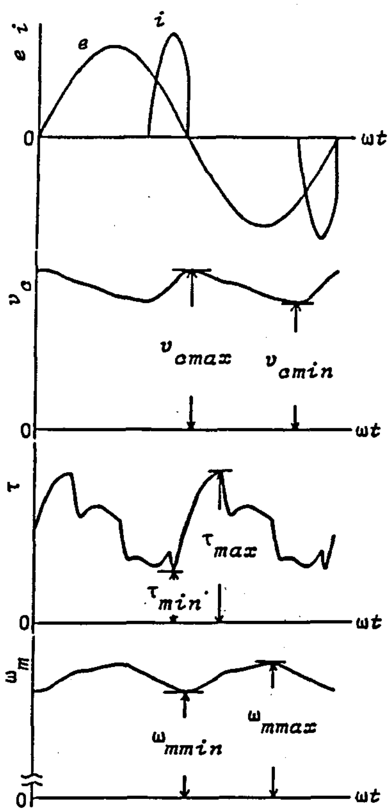


Fig.2 Voltage and current waveforms and speed variation

frequency and $2n$ multiple of source frequency. Then, this torque pulsation induces the speed variation of motor. In industrial applications, the speed variation rather than the torque pulsation becomes important. Thus, it is necessary to develop the analysis of speed variation taking the whole system into account.

3 Procedure of Analysis

3.1 Assumptions and Mathematical Expression of Motor

For the discussion of behaviors of converter and inverter on the performance of system, the next assumptions are introduced.

(1) The source voltage is a sinusoidal waveform, and the source internal impedance is ignored.
(2) The thyristor and diode are ideal switches.
(3) The smoothing reactor has linear characteristics.
(4) The motor constants are balanced in three phase.
(5) The load has a square of speed-torque characteristic.

In order to calculate the performance of system in steady state, the next relation is also satisfied.

$$T_s \cdot p = \frac{T_i}{6} \cdot q \tag{1}$$

According to the assumptions, the PMSM is expressed by the next equation [11].

$$\begin{aligned} v_{ma} &= r_m i_{ma} + L_m \frac{di_{ma}}{dt} + e_{ma} \\ v_{mb} &= r_m i_{mb} + L_m \frac{di_{mb}}{dt} + e_{mb} \\ v_{mc} &= r_m i_{mc} + L_m \frac{di_{mc}}{dt} + e_{mc} \end{aligned} \tag{2}$$

The EMF's are also expressed as follows:

$$\begin{aligned} e_{ma} &= \sqrt{2}\, E_m \sin(\omega_i t+\gamma) \\ e_{mb} &= \sqrt{2}\, E_m \sin(\omega_i t+\gamma-2\pi/3) \\ e_{mc} &= \sqrt{2}\, E_m \sin(\omega_i t+\gamma-4\pi/3) \end{aligned} \tag{3}$$

Where, E_m is denoted by

$$E_m = \sqrt{2}\ \omega_m \cdot k_\omega \cdot \omega_1 \cdot k_L \cdot \phi/(2k_\phi)$$
$$= k_e \cdot \omega_m \tag{4}$$

On the other hand, the dynamic equation of motor-load is given by

$$J\frac{d\omega_m}{dt} = \tau - \tau_l \tag{5}$$

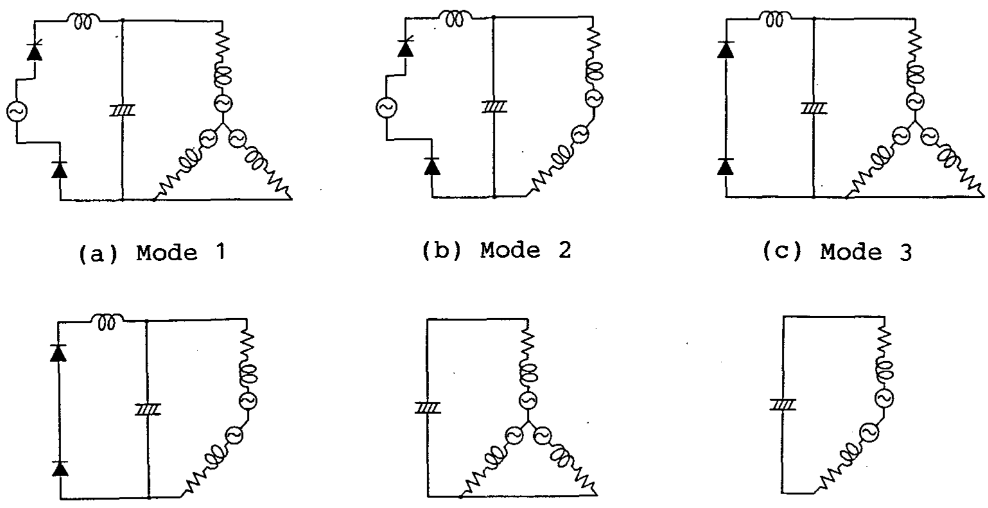
The load is expressed by

$$\tau_l = k_1 \cdot \omega_m^{\ 2} \tag{6}$$

Although only the system with a load of a square of speed-torque characteristic is analyzed, this analysis can deal with the system having any torque characteristics of load.

3.2 Mathematical Expression of System Performance

The performance of system is divided into fifteen modes according to the behaviors of converter and inverter [8]. Then, the inverter repeats one-sixth of performance of its cycle. Thus, the equivalent



(a) Mode 1 (b) Mode 2 (c) Mode 3

(d) Mode 4 (e) Mode 5 (f) Mode 6

Fig.3 Operating modes

circuits for analysis are six ones shown in Fig.3. The circuit equation of modes in matrix form is

$$\boldsymbol{x}_j = \boldsymbol{A}_j \boldsymbol{x}_j + \boldsymbol{B}_j \boldsymbol{u}_j \tag{7}$$

This equation is rewritten in a discrete state equation with a time step Δt as follows:

$$\boldsymbol{x}_j[(k+1)\Delta t] = \boldsymbol{\Phi}_j(\Delta t)\boldsymbol{x}_j(k\Delta t) + \boldsymbol{\Theta}_j(\Delta t)\boldsymbol{u}_j(k\Delta t) \tag{8}$$

For example, the circuit equation in Mode 1 is described in the following. The matrixes of Eq.(7) are given by

$$\boldsymbol{x}_2 = [\ i \quad i_{INV} \quad v_c \quad \omega_m\ ]^T \tag{9}$$

$$\boldsymbol{u}_1 = [\ e \quad e_{ma} \quad e_{mb} \quad \tau_l\ ]^T \tag{10}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -r_f/L_f & 0 & -1/L_f & 0 \\ 0 & -r_m/L_m & 1/2L_m & 0 \\ 1/C_f & -1/C_f & 0 & 0 \\ 0 & e_{mab}/(J\omega_m) & 0 & 0 \end{bmatrix} \tag{11}$$

$$\boldsymbol{B}_2 = \begin{bmatrix} 1/L_f & 0 & 0 & 0 \\ 0 & -1/2L_m & 1/2L_m & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/J \end{bmatrix} \tag{12}$$

Since e_{ma}, e_{mb}, and τ_l as components of the input vector $\boldsymbol{u}_1$ are functions of a state variable ω_m, it is impossible to calculate Eq.(8) directly. Therefore, the value of ω_m calculated before one step is used to solve the values of e_{ma}, e_{mb} and τ_l. The system matrix $\boldsymbol{A}_1$ has

$e_{mab}(=e_{ma}-e_{mb})$ and ω_m, and then this is also calculated using the values before one step. The calculation in the other modes is also performed in the same manner as the aboved.

The produced torque is calculated.

$$\tau = (\boldsymbol{e}_m \cdot \boldsymbol{i}_m)/\omega_m \tag{13}$$

Where,

$$\boldsymbol{e}_m = [\ e_{ma}\ e_{mb}\ e_{mc}\ ]$$

$$\boldsymbol{i}_m = \begin{bmatrix} i_{ma} \\ i_{mb} \\ i_{mc} \end{bmatrix}$$

4 Analytical Results

4.1 Time Step and Accuracy of Analysis

Since the analysis is carried out by using the discrete state equation with a time step Δt and the variables are lied in the system matrix, the components in the matrix are calculated by the values in one step before. Therefore, the analyzed results are affected by a time step Δt. In such analyses, generally the accuracy becomes better but the cpu time becomes longer as a time step is smaller.

Fig.4 shows the waveforms calculated with Δt=5/60/360 sec. and 1/60/360 sec. under the condition of Table 1. Table 2 shows the parameters of tested motor. The small deviation is

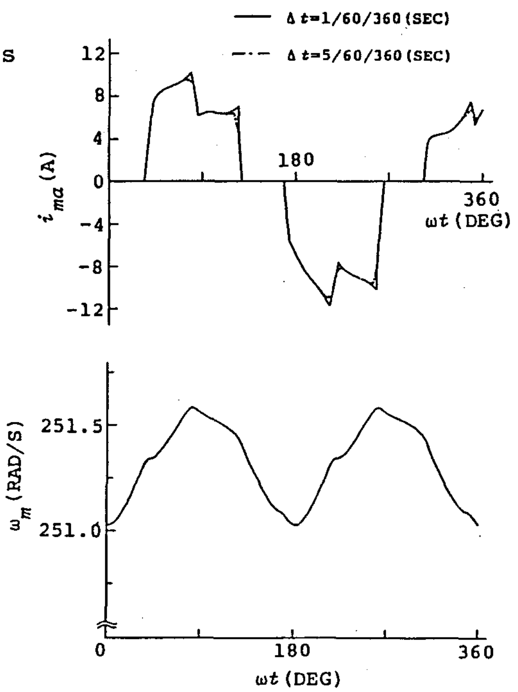


Fig.4 Calculated waveforms

observed between two waveforms in Fig.4. Further, the waveforms with Δt=2/60/360 sec. and 0.5/60/360 sec. are also calculated, and found to coincide with the waveform with Δt=1/60/360 sec. Thus, the time step of 1/60/360 sec. is considered appropriate for this analysis and this time step is used in the following analysis.

Table 1 Circuit constants(I)

constants	values
τ_0 (kg·m)	0.297
f_i (Hz)	80.0
γ (DEG)	30.0
J (kg·m^2)	0.005
δ (DEG)	33.0
L_f (mH)	3.0
C_f (μF)	2100.0
r_f (Ω)	0.0

4.2 Torque Pulsation and Speed Variation

Table 2 Constants of tested motor

constants	values
r_m' (Ω)	0.471
L_m (mH)	2.206
k_e	0.08507

In this drive system, the following factors effect the torque waveforms, or the speed variation.

(1) The phase difference δ between the source cycle and the inverter cycle (referred to degrees on the ac source voltage waveform),

(2) the load torque,

(3) the total inertia of load and motor,

(4) the filter constants (L_f and C_f).

Figs.5, 6 and 7 show the Fourier analysis of torque waveforms for the aboved factors in the inverter frequency f_i=80 Hz. In these figures, τ_n indicates the harmonics of torque (n=2,4,6,8,10,12). The circuit constants are shown in Table 1. The torque component of n=8 is due to the behavior of inverter and the torque components of n=2,4,6,... are due to the behavior of converter. It is found from these figures that the latter is larger than the former. Further, it is found in Fig.5 that the harmonic torque components due to the converter are influenced by δ. The component of torque n=2, which is produced by the behavior of converter,

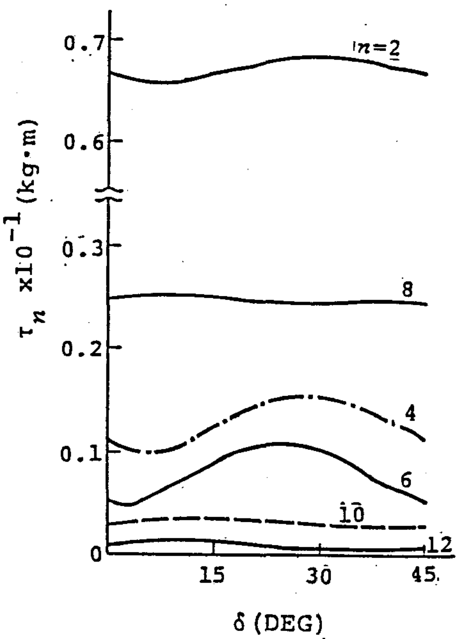
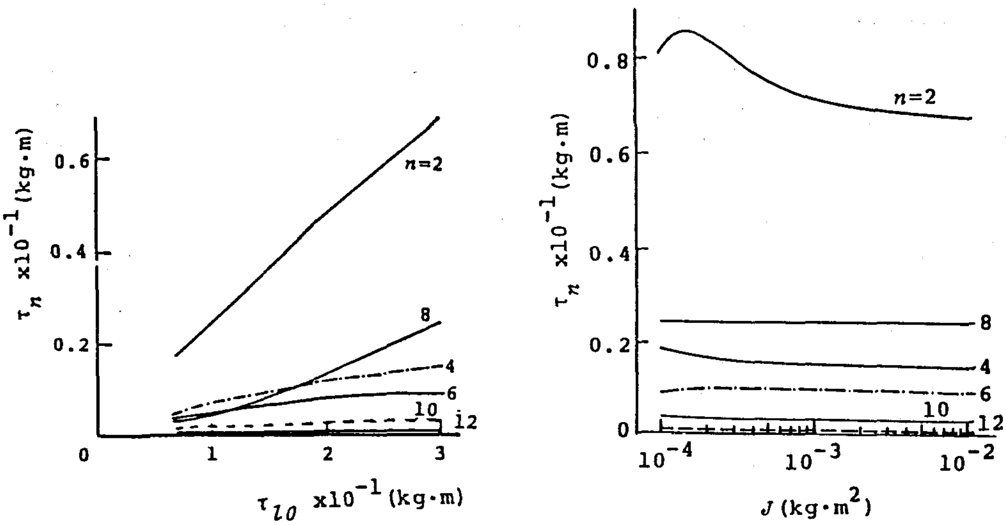
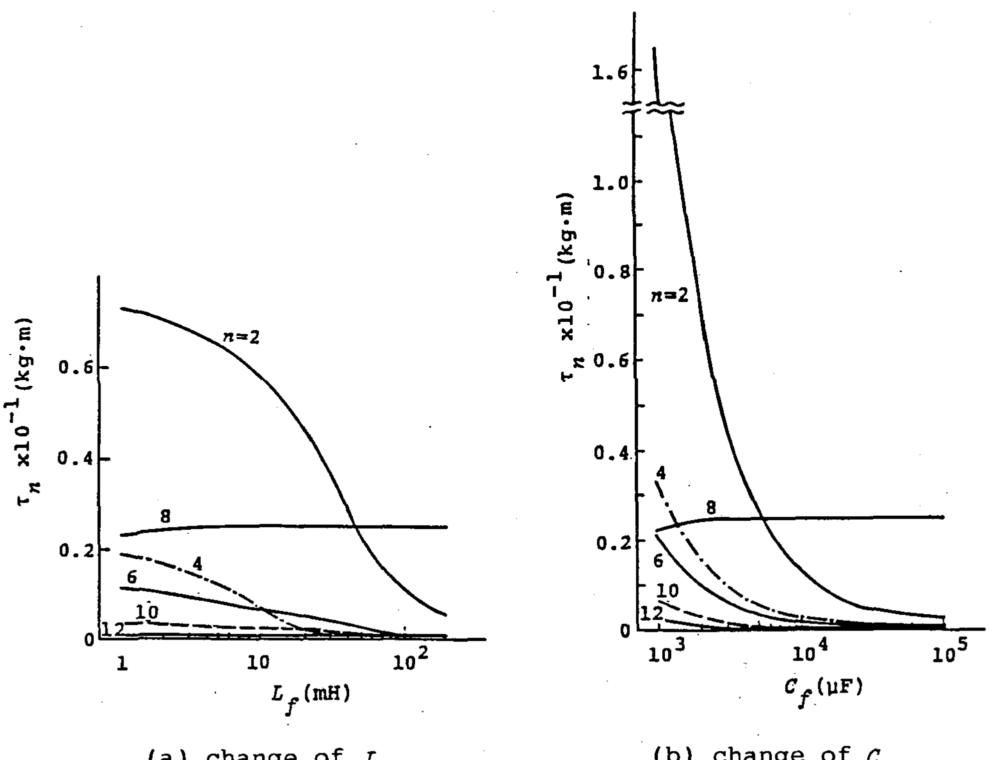


Fig.5 Harmonics of torque for δ



(a) change of load torque (b) change of inertia

Fig.6 Harmonics of torque for loads



(a) change of L_f (b) change of C_f

Fig.7 Harmonics of torque for filter constants is observed in Fig.6 to increase with the decrease of inertia. The components of torque due to the converter decrease with the increase of filter constants. However, the effect of L_f on the reduction of torque harmonics is not the same as that of C_f. That is because the increase of L_f or C_f does not indicate the same effect on the converter performance. On the other hand, the component of torque due to the inverter are almost constant.

Figs.8, 9 and 10 show the effect of factors on the speed variation $\Delta\omega_m$ $(=\omega_{mmax}-\omega_{mmin})$ in f_i= 60 Hz and f_i=80 Hz. The circuit constants in f_i=60 Hz and 80 Hz are shown in Table 1 and 3, respectively. It is clear from these figures that the speed variation is mainly due to the harmonic of torque n=2. Then,

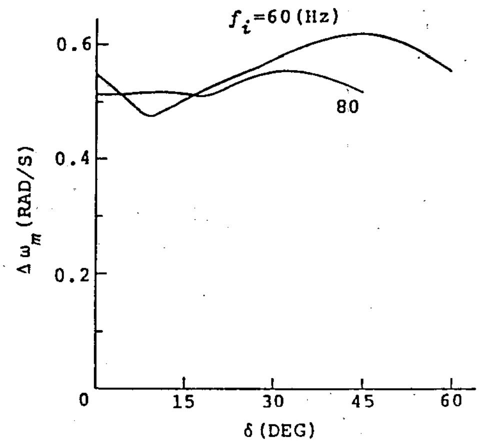
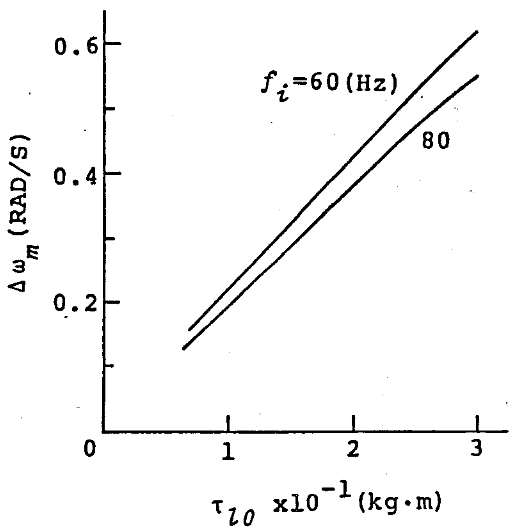
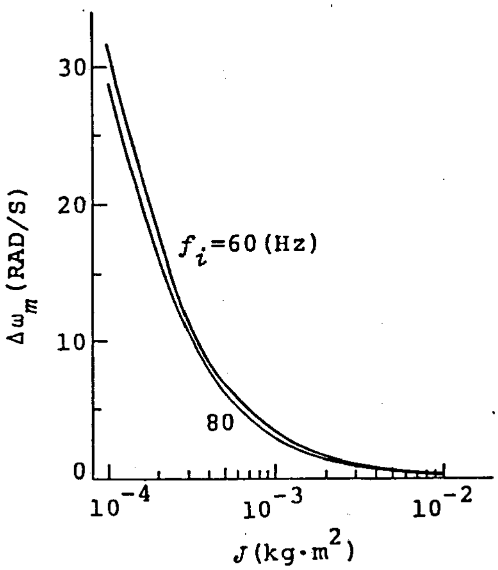


Fig.8 Speed variation for δ

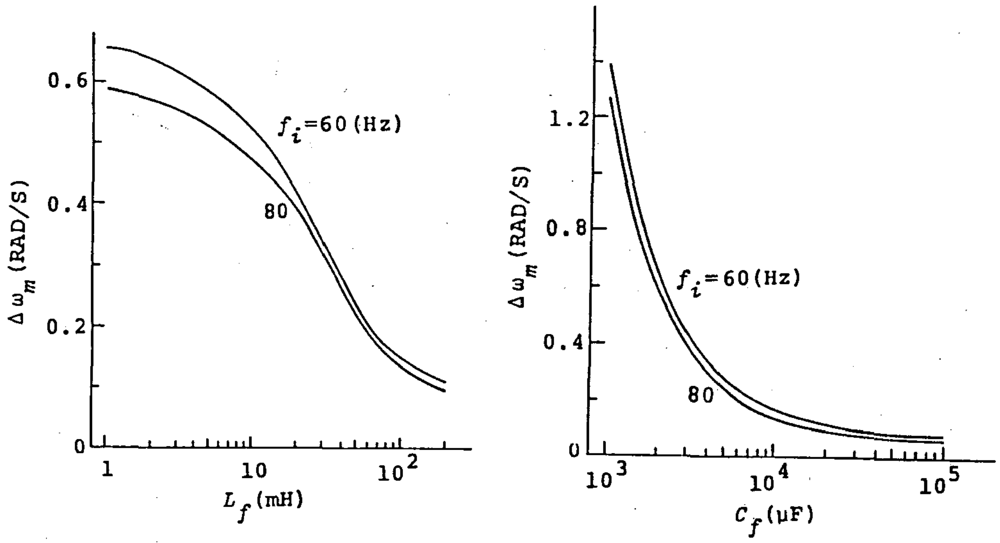


(a) change of load torque



(b) change of inertia

Fig.9 Speed variation for loads



(a) change of L_f (b) change of C_f

Fig.10 Speed variation for filter constants

the speed variation decreases exponentially with the increase of J and C_f. On the other hand, it decreases monotonously with the increase of L_f.

Table 3 Circuit constants(II)

constants	values
τ_0 (kg·m)	0.297
f_i (Hz)	60.0
γ (DEG)	30.0
J (kg·m^2)	0.005
δ (DEG)	45.0
L_f (mH)	3.0
C_f (μF)	2100.0
r_f (Ω)	0.0

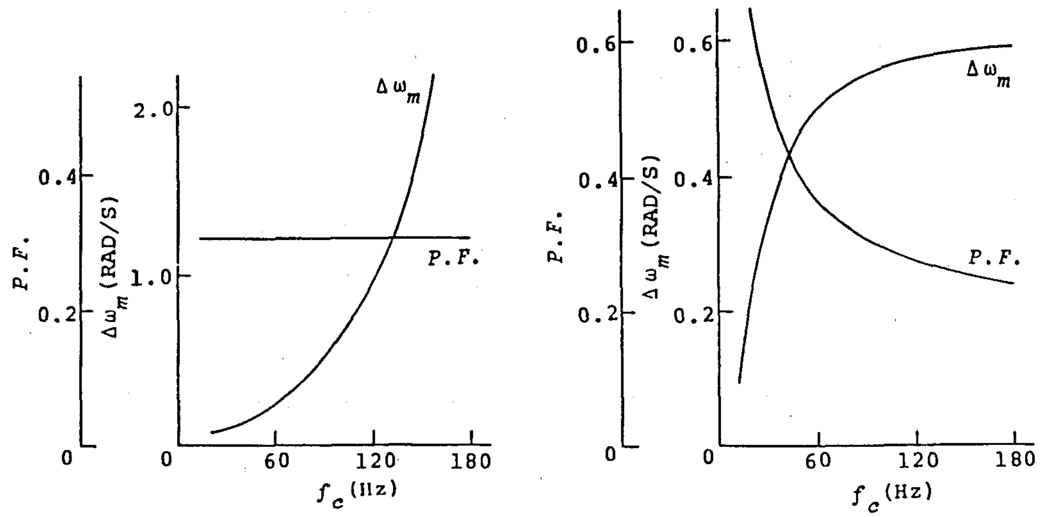
4.3 Discussion on Filter Constants

Let us discuss the effect of LC filter on the speed variation from a view of the cut-off frequency of filter. The cut-off frequency f_c of filter is denoted in the next expression.

$$20\ \log\left|\frac{1}{1-(2\ f_c)^2\ L_f\ C_f}\right| = -3 \tag{14}$$

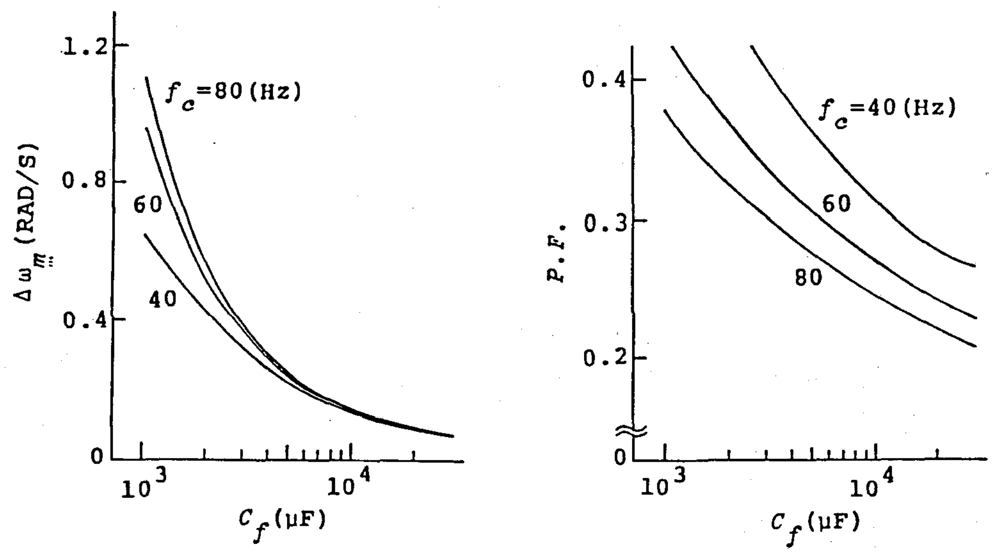
The input power factor is expressed by

$$P.F. = \sqrt{1-\mu}\ \cos\phi_1 \tag{15}$$



(a) L_f=3.0 mH (b) C_f=2100 μF

Fig.11 Speed variation and input power factor for cut-off frequency



(a) speed variation (b) input power factor

Fig.12 Speed variation and input power factor for constant cut-off frequency

Where,

$$\mu = 1 - (I_1/I)^2$$

Fig.11 shows the speed variation and the input power factor for the cut-off frequency f_c. When Cf is constant in this figure, the speed variation decreases rapidly in the region of low cut-off frequency but gently in the region of high cut-off frequency. On the other hand, when L_f is constant, the rate of decrease in the speed variation becomes smaller as the cut-off frequency becomes lower. The input power factor increases in Fig.11(b) as the cut-off frequency becomes lower. That is because the firing angle of converter proceeds and the displacement factor is improved as the cut-off frequency becomes lower with the constant torque and C_f.

Fig.12 shows the speed variation and the input power factor for C_f with a parameter of cut-off frequency. It is found in Fig.12(a) that the speed variation decreases as the cut-off frequency becomes smaller. On the other hand, the input power factor is improved for the same cut-off frequency when C_f is selected small. Thus, it is found that the input power factor is improved with a low cut-off frequency. As described above, the speed variation decreases as the cut-off frequency becomes lower. This is based on the low-pass type of filter and the reduction of harmonics which pass the filter as the cut-off frequency becomes lower.

5 Conclusion

In this paper, the analysis of a brushless-dc drive system taking account of a load is proposed. The following summary is obtained by the analysis.

(1) The harmonics in torque and the speed variation are principally due to the behavior of converter. Namely, the $2n$ multiple components of torque harmonics are produced by that behavior and the speed variation is generated.

(2) The speed variation decreases exponentially with the increase of inertia and C_f under the condition of constant torque. On the other hand, it decreases gently with the increase of L_f.

(3) When C_f is constant, the speed variation decreases in the region of low cut-off frequency and the input power factor is improved as f_c

becomes lower. When L_f is constant, the rate of decrease in the speed variation becomes smaller as f_c is selected lower and the input power factor is constant despite of the value of f_c.

(4) In the discussion of filter with a parameter f_c, it is found that the speed variation becomes smaller and the input power factor is improved as f_c is selected lower. The speed variation becomes smaller and the input power factor is improved in the same cut-off frequency as C_f is selected smaller.

This analysis is available for the estimation and design of system because all the system, e.g. an ac source, a converter, an inverter and a load, are taken into account.

Nomenclature

Th	a thyristor
D, D1, D2, D3, D4, D5, D6	diodes
T1, T2, T3, T4, T5, T6	transistors
t	a time
e	an instantaneous supply voltage
i	an instantaneous line current
i_{INV}	a current flowing into inverter
I, I_1	r.m.s. value and fundamental component of line current
v_{ma}, v_{mb}, v_{mc}	line to neutral voltages of the motor
$\boldsymbol{e}_m$	a vector of EMF's
i_{ma}, i_{mb}, i_{mc}	instantaneous motor currents in each phase
$\boldsymbol{i}_m$	a vector of motor currents
e_{ma}, e_{mb}, e_{mc}	instantaneous counter electromotive forces (EMF's) of the motor
E_m	a fundamental component of EMF
v_c	a capacitor voltage
V_{cmax}, V_{cmin}	maximum and minimum values of capacitor voltage
r_m	a resistance of motor winding
L_m	an inductance of motor converted in each phase
L_f	an inductance of filter
C_f	a capacitance of filter
r_f	a resistance of filter

f_c — a cut-off frequency of *LC* filter
J — an inertia of motor
ω_m — an instantaneous angular velocity of the motor
$\Delta\omega_m$ — a variation of motor angular velocity
ω_{mmax}, ω_{mmin} — maximum and minimum values of motor angular velocity
γ — a commutation advance
δ — a displacement angle between a cross point of the supply and the on-timing of transistor T1
T_s, T_i — periods of the supply and the inverter behavior
f, f_i — frequencies of the supply and the inverter
ω_i — an angular velocity of the inverter
n, p, q — positive integers
τ — an instantaneous torque
τ_n — harmonics in torque
τ_l — a load torque
τ_{max}, τ_{min} — a maximum and minimum values of instantaneous torque
$\boldsymbol{x}_j$ — a state vector of mode j
$\boldsymbol{A}_j$ — a system matrix of mode j
$\boldsymbol{B}_j$ — a control matrix of mode j
$\boldsymbol{u}_j$ — an input vector of mode j
$\boldsymbol{\Phi}_j$ — a state transition matrix of mode j
$\boldsymbol{\Theta}_j$ — a control transition matrix of mode j
j — the number of mode (j=1,2,3,4,5,6)
μ — a distortion factor
$\cos\phi_1$ — a displacement factor
P.F. — an input power factor
k_ω — a winding factor
ω_1 — the number of turn
k_L — a leakage factor
k_ϕ — a distribution factor
ϕ — a flux of field
k_e — $=\sqrt{2}\,\omega_m\cdot k_\omega\cdot\omega_1\cdot k_L\cdot\phi/(2k_\phi)$
k_1 — a constant

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