Memoirs of the School of Engineering, Okayama University Vol.13, February 1979

Longitudinal Properties of Two-Dimensional Classical Electron Liquids

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(Received February 3, 1979)

Synopsis

The dynamic form factor and the dispersion relation of the plasma oscillation of two-dimensional classical systems of electrons with ordinary Coulomb interaction are obtained by numerical experiments in the domain of the plasma parameter $2.24 \leq \Gamma = (\pi n)^{1/2} e^2/T \leq 70.7$, where n, e, and T are the areal number density, the electronic charge, and the temperature in energy units, respectively.

1. *Introduction*

The possibility to suppress one of three degrees of freedom of electrons by using surface bound states on some dielectric materials has made two-dimensional systems of electrons available for experiments as one component plasmas under almost ideal conditions [1]. With the number density easily changed by applying external electric field perpendicular to the surface, they have both the properties as systems of charged particles which interact via the ordinary (1/r) Coulomb potential and those originating in two-dimensionality of motions of their particles.

As the dielectric material the liquid helium has been used: In this case electrons in these systems obey classical mechanics and statistics under most of experimental conditions [2]. These two-dimensional classical electron liquids are characterized by the dimensionless plasma parameter Γ or ε defined by

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$$\Gamma=(\varepsilon/2)^{1/2}=(\pi n)^{1/2}e^2/T, \tag{1.1}$$

where n, e, and T are the areal number density, the electronic charge, and the temperature in energy units, respectively.

As well as real experiments on the surface of liquid helium numerical experiments have provided useful information on the properties of these systems [3,4,5]. In previous numerical experiments, however, static or thermodynamic properties have been mainly investigated and experimental information on dynamic properties seems to be still insufficient. We here report some results of numerical experiments on dynamic properties of these systems in the domain of the plasma parameter

$$2.24\leq\Gamma\leq 70.7. \tag{1.2}$$

We make dynamical simulations of these two-dimensional systems of electrons by the molecular dynamics method. We summarize briefly numerical processes in what follows; their details will be given elsewhere [6].

In order to cope with the long range nature of the Coulomb interaction we impose the periodic boundary condition on our system and the hexagonal symmetry is taken as the periodicity for the sake of consistency with possible formation of the Wigner crystal [3,5]. The number of independent particles is taken to be 81 which has been shown to be sufficient in the domain of our experiments [4]. For the domain of higher density, larger number of independent particles may be necessary in order to avoid spurious effects due to interference between oscillating long tail of the pair correlation function and the periodicity.

From initial configurations taken to be the two-dimensional lattice or a random configuration, equations of motions are numerically integrated for a sufficiently long time compared with typical time scales of fluctuations. In the process of numerical integration the force acting on each electron is calculated by the method of Ewald. We discard initial parts of simulations to remove effects of initial conditions observing that the system has relaxed to thermal equilibrium within the discarded initial part from behaviors of kinetic and correlation energies.

The accuracy of numerical integrations are examined by conservation laws of energy and momentum. The total energy is conserved with errors less than 0.05% in each step of time and through the whole experiment less than 4% (Γ=2.2), 0.2% (Γ=7,22) or 0.04% (Γ=50,71).

The total momentum is conserved with negligible errors compared with those in energy conservation.

2. Dynamic Form Factor and Dispersion Relation of Plasma Oscillation

Longitudinal properties of many body systems are most clearly shown in the dynamic form factor $S(\underset{\sim}{k},\omega)$ defined by

$$S(\underset{\sim}{k},\omega)=(1/2\pi)\int_{-\infty}^{\infty} dt<\rho_{\underset{\sim}{k}}(t)\rho_{-\underset{\sim}{k}}(t=0)>\exp(i\omega t), \tag{2.1}$$

where $\rho_{\underset{\sim}{k}}(t)$ denotes the density fluctuation

$$\rho_{\underset{\sim}{k}}(t)=\sum_i \exp[-i\underset{\sim}{k}\cdot\underset{\sim}{r}_i(t)] \quad (k\neq 0) \tag{2.2}$$

and $<\ >$ the statistical average. The dynamic form factor is related to the longitudinal part $m^2C_l(\underset{\sim}{k},\omega)$ of the fluctuation spectrum of the momentum density $m^2\underset{\approx}{C}(\underset{\sim}{k},\omega)$ as

$$S(\underset{\sim}{k},\omega)=(k/\omega)^2C_l(\underset{\sim}{k},\omega), \tag{2.3}$$

where m is the mass of an electron,

$$\underset{\approx}{C}(\underset{\sim}{k},\omega)=(1/2\pi)\int_{-\infty}^{\infty} dt<\underset{\sim}{g}_{\underset{\sim}{k}}(t)\underset{\sim}{g}_{-\underset{\sim}{k}}(t=0)>\exp(i\omega t), \tag{2.4}$$

$$\underset{\sim}{g}_{\underset{\sim}{k}}(t)=\sum_i [d\underset{\sim}{r}_i(t)/dt]\exp[-i\underset{\sim}{k}\cdot\underset{\sim}{r}_i(t)], \tag{2.5}$$

$$\underset{\approx}{C}(\underset{\sim}{k},\omega)=(\underset{\sim}{k}\underset{\sim}{k}/k^2)C_l(k,\omega)+(\underset{\approx}{I}-\underset{\sim}{k}\underset{\sim}{k}/k^2)C_t(k,\omega). \tag{2.6}$$

We show in Fig.1 the behavior of the dynamic form factor for several values of the wave number and the plasma parameter. Though the value at each $\underset{\sim}{k}$ and ω is rather noisy, we see that for small wave numbers the dynamic form factor is dominated by the contribution of the collective mode, the plasma oscillation, and its relative importance reduces with the increase of the wave number. As the plasma parameter increases, the range of existence of this peak structure extends to larger wave numbers.

This observation is more clearly shown in the dispersion relation in Fig.2 obtained from the dynamic form factor as the position and the full width at half maximum of the peak.

Simulations are made for durations of 10 (Γ=7.1) to 15 (Γ=2.2) times

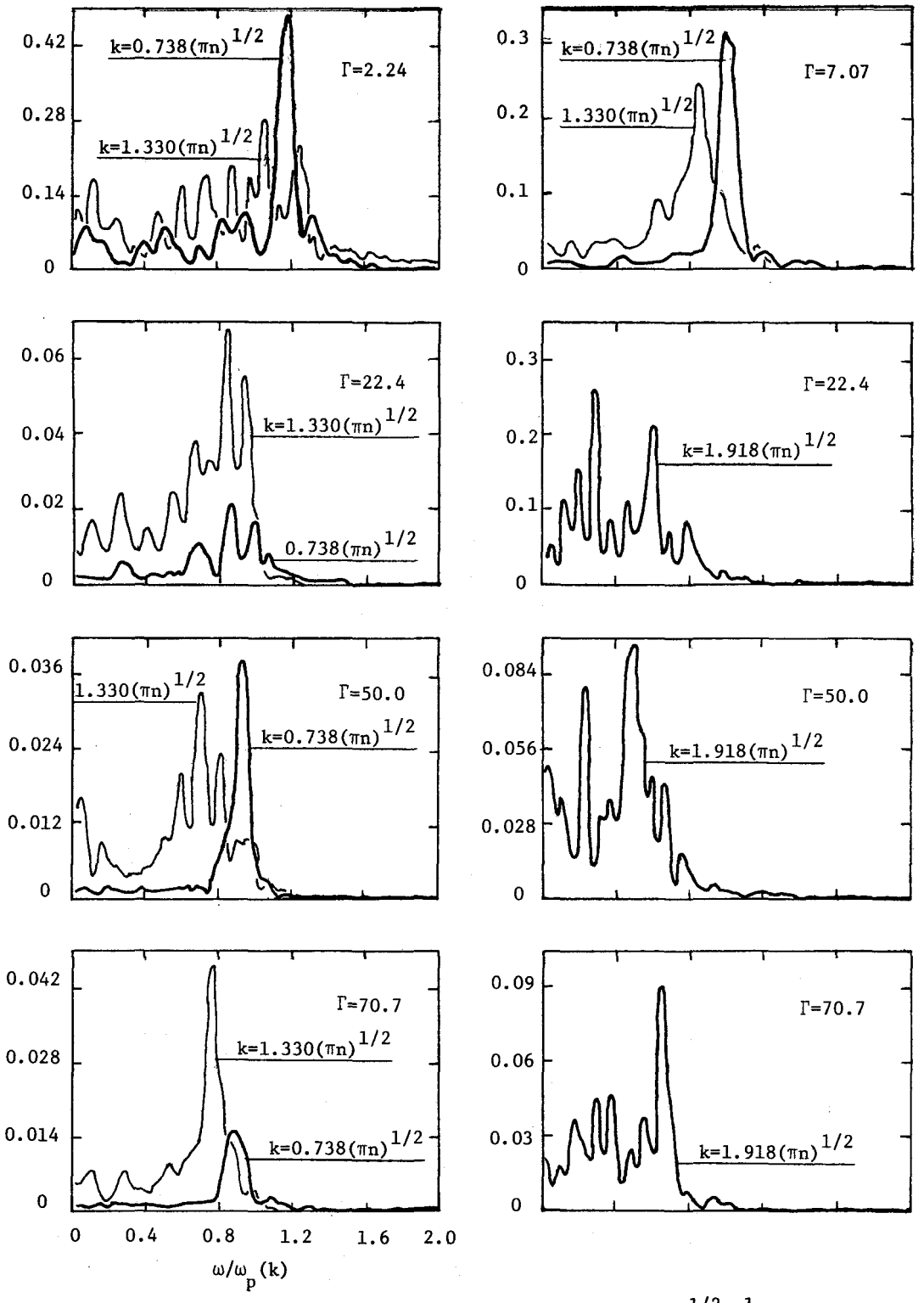

Fig.1. The dynamic form factor in the unit of $86.92 \cdot \omega_p[(\pi n)^{1/2}]^{-1}$ vs. $\omega/\omega_p(k)$.

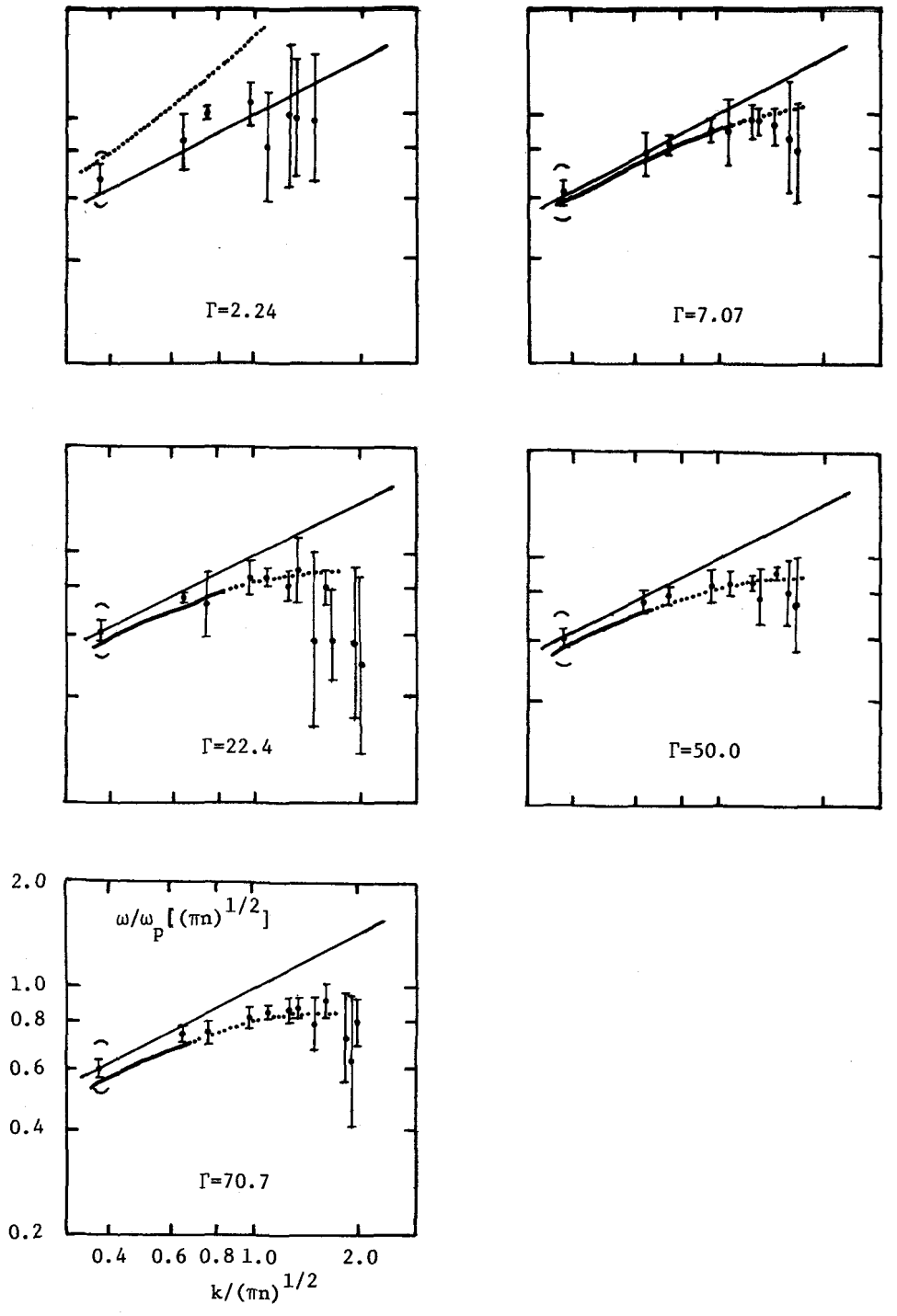

Fig.2. The dispersion relation of the plasma oscillation of two-dimensional classical electron liquids. Vertical lines indicate full widths at half maximum of peaks of $S(\underset{\sim}{k},\omega)$. Eq.(2.7) with $\delta=0$ and with δ given by known thermodynamic quantities are shown by thin and thick (or dotted when the term with δ is not small) lines.

the typical period of plasma oscillation for $k=(\pi n)^{1/2}$ without including discarded initial parts of about a quarter of them.

It has been shown that in the limit of long wavelengths the dispersion relation is given by [7]

$$\omega(k)\tilde{=}\omega_p(k)\,[1+\delta\,(k/k_D)]\,,$$
$$\omega_p(k)=(2\pi ne^2k/m)^{1/2},\quad k_D=2\pi ne^2/T,\quad \delta=(1/2)\,(c_p/c_V)\,(\chi_T{}^0/\chi_T)\,. \tag{2.7}$$

Here c_p, c_V, χ_T, and $\chi_T{}^0$ are the specific heat at constant pressure, the one at constant volume, the isothermal compressibility of our system, and that of the ideal gas, respectively. Eq.(2.7) with $\delta=0$ and with δ calculated from known thermodynamic properties [4] are plotted in Fig.2.

Since we have used the periodic boundary condition, our experiment does not give the dispersion relation of the plasma oscillation in the limit of very long wavelengths which are comparable with the period and we have no theoretical results which can be applied strictly in the domain of the wave number of our experiments. We see, however, the predictions of Eq.(2.7) in the limit of long wavelengths are consistent with the results of our experiments.

Acknowledgements

Numerical computations have been done at the Okayama University Computer Center and the Computer Center of the University of Tokyo. This work is partially supported by the Grant-in-Aids of the Ministry of Education.

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