

Allocation Technique Using Relative Distance and Relative Volume Estimated by Statistical Procedure

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(Received May 30, 1973)

Synopsis

This paper deals with the allocation technique of the layout whose solution it takes as a little computation time as possible to obtain and which becomes as near to the optimal method as possible. In this method, the relative transport distance of each location and the relative transport volume of each department are calculated from the distance matrix and the volume matrix by the statistical procedures. And allocating departments to locations is determined by one to one correspondence between the arranged relative transport distances and the arranged relative transport volumes. This method was called the allocation technique by the statistical procedure (ATSP in short).

This method doesn't use the heuristic algorithm. Therefore the calculation time can be reduced much in comparison with any other methods. As the algorithm of ATSP method is very simple, the sub-optimal layout can be determined easily by using the desk-calculator in cases of any layout problems.

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1. Introduction

The appropriate arrangement of workers, machines and materials into the factory is the most important to combine these elements systematically and rationally and further to improve the efficiency of the production. Especially allocating machines to locations much influences the transport distance of materials. Therefore many quantitative allocation techniques to solve the layout problem have been developed until now. Most of these techniques improve the layout by the heuristic algorithm.

In this paper, authors attempt to study the allocation technique which doesn't use the heuristic algorithm. The relative transport distance of each location and the relative transport volume of each department play an important role in this technique. These relative values are calculated by the statistical procedure.

2. Restriction of the Layout Problem

The various factors which influence the layout should be taken into consideration in determining the optimal layout planning. In general the factors of transport distances among locations and transport volumes among departments are frequently used to determine the layout. Then in this paper, the technique of allocating departments to locations is considered under the following restrictions.

(Restriction 1)

It is assumed that the number of departments is equal to that of locations. That is,

$$\text{number of departments} = \text{number of locations} = n .$$

n departments are equal to each other in area. n locations are equal to each other in area, too. And the area of a department is equal to that of a location.

(Restriction 2)

The distance between location I_i and I_j is put as d_{ij} . And it is assumed that the distance from I_i to I_j is equal to that from I_j to I_i . Then

$$\begin{cases} d_{ij} = d_{ji} \\ d_{ii} = 0 \end{cases} \quad i, j = 1, 2, \dots, n$$

(Restriction 3)

The transport volume v_{ij} between department B_i and B_j is put as the sum of the volume from B_i to B_j and that from B_j to B_i .

Therefore

$$\begin{cases} v_{ij} = v_{ji} \\ v_{ii} = 0 \end{cases} \quad i, j = 1, 2, \dots, n$$

(Restriction 4)

The efficiency of the obtained layout was evaluated by the following equation.

$$EV = \sum_{i,j} d_{ij} \sum_{i,j} v_{s(i)s(j)}$$

$s(i)$ and $s(j)$ are the department numbers
which are allocated to the location I_i and I_j

This evaluated value EV is generally used in order to approximate the material handling cost closely.⁽¹⁾

The smaller the evaluated value becomes, the better the layout does. And layout which has minimum value of EV is called the optimal layout.

3. Allocation Technique by the Statistical Procedure

3.1. Optimal Producing Method (OPM in short)⁽²⁾

It is assumed that n departments are assigned to n available locations and that any departments are compatible with any locations. Then there are $n!$ ways of allocating departments to locations in this case. The allocation which has minimum EV among $n!$ allocations is the optimal layout.

3.2. Allocation Technique by the Statistical Procedure (ATSP in short)

In OPM method, as n becomes large, the huge number of times of the repetitive calculation are necessary to determine the optimal layout. For example, when $n = 10$, about four million times of calculation are necessary. Therefore it is impossible to determine the optimal layout in the case of layout problem having more than ten departments. Then various types of the technique to obtain the sub-optimal layout have been developed by Buffa, etc.⁽³⁾ But the algorithm of these techniques uses mainly the heuristic algorithm.

Therefore the authors attempt to study the allocation technique

which doesn't use the heuristic algorithm in order to obtain the sub-optimal layout. In this technique, the relative distance of each location and the relative volume of each department are defined by the statistical method. This technique is constructed of ATSP1, ATSP2 and ATSP3 distinguished by the difference of the definition of the relative distance and the relative volume.

3.2.1. ATSP1

In this technique, the relative distance of each location and the relative volume of each department are defined by the mean value and the standard deviation as follows.

(Step 1)

The mean value of distance of each location is calculated as follows.

$$D_i = \frac{\sum_{j \neq i}^n d_{ij}}{(n-1)}, \quad i = 1, 2, \dots, n$$

The standard deviation of distance of each location is calculated as follows.

$$SD_i = \sqrt{\frac{\sum_{j \neq i}^n (d_{ij} - D_i)^2}{(n-2)}}, \quad i = 1, 2, \dots, n$$

And the relative distance P_i of the location I_i is defined by the following equation.

$$P_i = D_i + C \times SD_i \quad \dots \dots \dots (1)$$

$i = 1, 2, \dots, n$

C is the constant which indicates the scale of the statistical distribution

(Step 2)

The mean value and the standard deviation of volume of each department are calculated as follows.

$$V_i = \frac{\sum_{j \neq i}^n v_{ij}}{(n-1)}$$

$$SV_i = \sqrt{\frac{\sum_{j \neq i}^n (v_{ij} - V_i)^2}{(n-2)}} \quad i = 1, 2, \dots, n$$

And the relative volume Q_i of the department B_i is defined by the following equation.

$$Q_i = V_i + C \times SV_i \quad \dots \dots \dots (2)$$

$i = 1, 2, \dots, n$

C is the same value in the equation (1)

(Step 3)

The relative distances P_i , $i=1, 2, \dots, n$ are arranged in order from

the smallest of them. And the relative volumes Q_i , $i=1,2,\dots,n$ are arranged in order from the largest of them. It is assumed that these results are as follows.

$$P_{k_1} < P_{k_2} < P_{k_3} < \dots < P_{k_j} < \dots < P_{k_n} \quad \dots \quad (3)$$

$$Q_{m_1} > Q_{m_2} > Q_{m_3} > \dots > Q_{m_j} > \dots > Q_{m_n} \quad \dots \quad (4)$$

From the relation (3) and (4), it becomes possible to make P_{k_j} and Q_{m_j} , $j = 1,2,\dots,n$ correspond one to one.

Therefore the department B_{m_j} is allocated to the location I_{k_j} , $j = 1,2,\dots,n$.

The sub-optimal layout can be determined from this allocation. And the evaluated value of EV of this allocation is calculated.

3.2.2. ATSP2

The standard deviation used in ATSP1 needs the complex calculation. Therefore the range is used to estimate the standard deviation. The range RD_i of distance of the location I_i is calculated by the following equation.

$$RD_i = \max_{1 \leq j \leq n-1} d_{ij} - \min_{1 \leq j \leq n-1} d_{ij}, \quad i = 1,2,\dots,n$$

And the relative distance is defined as follows.

$$P_i = D_i + RD_i \quad \dots \quad (1)'$$

Similarily RV_i is put as the range of volume of the department B_i . And the relative volume is defined as follows.

$$Q_i = V_i + RV_i \quad \dots \quad (2)'$$

ATSP2 is the technique in which the equation (1)' and (2)' are used in step (1) and (2) in ATSP1 instead of equation (1) and (2).

3.2.3. ATSP3

The mean value is calculated by the statistically weighted average, incorporating the maximum, most likely and minimum in PERT. (4), (5)

The relative distance is defined as follows.

$$P_i = (AD_i + 4 \times MD_i + ID_i) / 6 \quad \dots \quad (1)''$$

$$AD_i = \max_{1 \leq j \leq n-1} d_{ij}, \quad MD_i = \text{mode } d_{ij}, \quad ID_i = \min_{1 \leq j \leq n-1} d_{ij}$$

Similarly the relative volume is defined as follows.

$$Q_i = (AV_i + 4 \times MV_i + IV_i) / 6 \dots\dots (2)''$$

$$AV_i = \max_{1 \leq k \leq n-1} v_{ij}, \quad MV_i = \text{mode } v_{ij}, \quad IV_i = \min_{1 \leq j \leq n-1} v_{ij}$$

ATSP3 is the technique in which the equation (1)'' and (2)'' are used in step (1) and (2) in ATSP1 instead of equation (1) and (2).

4. Results

First, OPM and ATSP method were applied to the layout problem⁽⁶⁾ of $n = 4$ having the distance matrix $D = (d_{ij})$ in Table 1 and the volume matrix $V = (v_{ij})$ in Table 2.

4.1. The Optimal Layout by OPM Method

There were four departments and four available locations in this problem. Therefore the optimal layout was selected among $4! = 24$ ways of the allocation. And the result was the allocation of the department B_2 to the location I_1 , B_1 to I_2 , B_4 to I_3 , and B_3 to I_4 . The evaluated value of EV of this optimal layout was 11048.

4.2. The Sub-optimal Layout by ATSP Method

The mean value, the standard deviation, the range and the weighted average of each location and department were calculated from the distance matrix and the volume matrix. These values were shown in the right half of Table 1 and Table 2.

The mean value or the sum of mean value and standard deviation, or two times of it, or three times of it was used generally in the statistical method. Then the constant C in equation (1) and (2) in ATSP1 was determined as follows.

$$C = 0, \quad C = 1, \quad C = 2 \text{ and } C = 3$$

The solution by ATSP1 in the case of $C = 0$ was shown in accordance with its algorithm.

The relative distance of each location was as follows from Table 1.

$$P_1 = 26.00, \quad P_2 = 30.67, \quad P_3 = 18.00, \quad P_4 = 17.33$$

The relative volume of each department was as follows from Table 2.

$$Q_1 = 80.00, \quad Q_2 = 77.77, \quad Q_3 = 120.00, \quad Q_4 = 87.33$$

The results of the above mentioned arrangement of P_i and Q_i became as follows.

$$P_4 < P_3 < P_1 < P_2$$

$$Q_3 > Q_4 > Q_1 > Q_2$$

Therefore the correspondence between P_4 and Q_3 , P_3 and Q_4 , P_1 and Q_1 , and P_2 and Q_2 were made. Then the sub-optimal layout was the allocation of the department B_3 to the location I_4 , B_4 to I_3 , B_1 to I_1 , and B_2 to I_2 . And the evaluated value of this layout was 11640. This value was larger by about 500 than that of the optimal layout by OPM method.

The results of the sub-optimal layout by ATSP1 in cases of $C = 1$,

	I_1	I_2	I_3	I_4	mean	s.d.	range	w.m.
I_1	0	42	14	22	26.00	14.43	28	18.67
I_2	42	0	30	20	30.67	11.02	22	29.44
I_3	14	30	0	10	18.00	10.58	20	10.67
I_4	22	20	10	0	17.33	6.43	12	22.22

s.d. = standard deviation

w.m. = weighted mean

Table 1, Distance matrix, mean value, standard deviation, range and weighted mean

	B_1	B_2	B_3	B_4	mean	s.d.	range	w.m.
B_1	0	55	135	50	80.00	47.70	85	34.17
B_2	55	0	95	82	77.33	20.40	40	85.89
B_3	135	95	0	130	120.00	21.79	40	138.33
B_4	50	82	130	0	87.33	40.27	80	77.56

s.d. = standard deviation

w.m. = weighted mean

Table 2, Volume matrix, mean value, standard deviation, range and weighted mean

$C = 2$ and $C = 3$ were shown in Table 3. From these results, the layout by ATSP1 in the case of $C = 0$ had the smallest evaluated value. Then this solution was the most efficient among other solutions by ATSP.

ATSP \ location	I_1	I_2	I_3	I_4	EV
ATSP1 ($C = 1$)	B_4	B_2	B_1	B_3	11904
($C = 2$)	B_2	B_3	B_4	B_1	13448
($C = 3$)	B_2	B_3	B_4	B_1	13448
ATSP2	B_2	B_3	B_1	B_4	13714
ATSP3	B_2	B_1	B_3	B_4	11794

Table 3, Results by ATSP method

4.3. Comparison of Various Techniques

Second, OPM and ATSP method were applied to the layout problems in the paper written by Nugent, Vollmann and Ruml⁽²⁾. The evaluated value of the determined layout and the computation time and the results by Nugent etc.⁽²⁾ were shown in Table 4 and Table 5.

It was impossible to obtain the optimal layout by OPM in the case of the layout problem having more than twelve departments, as the huge computation time was needed. The minimum evaluated value by ATSP1, ATSP2 and ATSP3 was shown by the symbol "*" in Table 4.

It made clear that the evaluated value became minimum almost in

n		5	6	7	8	12	15	20	30
	H63	27.6	44.2	78.8	114.4	317.4	632.6	1400.4	3267.2
	HC 63-66	29.4	44.2	78.4	110.2	310.2	600.2	1345.0	3206.8
	CRAFT	28.2	44.2	79.6	113.4	296.2	600.6	1339.0	3189.6
	B.S.	26.8	43.6	74.8	107.0	293.0	480.2	1313.0	3124.0
	OPM	25	43	74	107				
A T S P 1	C=0	26*	57*	86*	141	383	776	1483*	3756
	C=1	37	57	96	141	384	706*	1602	3698*
	C=2	30	57	97	138	357*	721	1620	3784
	C=3	34	57	98	137	374	751	1623	3825
	ATSP2	30	57	86	141	397	738	1573	3773
	ATSP3	38	61	103	123*	365	828	1536	3872

Table 4, Evaluated values by various allocation techniques

cases of $C = 0$, $C = 1$ and $C = 2$ of ATSP1 except the layout problem of eight departments. Therefore it made clear that ATSP1 in cases of $C = 0$, $C = 1$ and $C = 2$ was the most useful to solve the layout problem.

n	HC		CRAFT	B.S.	OPM	ATSP
	H63	63-66				
5	6	10	1	11	27	47
6	7	9	2	21	40	45
7	15	12	5	57	146	45
8	14	14	10	109	1372	48
12	55	19	70	658		46
15	78	40	160	2192		47
20	168	75	528	6915		49
30	398	285	3150	44224		56
used com- puter	G	E	2	6	5	NEAG2200-500

Table 5, Computation time of each allocation technique (sec)

Further from the results of Nugent etc., it

made clear that the layout by Biased Sampling method (B.S. in short) was more efficient than that by CRAFT. But the computation time was much longer than that by CRAFT. The smallest evaluated values by ATSP were averagely larger by 17 per cent than those by OPM in cases of $n = 5, 6, 7$ and 8. And they were averagely larger by 20 per cent than those by B.S.. Further they were averagely larger by 13 per cent than those by CRAFT. On the other hand, the computation time by ATSP was not influenced by the number of department till $n = 30$. And it was almost equal to 50 sec. . But the computation time by CRAFT and B.S. extremely increased with the increase of the number of department.

From these results, it made clear that ATSP method was useful to obtain the sub-optimal layout in the layout problem having more than twelve departments. Further the sub-optimal layout by this algorithm could be obtained easily by using the desk-calculator.

5. Conclusion

For the purpose of the determination of the efficient layout in the layout problem, the allocation technique which didn't use the heuristic algorithm was examined in this paper. The following results were obtained.

1) The relative distance of each location was estimated by the mean value or the sum of mean value and standard deviation, or two times of it, or three times of it calculated from the distance matrix. The relative volume of each department was estimated by the above mentioned

values calculated from the volume matrix.

The allocation of departments to locations was determined by one to one correspondence between the arranged relative distances and the arranged relative volumes. This allocation technique was called the allocation technique by the statistical procedure (ATSP in short).

2) The layout determined by using the relative distance and the relative volume estimated from the mean value or the sum of mean value and standard deviation was more useful than any other layouts determined by using other relative distances and relative volumes in ATSP. And the computation time of ATSP was not influenced by the number of department.

3) It was possible to determine the optimal layout by the Optimal producing method to the layout problem having less than eight departments. And the sub-optimal layout could be determined economically by ATSP to the layout problem having more than eight departments.

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