MEMOIRS OF THE SCHOOL OF ENGINEERING, OKAYAMA UNIVERSITY, Vol. 7, No. 2, OCTOBER 1972

A New Representation of Distorted Wave Forms

Takayoshi Nakata* and Yoshiyuki Ishihara*

(Received September 16, 1972)

Synopsis

A new method for representing distorted wave forms is investigated. The method suggested by us is a modified vector symbolic method. It has been hitherto thought that the vector symbolic method cannot be applied to the distorted waves, because the rotating speeds of each harmonic vector are not identical. Representing the argument of the n-th harmonic vector by 1/n times as large as the phase angle of the harmonic component, the relative positions of respective harmonic vectors are invariable wherever the standard vector is put, and the wave shape can be deduced from the vector diagram.

We found various correspondences between the wave forms and the vector diagrams. Therefore, the wave shape can be estimated from the vector diagram, and the mutual relationships between two wave forms can also be known.

In electric or magnetic circuits, the causes of distorted wave forms are in general obvious. Therefore, there are very often the fixed relationships between the amplitudes and phase angles of the harmonics. Further, in polyphase a.c. circuits, there are often the fixed relationships between corresponding harmonics in the wave forms of the respective phases. When the wave forms of those circuits are discussed, the new method investigated in this paper may offer a useful key.

Let us represent the distorted wave ϕ which is defined by the Eq.(1), using the vectors as shown in Fig.1.

$$\phi=\sum_{n=1,3,5,\cdots}\Phi_n\cdot \sin n(\omega t+\theta n). \tag{1}$$

The method for representing the fundamental wave is the same as that in the usual vector diagram of sinusoidal quantities. However, the argument of the n-th harmonic vector is 1/n times as large as the phase

* Department of Electrical Engineering

angle ($n \cdot \theta n$) of the n-th harmonic component. In contrast to the instantaneous value of the distorted wave, the instantaneous value of the sinusoidal wave can be directly obtained by the usual vector diagram. As we have the following various relationships between the wave form and the vector diagram, the wave shape can be estimated from the vector diagram. The mutual relationships between two wave forms can also be known.

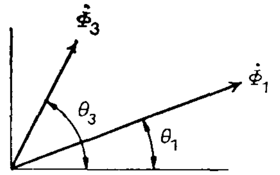
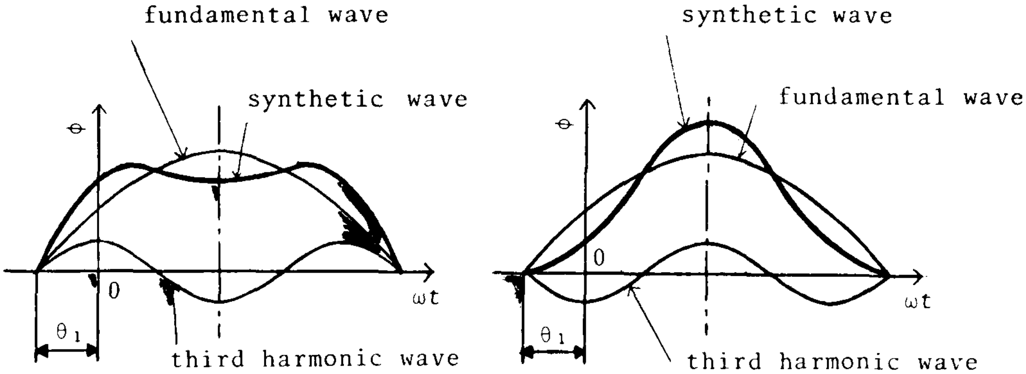


Fig.1 Vector representative method for distorted wave on the vector-plane (The figure shows an example consisting of the fundamental and the third harmonics).

(1) Two vectors the phase angles of which only differ by $2m\pi/n$ ($m=\pm1,\pm2,--$), are absolutely equal.

(2) In the Eq.(1), if a relationship $\theta_{k+i}-\theta_k=2m\pi/(k+i)$ is satisfied, the (k+i)-th harmonic is in-phase with the k-th harmonic. If a relationship $\theta_{k+i}-\theta_k=(2m-1)\pi/(k+i)$ is present, the (k+i)-th harmonic is anti-phase with the k-th harmonic. Here i/k is an integer, and the in- and anti-phases are defined by Fig.2.



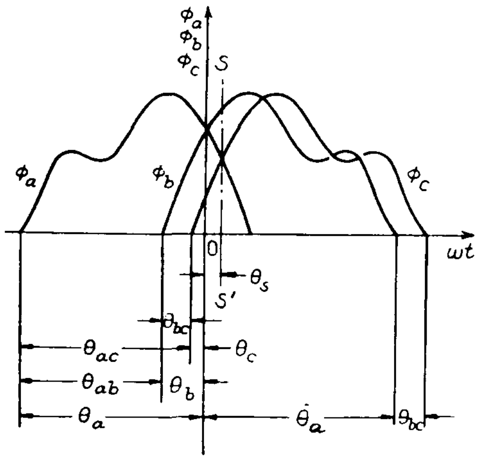
(a) In-phase (b) Anti-phase

Fig.2 Diagrams illustrating "in-phase" and "anti-phase".

(3) There are the following relationships between the vectors of the two distorted waves such as ϕ_k and ϕ_c shown in Fig.3(a). These vectors are otherwise the same but the phases are displaced by a value of θ_{kc} from each other; that is, the arguments of the corresponding harmonics in the both waves differ by θ_{kc} from each other and their magnitudes are equal.

When the distorted wave form is denoted using a conventional method for representing a vector which is used for sinusoidal wave, the relative positions of each harmonic will differ depending on how the standard vector is set. On the other hand, using the vector symbolic method shown in Fig.1, the relative positions of each harmonic vector are invariable because of this characteristic (3).

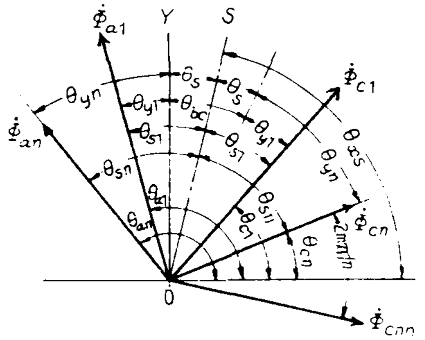
(4) The vector diagram of the two distorted waves ϕ_a and ϕ_c shown in Fig.3(a), which are symmetric with respect to the axis SS', has the following relationships as shown in Fig.3(b).



ϕ_a, ϕ_b ; Inversed waves which are symmetric with respect to the Y-axis

ϕ_a, ϕ_c ; Inversed waves phase angles of which are displaced θ_s each other

(a) Various wave forms



$\phi_a = \sum \Phi_{an} \sin n(\omega t + \theta_{xs} + \theta_{sn})$

$\phi_c = \sum \Phi_{an} \sin n(\omega t + \theta_{xs} - \theta_{sn})$

(b) Vector diagram for the "inversed waves with phase difference θ_s"

Fig.3 Various wave forms and their vector diagram.

When a segment $\overline{OS}$ is drawn in the center of the fundamental vectors $\dot{\Phi}_{a1}$ and $\dot{\Phi}_{c1}$, both vectors of the n-th order are symmetric with respect to the axis $\overline{OS}$, or are symmetric by a value of $2m\pi/n$ as shown in $\dot{\Phi}_{cnn}$ of Fig.3(b). The angle θ_s , measuring anticlockwise from the segment $\overline{OS}$ to the imaginary axis, is equivalent to θ_s shown in Fig.3(a). Hereafter, we call such two wave forms the "inversed waves with phase difference θ_s".

If ϕ_a and ϕ_c are the inversed waves and their instantaneous values are already known in an interval of ωt between θ_s and $(\theta_s + \pi/2)$, all the other intervals can be obtained by substituting the following equations.

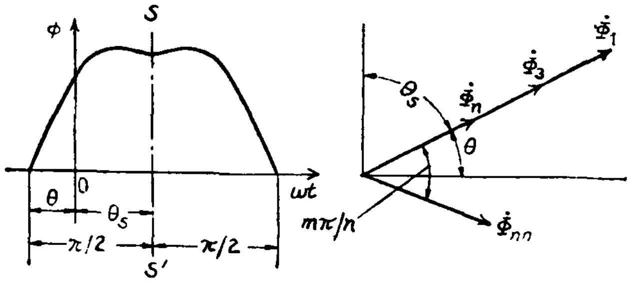
$$\left.\begin{aligned} \phi_a(\omega t) &= \phi_c(2\theta_s - \omega t) = -\phi_c(2\theta_s \pm \pi - \omega t), \\ \phi_c(\omega t) &= \phi_a(2\theta_s - \omega t) = -\phi_a(2\theta_s \pm \pi - \omega t). \end{aligned}\right\} \quad (2)$$

(5) If the distorted wave is a "symmetrical wave at 90°" as shown in Fig.4(a), the arguments of all harmonic vectors are entirely identical or will differ from each other by a value of $m\pi/n$ like that shown in $\dot{\Phi}_{nn}$ of Fig.4(b); that is, each harmonic vector is in-phase or anti-phase with the fundamental vector. Moreover, in Fig.4(b), the angle θ_s measuring anticlockwise from $\dot{\Phi}_1$ to the imaginary axis is the same as θ_s in Fig.4(a). Here, the "symmetrical wave at 90°" is defined as the symmetric wave form with respect to the axis SS' as shown in Fig.4(a). In this case, like the "inversed waves", wave form can be calculated by substituting the following equation.

$$\phi(\omega t) = \phi(2\theta_s - \omega t) = -\phi(2\theta_s \pm \pi - \omega t). \quad (3)$$

In electric or magnetic circuits, the causes of distorted wave forms are in general obvious. Therefore, there are very often the various relationships as mentioned above between the magnitudes and the phase angles of the harmonics. Hence, the wave shape can be estimated qualitatively from characteristics of the vector diagram as mentioned above without analysing the circuits.

In the reference (3), several other characteristics of vectors for the distorted wave and the practical examples are shown.



(a) Symmetrical wave at 90° (b) Vector diagram

$$\phi=\sum\Phi_n \cdot \sin n(\omega t+\theta_n+m\pi/n)$$

Fig.4 "Symmetrical wave at 90°" and its vector diagram

It has been hitherto thought that the vector symbolic method cannot be applied to the distorted wave, because the rotating speeds of each harmonic vector are not identical. Representing the argument of the n-th harmonic vector by 1/n times as large as the phase angle of the harmonic component as shown in Fig.1, the relative positions of respective harmonic vectors are invariable wherever the standard vector is put, and the wave shape can be deduced from the vector diagram. This new method for representing the distorted wave forms is especially useful when we want to know the relationships between two wave forms.

References

(1) T.Nakata, Y.Ishihara, M.Nakano: Electrical Engineering in Japan, 90 (1970), 1, 10-20.
(2) T.Nakata, Y.Ishihara, M.Nakano: Ibid., 91 (1971), 3, 17-28.
(3) T.Nakata, Y.Ishihara: Jour.I.E.E., Japan, 92-A (1972), 5, 241-245.