

Magnetoacoustic Amplification by Conduction Electrons ★

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Synopsis

A theory, based on Chambers' method to the classical Boltzmann equation, is developed for an acoustic amplification in both degenerate and nondegenerate piezoelectric semiconductors subjected to the Hall-geometrically configured electric and magnetic fields. It is found that an amplification constant for $qR > 1$ holds not only for a magnetic field $\omega_c \tau > 1$ but for $\omega_c \tau \leq 1$ under $ql > 1$ while the amplification constant for $qR < 1$ does for $ql \leq 1$ under $\omega_c \tau > 1$; q is the wave number vector of sound, R the cyclotron radius, ω_c the cyclotron frequency, l the mean free path and τ the relaxation time.

A generalized attenuation (amplification) constant is presented through an energy conservation law, being applicable to the sounds propagating at any angle with respect to the particle drift so the off-axis as well as on-axis amplifications are surely involved.

An application of the present theory to n-InSb reveals a threshold dependence for the acoustic amplification, which is semi-quantitative agreement with the experimental result of Arizumi et al.. The amplification constant by the nondegenerate particles is found to be almost equal to that by the degenerate ones, provided that the former carrier density should be replaced by its three times as much.

1. Introduction

In conformity with the Boltzmann equation technique of Chambers (1), Spector demonstrated an amplification of acoustic sound by degenerate conduction electrons in a case where crossed dc electric \underline{E}_0 and magnetic \underline{B}_0 fields are applied, with the sounds traveling in the $\underline{E}_0 \times \underline{B}_0$ direction (2). More recently Weller and Duzer have given almost the same results for a nondegenerate piezoelectric semiconductor in the same geometrical configuration (3). In the above two calculations, the product of the particle cyclotron frequency ω_c and its relaxation time τ , $\omega_c \tau$, was at least several times unity. It is readily comprehended that this magnetic field restriction comes from a geometric configuration of the fields and the propagation vector q of a sound, parallel to V_d^H the drift velocity of particles expressed as $\underline{E}_0 \times \underline{B}_0 / B_0 \cdot B_0$. Therefore by a choice of the Hall geometric configuration we may expect to lift this magnetic field restriction $\omega_c \tau > 1$. Abe and Mikoshiba (4) have tried to develop a theory in the Hall geometric nondegenerate semiconductors but obtained the amplification constant for $\omega_c \tau > 1$ as well, because the first-order particle trajectory, thus the conductivity tensor, may be incorrect so it can be seen that the magnetic field restriction occurred (5). Independently Ishii (6) discussed it briefly in a degenerate case and found the amplification constant which

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is applicable for all magnetic field.

Thus there remains the task of providing the detailed theoretical argument to remove the magnetic field restriction $\omega_c \tau > 1$. Firstly we set up an exact formulation of attenuation of acoustic sound by an energy conservation law in piezoelectric materials (7). It is a generalized formalism of the power theorem (8) which clearly brings out the contribution of various mechanism to energy flow, stored energy and energy dissipation in solids. Generally an attenuation (amplification) constant of acoustic sound is given, in the absence of collisional drag effects, by intuitively dividing the joule heat P by the energy flux of a sound as (9),

$$\alpha = P / (\rho \dot{\mathbf{u}}^* \dot{\mathbf{u}} V_s / 2), \quad (1)$$

where ρ is the mass density of lattice and \mathbf{u} is the displacement of lattice. This constant, however, should be self-consistently procured by the aid of an energy conservation law. In our previous paper (8) the energy conservation law has led us to the attenuation of the acoustic sounds in semi-isotropic media for any direction of propagation of the sound with respect to the direction of the conduction-carrier drift. But really media should be anisotropic so the so the formula can not always result in correct informations about the attenuation of the sounds propagating in all directions. Equation (1) is proved in a general fashion in Sec.2. in anisotropic piezoelectric-semiconductors. On the other hand the attenuation constant can be obtained by a dispersion relation as well in the interacting frame between sounds and conduction carriers (10), but this approach sometimes loses physical pictures. Sec.3. will be devoted to discussion on the conductivity tensors which hold a key to remove the magnetic field restriction, and on the first-order particle trajectory. Lastly some discussion on the results of the present theory, whose comparison with the experiments support a propriety of the present theory, and on an amplification by nondegenerate particles will be made.

2. General Formulation of Sound Attenuation

A Debye continuum is assumed, but, in order to allow for the variation of piezoelectric constant with directions of propagation and polarization in general, an anisotropic Debye approximation is used. Each set of directions has a separate sound velocity V_s , but each velocity is now assumed independent of frequency. A set of piezoelectric equations are written as (11),

$$T_{\alpha\tau} = \sum_{\beta\lambda} C_{\alpha\tau,\beta\lambda} S_{\beta\lambda} - \sum_{\beta} e_{\beta,\alpha\tau} E_{\beta}, \quad (2a)$$

$$P_{\alpha} = \sum_{\beta\tau} e_{\alpha,\beta\tau} S_{\beta\tau} + \sum_{\beta} a_{\alpha\beta} E_{\beta}, \quad (2b)$$

where $C_{\alpha\tau,\beta\lambda}$ is the elastic stiffness constant, $a_{\alpha\beta}$ the dielectric susceptibility constant, E_{α} the self-consistent field, $T_{\alpha\beta}$ the stress, $e_{\beta,\alpha\tau}$ the piezoelectric constant, $S_{\beta\lambda}$ the strain $\partial u / \partial x_{\lambda}$ and P_{α} the polarization of lattice. Equation (2a) gives an equation of motion for the lattices as

$$\rho \ddot{u}_{\alpha} = \sum_{\beta\tau\lambda} C_{\alpha\tau,\beta\lambda} \frac{\partial^2 u_{\beta}}{\partial x_{\tau} \partial x_{\lambda}} - \sum_{\beta\tau} e_{\beta,\alpha\tau} \frac{\partial E_{\beta}}{\partial x_{\tau}}. \quad (3)$$

The relations (2b) and (3) are combined with Maxwell's equations to yield the energy conservation law of second order quantities as is derived in the Appendix 1., in the following form

$$\sum_{\tau} \frac{\partial}{\partial x_{\tau}} S_{\tau} + \frac{\partial}{\partial t} W = - \sum_{\alpha\beta} (\frac{1}{2}) \text{Re} \alpha_{\alpha\beta} S'_{\alpha\beta} E_{\beta} E_{\alpha}^*, \quad (4)$$

$$S_{\tau} = (\frac{1}{2}) \text{Re} \sum_{\alpha\beta\lambda} \omega q_{\lambda} C_{\alpha\tau,\beta\lambda} u_{\alpha}^* u_{\beta}, \quad W = (\frac{1}{2}) \rho \omega^2 \mathbf{u} \cdot \mathbf{u}.$$

The above equations have been averaged over a period of vibration. Since the systems vary with $\exp(i\mathbf{q} \cdot \mathbf{r} - i\omega t)$, an energy flow density \mathbf{S} or an energy density W varies with a spatial dependence of $\exp(-2\mathbf{q}_1 \cdot \mathbf{r})$ if we take the propagation vector and the angular frequency as $\mathbf{q} = \mathbf{q}_r + i\mathbf{q}_i$ and $\omega = \omega_r + i0$ respectively. Accordingly $\partial / \partial x_{\tau} = -2q_{i\tau}$ and $\partial / \partial t = 0$ follow in eq.(3), in which α is defined as an attenuation (amplification)

constant of the sound. From eq.(4) together with the relations (7)-(8) in Sec.3., the exact α is expressed as

$$\frac{1}{2} \omega \operatorname{Re}(\underline{u} \cdot \underline{\alpha}) : \underline{C} : (\underline{u} \cdot \underline{q}_r) = \frac{\omega^2}{2 \alpha_0} \operatorname{Re} \sum_{\alpha} (\underline{e} \cdot (\underline{u} \cdot \underline{q}_r))_{\alpha} \left(\sum_{\beta} (\underline{S}' + \underline{B})_{\alpha\beta}^{-1} (\underline{e} \cdot (\underline{u} \cdot \underline{q}_r))_{\beta} \right)^* . \quad (5)$$

Equation (5) is just the energy conservation law and is the general expression of eq.(1), where the right hand side is equal to the joule heat P while the left hand side is equal to αx (the energy flux). If we assume $\omega = \omega_r + i \omega_i$ for $\underline{q} = \underline{q}_r + i0$, we can obtain the relation

$$- \frac{1}{2} \rho \omega^2 \underline{u} \cdot \underline{u} \cdot \underline{\gamma} = - \frac{\omega^2}{2 \alpha_0} \operatorname{Re} \sum_{\alpha} (\underline{e} \cdot (\underline{u} \cdot \underline{q}_r))_{\alpha} \left(\sum_{\beta} (\underline{S}' + \underline{B})_{\alpha\beta}^{-1} (\underline{e} \cdot (\underline{u} \cdot \underline{q}_r))_{\beta} \right)^* , \quad (6)$$

here $\underline{\gamma} = 2 \omega_i$ is defined as an attenuation constant in time. Equations (5) and (6) instruct us that if α is smaller than zero, then the constant $\underline{\gamma}$ becomes larger than zero, which means an amplification of the sound spatially occurs as well as in time. Therefore a convective instability prevails in the system of conduction carriers and sounds (12).

3. Formal Theory

3-1 Constitutive Equation

In the model developed by Cohen, Harrison and Harrison (13), the conduction electrons are replaced by the model of a free-electron gas of density N_0 , which is neutralized by a positive background of the same density N_0 . The sound of wave vector \underline{q} and frequency ω manifests itself as a velocity field, $\underline{u} \propto \exp(i \underline{q} \cdot \underline{r} - i \omega t)$, in the background. The interaction between the electron (or hole) gas and the sound can be presented in this paper only by a piezoelectric force, whose field \underline{E} is regarded as a self-consistent field involved in Maxwell's equations. The self-consistent electromagnetic field induced by the passage of the sound can be derived from Maxwell's equations. In our case it can be written in the form

$$\underline{J}_e + \frac{\partial}{\partial t} \underline{e} \cdot \underline{S} = - \alpha_0 \underline{B} \cdot \underline{E} , \quad (7)$$

where \underline{J}_e and \underline{E} are the electronic current and self-consistent field and \underline{B} is the tensor,

$$\underline{B} = i \frac{\omega}{\omega_p^2 \tau} (\epsilon_0 / \epsilon_1) \begin{vmatrix} -\epsilon_1 / \epsilon_0 & c^2 / V_x V_y & c^2 / V_x V_z \\ c^2 / V_y V_x & -\epsilon_2 / \epsilon_0 & c^2 / V_y V_z \\ c^2 / V_z V_x & c^2 / V_z V_y & -\epsilon_3 / \epsilon_0 \end{vmatrix} , \quad (8)$$

which has been used in eqs.(5)-(6). Here a plasma frequency is defined as $\omega_p = (N_0 e^2 / \pi \epsilon_1)^{1/2}$, c is the light velocity and V_i the sound velocity.

In this paper we make use of a particle having its charge e instead of an electron or hole. Thus the particle current can be obtained from the distribution function in the usual manner;

$$\underline{J}_e = e \int d^3 V \underline{V} f . \quad (9)$$

The Boltzmann equation from which the distribution function is determined in the presence of external electric and magnetic field is

$$\left(\frac{\partial f}{\partial t} \right) + \underline{V} \cdot \left(\frac{\partial f}{\partial \underline{r}} \right) + (e/m) (\underline{E} + \underline{E}_0 + \underline{E}_H + \underline{V} \times \underline{B}_0) \cdot \left(\frac{\partial f}{\partial \underline{V}} \right) = - (f - f_s) / \tau . \quad (10)$$

In equation (10), \underline{E}_H is the Hall field, \underline{E}_0 is the dc electric field and \underline{B}_0 is the dc magnetic field. The distribution function relaxes, in the presence of the sound, to an equilibrium distribution of

$$f_s(\underline{r}, \underline{V}, t) = f_0(\underline{V}, E_f(\underline{r}, t)) , \quad (11)$$

in the absence of impurities, where $f_0(\underline{V}, E_f)$ is the equilibrium Fermi-Dirac

distribution, and $E_f(\underline{r}, t)$ is the Fermi energy chosen to give the correct electron density. It has been shown that the Boltzmann equation can be solved by a method due to Chambers (1). This solution is given by

$$f(\underline{r}, \underline{v}, t) = \int_{-\infty}^t f_s(\underline{r}', \underline{v}', t') \exp(-(t-t')/\tau) dt' / \tau. \quad (12)$$

Expanding the distribution function to first order \underline{E} , \underline{u} and terms proportional to \underline{u} , and keeping terms that are of first order in both \underline{E} and \underline{u} , we have

$$f = f_0 + f_1^0 + f_1^1. \quad (13)$$

Here f is the unperturbed distribution function, f_1^0 is the part of the perturbed distribution function which is independent of the sound, and f_1^1 is the part of the perturbed distribution function which varies as $\exp(i\underline{q} \cdot \underline{r} - i\omega t)$. Thus we obtain, taking account of the Hall field in the x direction where $E_H = -\omega_c \tau E_0$, the solutions:

$$f_1^0 = -e \tau E_0 v_y (\partial f_0 / \partial \epsilon), \quad (14)$$

$$f_1^1 = - \int_{-\infty}^t \frac{dt'}{\tau} \exp(-(t-t')/\tau) \left((e \tau \underline{v}' \cdot \underline{E} + \frac{2N}{3N_0} E_f) (\partial f_0 / \partial \epsilon + m \underline{v}'_H \cdot (\underline{v}' - \underline{v}) \times \right. \\ \left. \times \partial^2 f_0 / \partial \epsilon^2) - e \tau v_d (E_y \partial f_0 / \partial \epsilon + m \underline{v}'_y \cdot \underline{E} \cdot \underline{v}' \partial^2 f_0 / \partial \epsilon^2) \right), \quad (15)$$

where $\underline{v}_H = \underline{v}_d^H + \underline{v}_d^{HH}$ with $\underline{v}_d^H = (\underline{E}_0 \times \underline{B}_0) / B_0^2$, and $\underline{v}_d^{HH} = (\underline{E}_H \times \underline{B}_0) / B_0^2$ is the seeming drift velocity of the particles in the crossed electric and magnetic fields of Hall geometry, and in fact the seeming velocity \underline{v}_d^H disappears later in the calculation of the conductivity tensors with the help of first-order particle trajectory; this is easily understood since in the Hall geometry there is no steady current in the $E_0 \times B_0$ direction and the effective drift velocity is \underline{v}_d^{HH} which is equal to $e \tau E_0 / m$ under the constant relaxation time. We have chosen our y axis to be in the direction of \underline{E} and z axis to be in the direction of \underline{B} . From eq.(9), we see that eq.(14) will only contribute to the dc current and, thus, can be neglected in considering the particle-sound interaction. The particle current which is proportional to the sound can be obtained from eqs.(9) and (15). The deduced constitutive equation is

$$\underline{J}_e = (\underline{\alpha} + \underline{\Sigma}) \cdot \underline{E} + R n e v_s, \quad (16)$$

where

$$\Sigma_{ij} = -e^2 \tau v_d \int d^3 v \int_{-\infty}^t \frac{dt'}{\tau} \exp(h(t')) v_i \left((-\partial f_0 / \partial \epsilon) \delta_{yj} + m v'_j v'_y (-\partial^2 f_0 / \partial \epsilon^2) \right), \quad (17)$$

$$\alpha_{ij} = e^2 \tau \int d^3 v \int_{-\infty}^t \frac{dt'}{\tau} \exp(h(t')) v_i v'_j \left((-\partial f_0 / \partial \epsilon) + m \underline{v}'_H \cdot (\underline{v}' - \underline{v}) (-\partial^2 f_0 / \partial \epsilon^2) \right), \quad (18)$$

$$R_{ii} = (2E_f / 3N_0 v_s) \int d^3 v \int_{-\infty}^t \frac{dt'}{\tau} \exp(h(t')) v_{ii} \left((-\partial f_0 / \partial \epsilon) + m \underline{v}'_H \cdot (\underline{v}' - \underline{v}) \times \right. \\ \left. \times (-\partial^2 f_0 / \partial \epsilon^2) \right), \quad (19)$$

$$h(t') = (1 - i\omega\tau)(t' - t) / \tau + i\underline{q} \cdot (\underline{r}' - \underline{r}). \quad (20)$$

It should be noted that the above conductivity tensors, especially Σ_{ij} , have no ~~magnetoresistance effects~~, which are different from the result of Spector's theory, and the above equations are more concise (6), (2). Whether one can remove the magnetic field restriction $\omega_c \tau > 1$ or not, surely depends on this difference for one part. The same arguments can be done in eq.(15), and furthermore can be applied to the nondegenerate case as in the last section.

The continuity equation relates the nonuniform part of the particle density N to the current along the direction of propagation; i.e., $\hat{q} \cdot \underline{J}_e = N v_s$. Defining a tensor \underline{R} by means of the relation $\underline{R} \cdot \underline{J}_e = \hat{q} \cdot \underline{J}_e$,

we can write eq.(16) in the form

$$\underline{J}_e = \alpha_0 (\underline{\alpha}' + \underline{\Sigma}') \underline{E} = \alpha_0 \underline{S}' \cdot \underline{E}, \quad (21)$$

where $\underline{\alpha}' = (\underline{1} - \underline{R})^{-1} (\underline{\alpha} / \alpha_0)$ and $\underline{\Sigma}' = (\underline{1} - \underline{R})^{-1} (\underline{\Sigma} / \alpha_0)$.

3-2 Attenuation (Amplification) Constant

In Sec.2., we have studied the generalized attenuation constant α of the sounds in the absence of collisional drag effects by impurities. In order that the better figure of attenuation of sound can be seen, eq.(5) should be furtherly simplified for a special geometric arrangement of \underline{u} and \underline{q} (or $\underline{\alpha}$). Thus the y axis is taken parallel to $\langle 110 \rangle // \underline{q}$ and the z axis parallel to $\langle 001 \rangle // \underline{u}$ in the piezoelectric semiconductors of zinc blende structure. Then eq.(5) directly leads us to

$$\alpha_y = \frac{\omega^2 K^2}{\omega_p^2 \epsilon V_s} \text{Re} \left(\underline{S}' + \underline{B} \right)_{yy}^{-1}, \quad (22)$$

therein B becomes $-i(\omega / \omega_p^2 \epsilon)$ because in eq.(8) the elements concerned with V_i for $i = x, y$ vanish. Here $\omega_p^2 = N_0 e^2 / m \epsilon$ for $\epsilon_i = \epsilon$ with e as the electronic charge and N_0 as the number density of the carriers, $V_s = (C_{44} / \rho)^{1/2}$, and $K^2 = e_{14}^2 / \epsilon C_{44}$ with e_{14} as the piezoelectric constant and C_{44} as the elastic stiffness constant in the Voigt representation. This final result is quite consistent with Spector's result given in the deformation coupling (2).

3-3 Conductivity Tensors

The attenuation constant is now specified in terms of the conductivity tensors \underline{S}' . The present task is to evaluate explicitly the integral expressions (17)-(19). It is noted that in the expressions for the attenuation constant (22), $\underline{\alpha}$ and $\underline{\Sigma}$ occur only in the combination $\underline{S} = \underline{\alpha} + \underline{\Sigma}$, $\underline{S}' = \underline{\alpha}' + \underline{\Sigma}'$. To calculate them, we have to know the first-order particle trajectory how the particles can go along an orbit before being scattered. It is chosen that a coordinate system has a y axis in the direction of \underline{q} and \underline{E}_0 , a z axis in the direction of \underline{B}_0 and a x axis in the direction of \underline{E}_H . In this coordinate system, the relation between $(\underline{r}, \underline{V})$ and $(\underline{r}', \underline{V}')$ is found, by solving an equation of motion,

$$\frac{d\underline{V}'}{dt'} \Big|_1 = (e/m) (\underline{E}_0 + \underline{E}_H + \underline{V}' \times \underline{B}_0), \quad (23)$$

for an electron affected by the force: $e(\underline{E}_0 + \underline{E}_H + \underline{V}' \times \underline{B}_0)$,

$$V'_x = V_L \cos(\omega_c s - \varphi) + V_d^H (1 - \cos \omega_c s) - V_d^{HH} \sin \omega_c s,$$

$$V'_y = -V_L \sin(\omega_c s - \varphi) + V_d^H \sin \omega_c s + V_d^{HH} (1 - \cos \omega_c s),$$

$$V'_z = V_{||}, \quad V_L = V \sin \theta, \quad V_{||} = V \cos \theta, \quad s = t' - t,$$

$$x' = x + (V_L / \omega_c) (\sin(\omega_c s - \varphi) + \sin \varphi) - (V_d^{HH} / \omega_c) (1 - \cos \omega_c s) + V_d^H s - (V_d^H / \omega_c) \sin \omega_c s, \quad (24)$$

$$y' = y + (V_L / \omega_c) (\cos(\omega_c s - \varphi) - \cos \varphi) + (V_d^H / \omega_c) (1 - \cos \omega_c s) + V_d^{HH} s - (V_d^{HH} / \omega_c) \sin \omega_c s, \quad z' = z + V_{||} s,$$

where θ and φ are the polar and azimuthal angles of \underline{V} . Equation (23) originates from the reason why the Hall field can be defined only in the stationary state so that it is switched on at time $t' = -\infty$ in the same fashion as \underline{E}_0 and \underline{B}_0 , and why an equation of motion must not include collision term such as V'/τ because in Chambers' method, a particle trajectory must be free from the collision-drag term during the time interval $s = t' - t$. Further discussion will be made in Sec.6. By use of eq.(24), we obtain the

following expressions for the components S and R in the Fermi-Dirac statistics:

$$S_{yy} = i \frac{3\alpha_0}{X_{q1}} \exp(iX_H) \sum_{mp1} \frac{m I_1(-iX_H) J_p(X_{HH})}{i(m-p-1)\omega_c \tau + \lambda} \left[\lambda g_m(X) + ((1-i\omega\tau) \left(\frac{V_d}{XV_f} \right) - i \frac{V_d}{q1V_f}) \right. \\ \left. - i \frac{\lambda V_d}{q1V_f} \right] J_{2m}(2X) + (1 - i\omega\tau) \frac{V_d}{V_f} (i2/\omega_c \tau - 1/m) J_{2m}'(2X) - i \frac{3\alpha_0}{(q1)^2} qV_d \tau, \quad (25)$$

$$R_{yy} = \frac{q1}{X\omega\tau} \exp(iX_H) \sum_{mp1} \frac{m I_1(-iX_H) J_p(X_{HH})}{i(m-p-1)\omega_c \tau + \lambda} \left[g_m(X) + \frac{V_d}{V_f} (m/X - i/q1 - i\lambda/q1) \times \right. \\ \left. \times J_{2m}(2X) + \frac{V_d}{V_f} (i2/\omega_c \tau - 1/m) J_{2m}'(2X) \right], \quad (26)$$

where $X=qV_f/\omega_c$, $X_H=qV_d^H/\omega_c$, $X_{XX}=qV_d^{HH}/\omega_c$, $\lambda = 1 - i\omega\mu\tau = 1 - i\omega\tau(1 - V_d/V_s)$. Furthermore $J_m(X)$ and $I_m(X)$ are the Bessel function and the modified Bessel function respectively, and $g_m(X)$ are those defined by Cohen, Harrison and Harrison in their paper (13). Equations (25)-(26) have been obtained by that we have dropped the terms of the order of $(V_s/V_f)^2$ and $(V_d/V_f)^2$ in equations in the Appendix 2., and they are valid for any values of magnetic fields under the present approximations. Since we are interested primarily in phenomena which occur when the magnetic fields satisfy the relations of $X \gg 1$ which would involve the cyclotron resonance effects, and of $X \ll 1$ which would do the geometric resonance effects, we are to drop the terms that are smaller by a factor of the order of $X^{-3/2}$ for $X \gg 1$ and X^4 for $X \ll 1$ than the remaining terms in $g_m(X)$.

4. Low Field Limit $X \gg 1$

It has been shown that the Doppler shifted cyclotron resonance effects occur when $\omega\mu$ is of the order of the cyclotron frequency (2). Under these conditions, X is very large. Thus we obtain the limiting expressions for S and R using the relations in the Appendix 2.:

$$S_{yy} = \frac{3\alpha_0}{(q1)^2} (1 - i\omega\tau) \left(1 - \frac{\lambda\pi}{2q1} \coth \frac{\lambda\pi}{\omega_c \tau} \right), \quad (27)$$

$$1 - R_{yy} = (i/\omega\tau) (1 - i\omega\tau) \left(1 - \frac{\pi}{2q1} \coth \frac{\lambda\pi}{\omega_c \tau} \right), \quad (28)$$

$$\alpha_y = \frac{\mu\tau}{3V_s} \left(\frac{\omega^2 V_f K}{\omega_p V_s} \right)^2 \frac{A(1-A) - B^2}{\left((1-A) \left(1 + \frac{1}{3} \left(\frac{\omega V_f}{\omega_p V_s} \right)^2 \right) - B\omega\mu\tau \right)^2 + \left(A\omega\mu\tau - B \left(1 + \frac{1}{3} \left(\frac{\omega V_f}{\omega_p V_s} \right)^2 \right) \right)^2}, \quad (29)$$

where $A + iB = (\pi/2q1) \coth(\lambda\pi/\omega_c \tau)$. Above all equations can be applicable for any magnetic fields for $X \gg 1$. Again we would rather insist on a point that no magnetoresistance effects in equations (14)-(15) or (17) have been conserved so that its effects are not contained in eqs.(27)-(29). If we are interested in the limit $q1 \gg 1$, we can neglect A and B with respect to unity, and eq.(28) reduces to

$$\alpha_y = \frac{\mu\tau}{3V_s} \left(\frac{\omega^2 V_f K}{\omega_p V_s} \right)^2 \frac{A}{\left(1 + \frac{1}{3} \left(\frac{\omega V_f}{\omega_p V_s} \right)^2 \right)^2}. \quad (30)$$

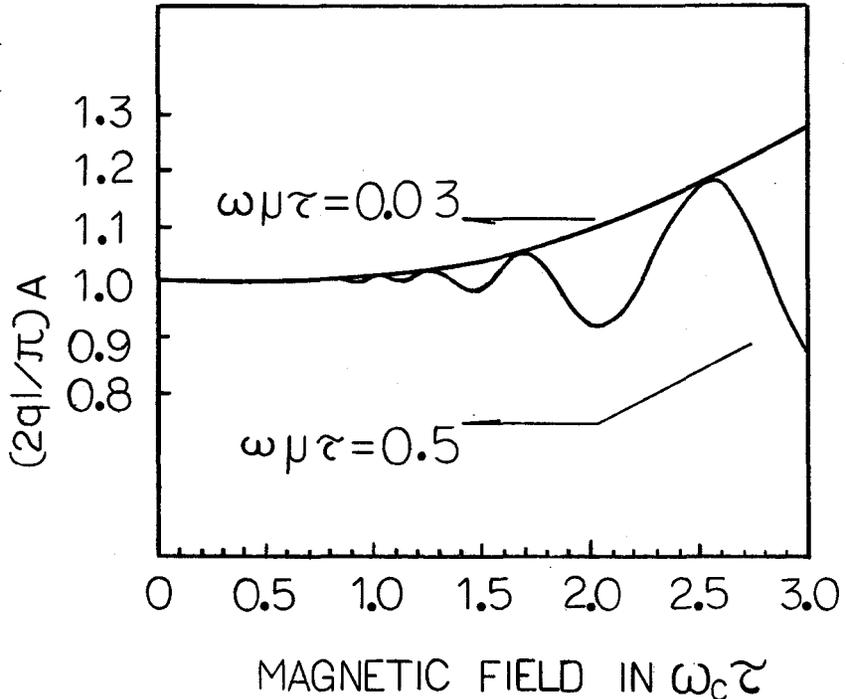
In order to clarify a magnetic field dependency of α_y for the small magnetic-field intensity, we are to trace a normalized relative amplification function $(2q1/\pi)A$ for the quantity $\omega_c \tau$. In Fig.1 is shown its dependency for the cases of $\omega\mu\tau = 0.03$ and 5.0 , where, in the vicinity of $\omega_c \tau \approx 1$, an oscillatory behaviour can be seen. When ω_c approaches to zero,

we have

$$\alpha_y = \frac{\pi K^2}{6} (\omega/v_f) \left(\frac{(\omega/\omega_p)(v_f/v_s)}{1 + \frac{1}{3} \left(\frac{\omega v_f}{\omega_p v_s} \right)^2} \right)^2, \quad \text{for } ql \gg 1. \quad (31)$$

If the sounds couple with the particles through the deformation potential energy, a correspondent equation to eq.(31) is exactly equal to the expression obtained by Spector (14) in the case where no external magnetic field is present.

Fig.1
Magnetic field dependence of the normalized relative amplification constant $(2ql/\pi)A$. The parameter $\omega\mu\tau$ is taken equal to 0.03 and 0.5.



5. High Field Limit $X < 1$

In the high field region of $X < 1$ and $\omega_c \tau > 1$, the yy -component of the conductivity tensors \underline{S} and \underline{R} are obtained on the arguments in the Appendix 2.,

$$S_{yy} = \frac{3\sigma_0}{(ql)^2} \left[\lambda (1 - g_0(X)) - iqV_d \tau (1 - g_1(X) + \frac{1 - i\omega\tau}{\lambda} \frac{J'_0(2X)}{X}) \right], \quad (32)$$

$$1 - R_{yy} = (i/\omega\tau) \left[1 - i\omega\tau - g_0(X) - \frac{1 - iqV_d \tau}{\lambda} \left(\frac{J'_0(2X)}{X} - g_1(X) \right) \right], \quad (33)$$

$$\alpha_y = \frac{\mu\tau}{v_s} \left(\frac{\omega^2 v_f K}{\omega_p v_s} \right)^2 \frac{X^2 (1 - X^2/3)}{X^4 \left(1 + \frac{1}{3} \left(\frac{\omega v_f}{\omega_p v_s} \right)^2 \right)^2 + (\omega\mu\tau)^2 \left(X^2 + \left(\frac{\omega v_f}{\omega_p v_s} \right)^2 \right)^2}. \quad (34)$$

The attenuation constant has been derived by use of the limiting relation $g_0(X) = 1 - X^2/3$. The condition of $\omega_c \tau > 1$ has been imposed in order to make the calculation easier. So it is not the condition imposed in order to give the physical meaning as the drift velocity of particles to V_d^H unlike

the case of Spector's theory(2). Therefore equations (25)-(26), in its sense, include the solutions for the field $\omega_c \tau \approx 1$ in the region $X < 1$ as well. Moreover eqs.(32)-(34) imply the validity for any value of q_1 under $X < 1$ and $\omega_c \tau > 1$, which means that q_1 is allowed to take a value even larger than unity (see Appendix 2.).

6. Discussion and Conclusion

We have presented the theory of sound amplification in the Hall geometric configuration, where it is applicable for any values of magnetic field especially for the case $X > 1$ and is also applicable for any value of q_1 for the case $X < 1$. Furthermore it is proved that the present theory embraces Spector's theory (2,14), so it is seen that the curve of his cyclotron resonance is identical with our result, which is, however in the present theory, possibly extended to the region $\omega_c \tau \lesssim 1$.

Now let us examine the present theory by its application to n-InSb. Kikuchi, Hayakawa and Abe (15) found two branches of the threshold fields for an amplification of the sounds. Since the first branch is involved in the region of low magnetic field, only the result (28) can be significant. In our preliminary report (16), the calculation has depicted an oscillation on the threshold curve. Just the same calculation has been carried out by changing the parameters ω and V_d in smaller bits to obtain the result shown in Fig.2, which is rather satisfactory. A magnetoresistance effect has

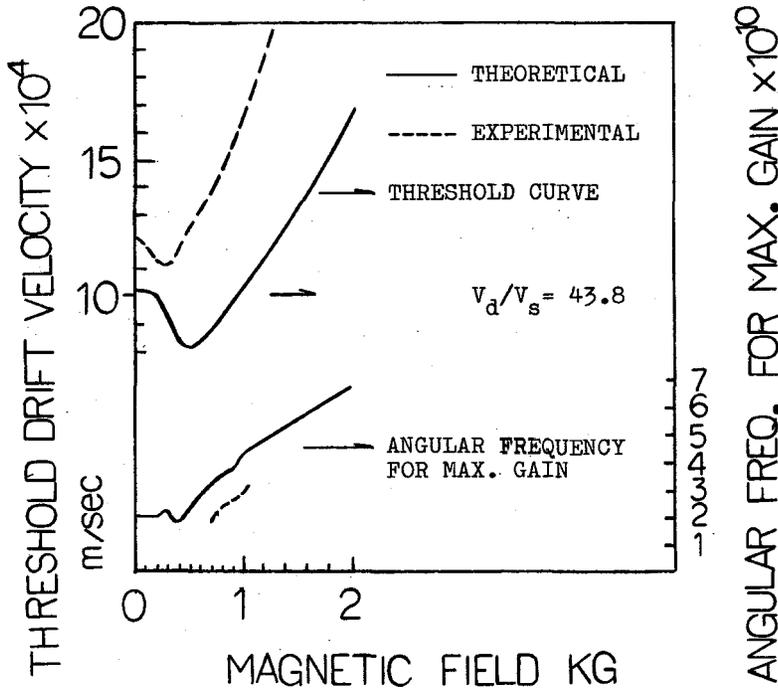


Fig.2 Theoretical threshold curve of sound amplification for n-InSb ($\tau = 6.14 \times 10^{-12}$ sec, $V_f = 2.14 \times 10^7$ m/sec and $K_p^2 = 0.00462$ as were taken in Ref.16) at 77K is indicated in solid line while the broken line shows the experimental result by Arizumi et al.(17). The lower solid line shows a shift of angular frequency for the maximum gain just on the upper threshold curve, whereas the experimental curve obtained at 55 V/cm is compared with the broken line. As for the frequencies and the drift velocities in computations, $\omega \times 10^{-10} = 1+0.25m$ for $m=1,2,---,24$ and $V_d = (20.5 - 0.5n) \times 10^4$ for $n=1, 2, ---, 25$ are prepared respectively.

been taken into account in the above calculations, which will be discussed

in subsequent discussion.

First-order Particle Trajectory

The important point in eq.(12) is that τ is assumed to be independent of t and t' . Furthermore τ is also assumed to be independent of a particle energy $\frac{1}{2}mV^2$, otherwise it is almost impossible for the nondegenerate distribution function or even for the degenerate one to analyse the current integral, eq.(9), in the presence of the acoustic sounds; see the discussion on a magnetoresistance effect. On the above assumptions we have taken eq.(23) as the Equation of motion for first-order particle trajectory. Now let us investigate its validity. In the absence of the acoustic sounds, we obtain the distribution function of particles to first order in \underline{E} via eqs.(12) and (23) as

$$f_1^0 = \underline{V} \left\{ \underline{E}_0 + \underline{B}_0 (\underline{B}_0 \cdot \underline{E}_0) (e\tau/m)^2 + (\underline{E}_0 \times \underline{B}_0) (e\tau/m) \right\} \frac{e}{1 + \omega_c^2 \tau^2} \left(- \frac{\partial f_0}{\partial \epsilon} \right), \quad (35)$$

which is identical with the usual result (18). If we take \underline{B}_0 in the z direction and \underline{E}_0 in the y direction, the Hall field \underline{E}_H is defined in the stationary state as in eq.(36) from the requirement $\underline{J}_{ox}=0$,

$$\begin{aligned} \underline{E}_H &= - \underline{E}_0 \omega_c \tau^2 / (1 + \omega_c^2 \tau^2) \rangle / \langle \tau / (1 + \omega_c^2 \tau^2) \rangle \\ &= - \underline{E}_0 \omega_c \tau \quad \text{for our condition assumed above.} \end{aligned} \quad (36)$$

The foregoing discussion guarantees that eq.(23) is the correct expression of first-order particle trajectory. On the other hand Abe and Mikoshiba (4) started from an equation of motion for the first-order particle trajectory given by

$$\left. \frac{d\underline{V}'}{dt'} \right|_1 = (e/m) (\underline{E}_0 + \underline{E}_H + \underline{V}' \times \underline{B}_0) - \underline{V}'/\tau, \quad \text{for } s \gg \tau. \quad (37)$$

It is, however, an error partly because eqs.(12) and (37) can not result in eq.(35) as was discussed in Ref.(5).

Amplification by Nondegenerate Conduction Electrons

In the expressions of conductivity tensors, \sum_{ij} in Refs.(2)-(4) includes a term associated with $(1 + \omega_c^2 \tau^2)^{-1}$. It can be seen from eq.(17) that its term would not appear as far as the conductivity tensors derived from eqs.(12) and (23) are concerned. This is the same reason eq.(35) is not a function of $(1 + \omega_c^2 \tau^2)^{-1}$ in the Hall geometry. This argument is applicable to the nondegenerate case as well. Equations (17)-(19) lead to the conductivity tensors for the distribution function

$$f_0(V) = (N_0/\pi^{3/2} V_0^3) \exp(-mV^2/2k_B T), \quad (38)$$

in the form

$$\Sigma_{ij} = - \frac{e^2 \tau V_d}{k_B T} \int d^3V f_0(V) \int_{-\infty}^t \frac{dt'}{\tau} \exp(h(t')) V_i (\delta_{yj} - \frac{m}{k_B T} V'_j V'_y), \quad (39)$$

$$\alpha_{ij} = \frac{e^2 \tau}{k_B T} \int d^3V f_0(V) \int_{-\infty}^t \frac{dt'}{\tau} \exp(h(t')) V_i V'_j \left(1 + \frac{m}{k_B T} \underline{V}_H \cdot (\underline{V} - \underline{V}') \right), \quad (40)$$

$$R_{ii} = (N_0 V_s)^{-1} \int d^3V f_0(V) \int_{-\infty}^t \frac{dt'}{\tau} \exp(h(t')) V_{ii} \left(1 + \frac{m}{k_B T} \underline{V}_H \cdot (\underline{V} - \underline{V}') \right), \quad (41)$$

where V_0 is the thermal velocity of particles. Equation (39) as well as eq.(17) include no term associated with $(1 + \omega_c^2 \tau^2)^{-1}$ and therefore one needs not use the approximation $\omega_c \tau > 1$ in the calculation of eqs.(17)-(19)

and (39)-(41), in order to obtain the amplification constant for $X > 1$.

The calculations base on the same method lead to the amplification constants in extrinsic nondegenerate piezoelectric-semiconductors:

I. $X > 1$ for $0 \leq B_0$,

$$\alpha_y = \frac{K^2 (\omega_R/V_s) (\omega/\omega_D) \omega \mu \tau (A(1-A) - B^2)}{((1-A)(\frac{\omega_R}{\omega} + \frac{\omega}{\omega_D}) - B \omega_R \mu \tau)^2 + (A \omega_R \mu \tau - B(\frac{\omega_R}{\omega} + \frac{\omega}{\omega_D}))^2}, \quad (42)$$

where $A + iB = (\pi/2)^{1/2} (1/q1) \coth(\lambda \pi / \omega_c \tau)$.

II. $X < 1$ for $1 < \omega_c \tau$,

$$\alpha_y = \frac{K^2 (\omega_R/V_s) \mu (\omega_c \tau)^2}{\mu^2 ((\omega_p \tau)^2 + (\omega_c \tau)^2)^2 + ((\omega_R/\omega) + (\omega/\omega_D))^2}, \quad (43)$$

here $\omega_p = \omega^2 \tau$, $\omega_D = V^2/D$ with D as the diffusion constant. These results are completely equivalent to the Weller and Van Duzer's results, for the region $\omega_c \tau > 1$. The equations are almost equal to eqs.(29) and (34) if the former carrier density is replaced by its three times as much.

Magnetoresistance Effect

Now let us discuss the magnetoresistance effects on the amplification of sound. We have not taken account of the magnetoresistance effects in the theory, namely τ has been assumed to be independent of the energy. But its effect is quite large (19) and can not be neglected in the calculation of the amplification of sound. Thus we rewrite Boltzmann equation with a newly defined relaxation time $\langle \tau \rangle$ self-consistently from eqs.(9)-(10), (12), (23), and (35)-(36),

$$\langle \tau \rangle = \langle \tau^2 / (1 + \omega_c^2 \tau^2) \rangle / \langle \tau / (1 + \omega_c^2 \tau^2) \rangle. \quad (44)$$

Then τ in eq.(44) may be a function of $\frac{1}{2} mV^2$. If τ is replaced by $\langle \tau \rangle$ in the calculations of this paper, all equations reflect the magnetoresistance effects correctly to first order in the distribution function. The above procedure is the one which has been taken in Ref.(16) and in the calculation of Fig.2. A large difference manifests itself between the cases when its effect is taken into account and not (20). Therefore the magnetoresistance effects should be taken into account in the calculations in Refs(3)-(4).

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Appendix 1.

Along the derivation of the power theorem (8), we obtain the following relation from Maxwell's equation of $\text{rot } \underline{E} = - \partial \underline{H} / \partial t$ and $\text{rot } \underline{H} = \alpha_0 \underline{S}' \cdot \underline{E} + \epsilon_0 \partial \underline{E} / \partial t + \partial \underline{P} / \partial t$, and eq.(2b),

$$\text{div}(\underline{E} \times \underline{H}) + \frac{\partial}{\partial t} (\mu_0 \underline{H}^2 + \underline{E} \cdot \underline{D}) = \frac{1}{2} \sum_{\alpha} ((\underline{e} \cdot \underline{S} + \underline{\epsilon} \cdot \underline{E})_{\alpha} \partial E_{\alpha} / \partial t - E_{\alpha} \frac{\partial}{\partial t} (\underline{e} \cdot \underline{S} + \underline{\epsilon} \cdot \underline{E})_{\alpha}) - \sum_{\alpha} \alpha_0 (\underline{S}' \cdot \underline{E})_{\alpha} E_{\alpha} \quad (1-1)$$

where $\underline{D} = \underline{P} + \epsilon_0 \underline{E}$ and $\underline{\epsilon} = \underline{a} + \frac{1}{2} \underline{\epsilon}_0$. If an energy density and an energy flow density of electromagnetic waves are enough smaller than those of the sounds, which is satisfied in most of the cases, the right hand side of eq.(1-1) gets to be zero. The substitution of the last term in the right hand side of eq.(3) for the first term of the right hand side of eq.(1-1) gives

$$\frac{1}{2} \frac{\partial}{\partial t} \rho ((\partial \underline{u} / \partial t)^2 - \underline{u} \cdot \partial^2 \underline{u} / \partial t^2) + \frac{1}{2} (\underline{u} \cdot \frac{\partial}{\partial t} \sum_f \frac{\partial}{\partial x_f} : \underline{C} : \underline{S} - \frac{\partial \underline{u}}{\partial t} \sum_f \frac{\partial}{\partial x_f} : \underline{C} : \underline{S}) = - \sum_{\alpha} \alpha_0 (\underline{S}' \cdot \underline{E})_{\alpha} E_{\alpha} \quad (1-2)$$

Now we assume that all the fields have a common phaser $\exp(iq \cdot \underline{r} - i\omega t)$, then eq.(4) of the text can be derived provided that eq.(1-2) is averaged over a period of vibration.

Appendix 2.

From eqs.(17)-(20) and (24), we have the exact conductivity tensors as

$$S_{yy} = - \frac{3\alpha_0}{4V_f^2 \tau} \left[\frac{\lambda V_f^2}{q_l} \int d\theta \sin^2 \theta Q_1(X_L) + \frac{1}{V_f^2} \frac{d}{dV_f} V_f^2 \int d\theta \sin^2 \theta x \right. \\ \times \left. i \frac{V_d}{q_l} \left\{ \left(\frac{1}{\omega_c \tau} (1 - i\omega \tau) V_L \frac{\partial}{\partial X_L} - \frac{\lambda V_f}{q_l} \right) V_f Q_1(X_L) - (1 - i\omega \tau) V_f V_L Q_2(X_L) \right\} + \right. \\ \left. + i 4V_d / q \right] \quad (2-1)$$

$$R_{yy} = i \frac{V_f}{4V_s \tau} \left[\int d\theta \sin^2 \theta Q_1(X_L) + \frac{1}{V_f^3} \frac{d}{dV_f} V_f^2 \int d\theta \sin^2 \theta \left[\left\{ i V_d^H V_L \frac{\partial}{\partial X_L} - \right. \right. \right. \\ \left. \left. \left. V_d^{HH} \left(i \frac{\lambda V_f}{q_l} + V_d^{HH} \right) \right\} Q_1(X_L) - i V_d^{HH} V_L Q_2(X_L) \right] \right] \quad (2-2)$$

where $X_L = qV_L / \omega_c$, and $Q_1(X)$, $Q_2(X)$ are written as

$$Q_1(X) = -i \frac{2\tau}{X} \sum_{mpl} \frac{\exp(iXV_d^H / V_L) I_1(-iXV_d^H / V_L) J_p(XV_d^{HH} / V_L) m J_m^2(X)}{i(m-p-1)\omega_c \tau + \lambda} \quad (2-4)$$

$$Q_2(X) = -2 \tau \sum_{m \neq 1} \frac{\exp(iXV_d^H/V_\perp) I_1(-iXV_d^H/V_\perp) J_p(XV_d^{HH}/V_\perp)}{i(m-p-1)\omega_c\tau + \lambda} \times$$

$$\times \left(\frac{J_m(X)J_m'(X)}{X} - \frac{m^2 J_m^2(X)}{X^2} \right). \quad (2-4)$$

If we drop the terms of the order of $(V_d/V_f)^2$ and $(V_d/V_f)^2$ in eqs.(2-1)-(2-2), we have eqs.(25)-(26) in the text.

2-1 Low Field Limit $X \gg 1$

On the assumption of $V_d/V_f < 1$ and $ql > 1$, we can neglect all terms except the first and the last terms in eq.(25), and except the first term in eq.(26). Further use of asymptotic form of eqs.(2-5)-(2-6) for a large $X \gg m$ makes eqs.(25)-(26) simplify to yield eqs.(27)-(29) in the text.

$$\sum_{m=-\infty}^{\infty} \frac{g_m(X)}{\lambda + i(m-p-1)\omega_c\tau} = \frac{1}{2X} \sum_{m=-\infty}^{\infty} \frac{1}{\lambda + i(m-p-1)\omega_c\tau} -$$

$$- 0 \left[\frac{1}{2X} \sum_{m=X}^{\infty} \frac{2(\lambda - i(p+1)\omega_c\tau)}{(\lambda - i(p+1)\omega_c\tau)^2 + (m\omega_c\tau)^2} \right], \quad (2-5)$$

$$\sum_{m \neq 1} \frac{m^J I_1 g_m}{i(m-p-1)\omega_c\tau + \lambda} = (-i/\omega_c\tau) \exp(-iX_H) (1 - (1 - i\omega\tau)) \times$$

$$\times (\pi/2ql) \coth(\lambda\pi/\omega_c\tau). \quad (2-6)$$

2-2 High Field Limit $X \ll 1$

In this case, it is complicated if the magnetic field varies from zero so that we impose the limit of the magnetic field to $\omega_c\tau > 1$. As for the region $X \ll 1$, we have the relation assumed $1 \ll ql \ll \omega_c\tau$ to obtain eqs.(32)-(34). If X is much less than unity as $X \ll 1$, eqs.(32)-(34) can be obtained for both regions of $ql > 1$ and $ql < 1$. Therefore it may be allowed that eqs.(32)-(34) for $X \ll 1$ is applicable for any values of ql .