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# On the Friction Factor for the Turbulent Flow of Dilute Aqueous Polymer Solutions

Kōji HIROSE and Takuma OKA

*Department of Mechanical Engineering*

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## Synopsis

A series of turbulent pipe flow experiments with dilute aqueous C. M. C. solutions are described.

The purpose of these experiments are to compare the anomalous viscous drag results from these experiments with the results using water (Newtonian viscous fluid).

The additive concentrations of the present experiments are from 0.01% to 0.5% by weight of C. M. C., giving power-law indexes 1 to 0.75, respectively.

All of the concentrations are found to give a reduction in turbulent friction factor, compared with Newtonian at the same Reynolds number.

A maximum friction factor reduction of 64% is obtained at a Reynolds number $2\times10^5$ for solutions having polymer concentration of 0.3% by weight.

## § 1. Introduction

The phenomenon of drag reduction in turbulent flow of polymer solution was discovered by B. A. Toms[1], and has been called "Toms phenomenon" or "Toms effect" after his investigation.

This phenomenon has been recently become of interest because of the drag reducing characteristics and contribution to analyzing the mechanism of the turbulent flow.

The present work is devoted to a verification of a particular flow using a single polymer.

The particular flow chosen is the fully developed turbulent flow through the smooth 13, 25 and 35 -mm-dia pipes, and measurements of friction factors at different Reynolds numbers and polymer concentrations are carried out.

The polymer used is carboxymethylcellulose (C. M. C.).

## § 2. Apparatus and Experimental Procedure

A schematic diagram of apparatus is shown in Fig. 1. The polymer is dissolved in the reservoir tank and then stored. In the case of small flow rate, the polymer solution is pumped up with single pump to the constant head tank which locates 6.5 meter high, and drawn through a pressure tank, from which it flows through the test pipe circuit and electromagnetic flow-meter, discharging into the reservoir tank.

In large flow rate, the fluid is pumped up directly into the pressure tank with the double-headered pumps in series.

The test pipes are made of acryl resin so that it is convenient to observe the state of flow with the aid of ink injected into the pipes. They have three different diameters 13 mm, 25mm and 35mm, respectively, and the each has pressure stations.

The pressure drop is measured with the inclined U-tube manometers.

The experiments are carried out at constant temperature 20±0.5°C with the aid of electric heaters.

The polymer used is a corboxymethylcellulose, manufactured by the Shōwa-Kōbunshi Corporation under the trade name of C. M. C. AP-H and C. M. C. AP-L. Each molecular weight is roughly 150,000 and 15,000.

The C. M. C. molecule is long-chained and unbranched.

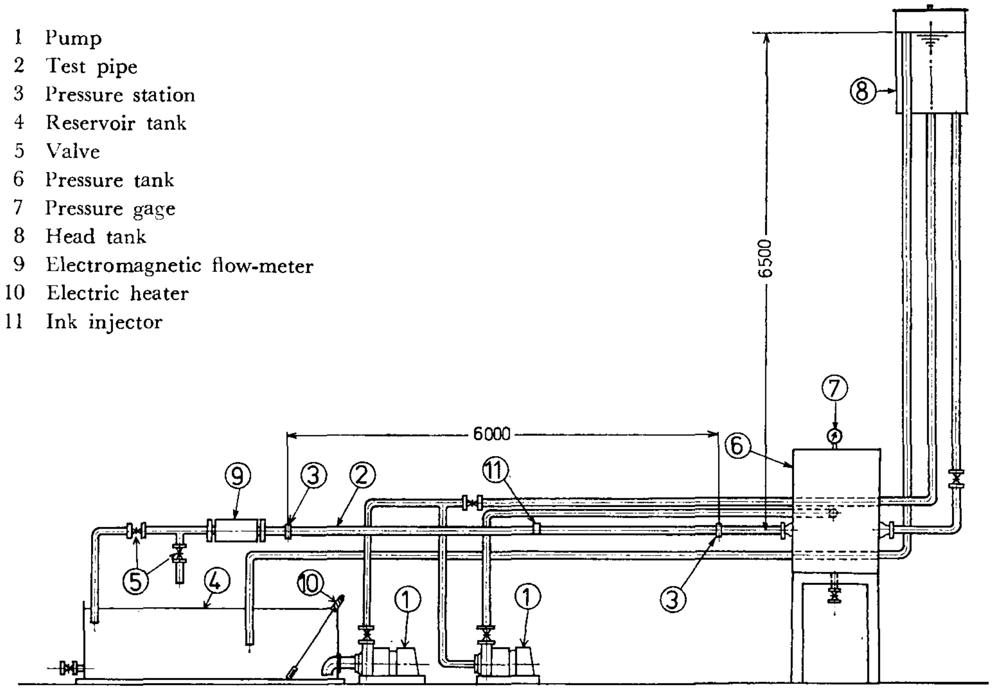

Fig. 1 Outline sketch of experimental apparatus.

## § 3. Analysis

### Nomenclature

$D$ = pipe diameter, cm
$K$ = fluid consistency index, g·sec$^n$/cm$^2$
$K'$ = non-Newtonian fluid property, defined by Eq. (6), g·sec$^n$/cm$^2$
$L$ = length along pipe, cm
$n$ = flow behavior index, power-law exponent, dimensionless
$\Delta P$ = static pressure difference, g/cm$^2$
$R$ = pipe radius, cm
$r$ = distance from pipe center, cm
$Re$ = Reynolds number, dimensionless
$Re^*$ = generalized Reynolds number, defined by Eq. (11), dimensionless
$u$ = velocity in the streamwise, cm/sec
$\bar{u}$ = mean flow velocity, cm/sec
$y$ = distance from pipe wall, cm
$\tau$ = shear stress, g/cm$^2$
$\tau_w$ = shear stress at the wall, g/cm$^2$
$\lambda$ = friction factor, defined by Eq. (10), dimensionless
$\rho$ = density, g·sec$^2$/cm
$\mu$ = viscosity, g·sec/cm$^2$

The nonlinear functional relation most often used to characterize the properties of a purely viscous liquid is the power-law relation :

$$\tau = K\left(\frac{du}{dy}\right)^n . \tag{1}$$

The steady laminar pipe flow of incompressible fluid through the straight circular pipe is considered. The forces, taking pressure and shear stress, balance in equilibrium, then

$$\tau = \frac{r}{2L}\Delta P, \tag{2}$$

and at the wall

$$\tau_w = \frac{D}{4L}\Delta P. \tag{2$'$}$$

Integrating, and determining the integration constant by the fact that $u=0$ at $r=R$ (fluid adheres to the pipe wall), we obtain

$$u = \frac{3n+1}{n+1}\bar{u}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]. \tag{3}$$

Differentiating the Eq. (3), we obtain

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2(3n+1)}{n}\frac{\bar{u}}{D}. \tag{4}$$

From Eq. (1) and Eq. (4),

$$\tau_w = K\left(\frac{3n+1}{4n}\right)^n\left(\frac{8\bar{u}}{D}\right)^n \tag{5}$$

is obtained. We can represent Eq. (5) in common logarithm as follows,

$$\log \tau_w = \log K' + \log\left(\frac{8\bar{u}}{D}\right)^n, \qquad (6)$$

where

$$K' = K\left(\frac{3n+1}{4n}\right)^n.$$

On Newtonian flow, it is defined that

$$Re = \frac{\rho D \bar{u}}{\mu}, \qquad (7)$$

$$\lambda = \frac{\Delta P}{L}\frac{D}{\frac{1}{2}\rho\bar{u}^2} = \tau_w \frac{8}{\rho\bar{u}^2}, \qquad (8)$$

where $Re$ and $\lambda$ are called Reynolds number and friction factor, respectively, and in the case of the laminar flow, a relation between $\lambda$ and $Re$ is, as well known,

$$\lambda = \frac{64}{Re}. \qquad (9)$$

On non-Newtonian purely viscous power-law fluid (as it is called pseudoplastic fluid), the friction factor $\lambda$ becomes as follows,

$$\lambda = \tau_w \frac{8}{\rho\bar{u}^2} = \frac{64 \cdot K' \cdot 8^{n-1}}{D^n \cdot \rho \cdot \bar{u}^{2-n}}. \qquad (10)$$

Compareing Eqs. (9) and (10), it is found that the generalized Reynolds number reduces to the following special form in the case of power-law fluids,

$$Re^* = \frac{D^n \cdot \bar{u}^{2-n} \cdot \rho}{8^{n-1} \cdot K'}. \qquad (11)$$

In the case of turbulent flow, an extensive theoretical and experimental study has been presented by Dodge and Metzner[2] and their result is as follows,

$$\frac{1}{\sqrt{\lambda}} = \frac{2}{n^{0.75}} \log\left[Re^*(\lambda)^{1-\frac{n}{2}}\right] - \left[\frac{0.2}{n^{1.2}} + \frac{2(2-n)}{n^{0.75}} \log 2\right]. \qquad (12)$$

In the Newtonian case ($n=1$), this reduces to

$$\frac{1}{\sqrt{\lambda}} = 2 \log [Re\sqrt{\lambda}] - 0.8, \qquad (13)$$

which coincids with the well-known expression given by Nikuradse.[3] Eq. (12) cannot be solved explicitly for $\lambda$, and has accordingly been used to prepare the chart in Fig. 2.[2] The solid lines on the figure refer to areas in which experimental data were obtained, and the broken lines represent extrapolations using Eq. (12).

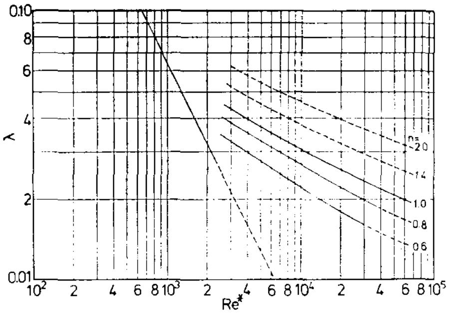

Fig. 2 Friction factors vs. generalized Reynolds number chart for non-Newtonian purely viscous power-law fluid and Newtonian fluid.

## § 4. Results and Discussion

### (1) Previous Experiment on Newtonian Fluid

In the first place the friction factors data of the pipe flow experiments with water are summarized and presented in Fig. 3.

The experimental values agree well with the well-known values, that is $\lambda = 64/Re$ in laminer region and $1/\sqrt{\lambda} = 2 \log [Re\sqrt{\lambda}] - 0.8$ in turbulent region.

It is found that the inner walls of test pipes are hydraulically smooth within this experiment ($Re < 2.3 \times 10^5$).

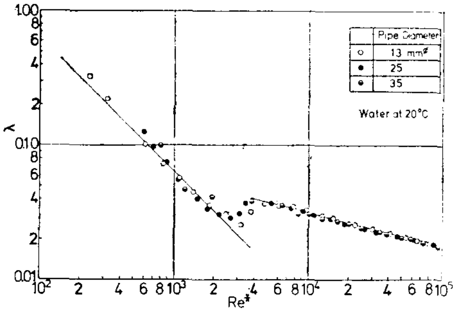

Fig. 3 Friction factors for Newtonian flow. (water)

### (2) Viscous Properties

The results of the laminar pipe flow test with solutions of various concentrations of C. M. C. AP-H and AP-L are shown in Fig. 4, where the shear stress on the wall is plotted vs. shear rate, logarithmically.

The rheological constants can be determined with Fig. (4). and are listed in table 1.

In the cases of the concentrations of 0.01, 0.03 and 0.05%, the flow behavior indexes $n$

are confined to 1, that is Newtonian behavior, whereas the indexes of 0.07% through 0.5% concentrations are progressively smaller values for increasing concentration.

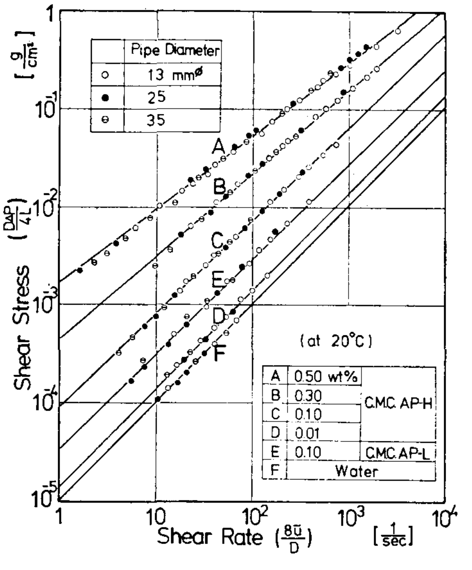

Fig. 4 Laminar pipe flow for several concentrations of polymer at 20°C.

Table 1 Rheological constants (power-law constants) for C. M. C. solutions. (at 20°C)

| Rheological Constants / Fluids | | $n$ | $K$ | $K'$ |
|---|---|---|---|---|
| Units | | | $\times 10^{-5}\ \frac{g \cdot sec^n}{cm}$ | $\times 10^{-5}\ \frac{g \cdot sec^n}{cm}$ |
| C. M. C. AP-H | 0.01wt% | 1.000 | 1.370 | 1.370 |
| | 0.03 | 1.000 | 2.760 | 2.760 |
| | 0.05 | 1.000 | 3.970 | 3.970 |
| | 0.07 | 0.9813 | 4.993 | 4.993 |
| | 0.10 | 0.9486 | 9.135 | 9.252 |
| | 0.30 | 0.8544 | 43.00 | 44.56 |
| | 0.50 | 0.7502 | 161.3 | 171.3 |
| C. M. C. AP-L | 0.10wt% | 0.9765 | 3.422 | 3.422 |
| water | | 1.000 | 1.028 | 1.028 |

### (3) Friction Factors

The friction factor data for C. M. C solutions are plotted in terms of *Re** in order to obtain comparisons with Newtonian fluids (the relations given by Eqs. (9) and (13)) and purely viscous fluid (power-law fluid, the relations given by Eq. (12)) in both the laminar and turbulent regions.

(influence of concentrations)

Figs. 5 through 7 show the friction factors vs. generalized Reynolds number for each diameter of test pipes. An interesting aspect of these figures is the trend that in turbulent region for a given *Re**, $\lambda$ decreases as concentration increases up to 0.5% by weight.

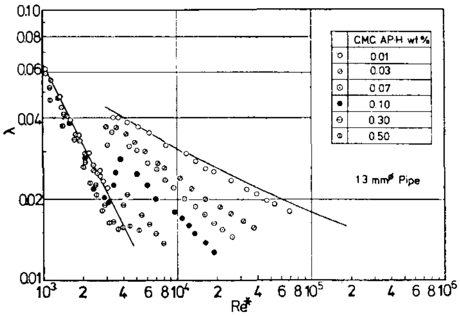

Fig. 5 Friction factors for several concentrations of C. M. C. AP-H vs. generalized Reynolds number. ($D$=13mmφ)

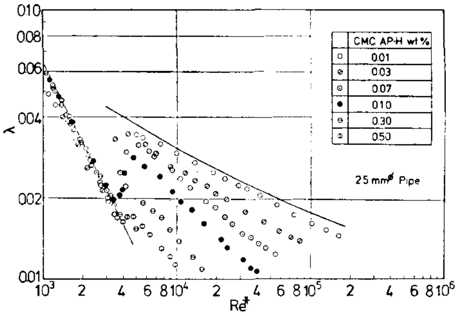

Fig. 6 Friction factors for several concentrations of C. M. C. AP-H vs. generalized Reynolds number. ($D$=25mmφ)

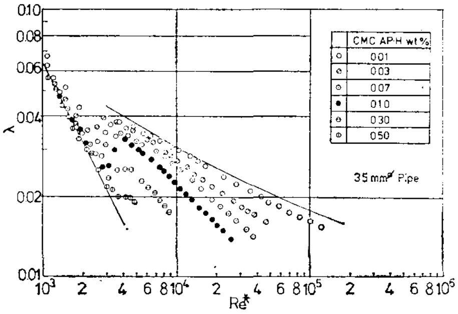

Fig. 7 Friction factors for several concentrations of C. M. C. AP-H vs. generalized Reynolds number. ($D$=35mmφ)

(Influence of diameters)

Figs. 8 through 10 show the friction factors vs. *Re** for each concentrations using parameter *D*.

In these figures each broken line shows the relations by Eq. (12) based on Dodge and Metzner's analysis.

The friction factors for each diameter are significantly lower than predicted for purely viscous fluid (power-law fluid). Further, the data of these experiments are not correlated by these parameters in the turbulent region, i. e., there is a diameter effect, whereas in the laminar region data for purely viscous fluid in pipes of various diameter are correlated by these parameters.

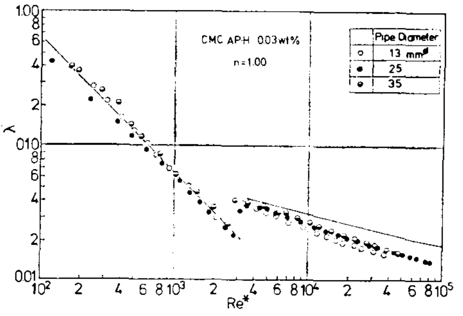

Fig. 8 Friction factors for 0.03 wt % of C. M. C. AP-H vs. generalized Reynolds number.

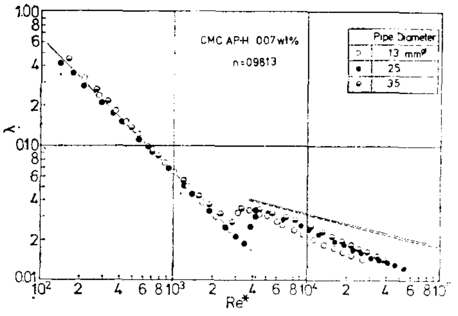

Fig. 9 Friction factors for 0.07wt% of C. M. C. AP-H.

(Influence of polymer molecular weight)

The C. M. C. solutions of two different molecular weight is tested, i. e., C. M. C. AP-H and AP-L, each molecular weight is roughly 150,000 and 15,000, respctively.

For the same concentration of 0.1% by weight, the results are shown in Figs. 10 and 11.

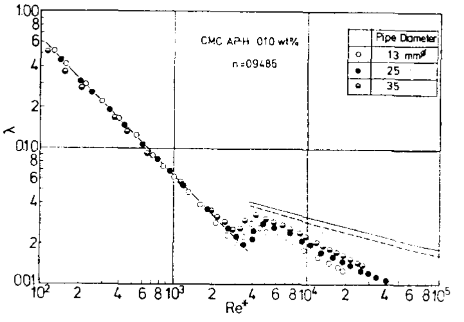

Fig. 10 Friction factors for 0.10wt% of C. M. C. AP-H.

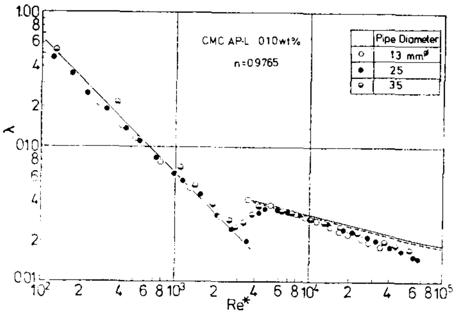

Fig. 11 Friction factors for 0.10wt% of C. M. C. AP-L.

It is recognized that the more reduction in turbulent shear stress appeares with the larger molecular weight polymer.

## § 5. Conclnsions

The following conclusions may be drawn from the experimental results:

(1) The wall shear stress in turbulent flow is significantly reduced by concentrations of the polymer additive as low as 0.01% by weight.

(2) In turbulent flow, for concentrations up to 0.5%, the friction factors for different pipe diameters are not correlated by the generalized Reynolds number appropriate for purely viscous fluid. But in laminar flow, they are correlated by the generalized Reynolds number only.

(3) The flow behavior index *n* does not always become a factor causing reduced friction factors.

(4) Reductions in friction factor for polymer solutions as the same Reynold number increases with increasing polymer moleclar weight.

(5) A complete understanding of this flow

type cannot be achieved without information about another properties of the fluid, such as the elasticity. We must wait for the further detailed experimental results to understand these relation quantitatively.

## References

1) B. A. Toms : Proc. 1st Intern. Congr. on Rheology, **2** (1948), 135.
2) D. W. Dodge and A. B. Metzner : J. Am. Inst. Chem. Engrs., **5** (1959), 189.
3) H. Schlighting : "Boundary Layer Theory", McGraw-Hill Book Co., New York (1968), 574.