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On the Friction Coefficient for Turbulent Flow Through Sectionally Roughened Square Ducts (Continued)*

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In this study, friction coefficients for sectionally roughened square ducts were measured for transional and turbulent flow. The velocity fields in the cross section were mesured for turbulent flow, and the predicting equation in the previous report was corected.

§ 1. Introduction

In the previous report[1], the friction coefficients for turbulent flow through sectionally roughened square ducts were experimentally studied. Their measurements were made for air flow in the Reynolds Number range from 9×10^4 to 3.5×10^5. The measured friction coefficients for alloverly rough and smooth square ducts were compared with the Kármán-Nikuradse formula and the Nikuradse formula for rough and smooth circular pipes respectively. The prediction of the friction coefficients for sectionally roughened square ducts was made.

In this report, to extend the Reynolds Number range, the measurements of the friction coefficients were made for air flow by improving the previous apparatus and for water flow by the new apparatus. The obtained Reynolds Number ranges were from 3×10^4 to 4×10^5 for air flow and from 6×10^3 to 7×10^4 for water flow.

The velocity fields in the cross section were measured, and using these results the equation in the previous report, from which the friction coefficients for the sectionally roughened square ducts were predicted, was corected.

§ 2. Experimental Studies

The experimental apparatus in the previous report for air flow was improved at the following three points.

1) The length of the test section was changed from 1.8 m to 3.6 m to obtain the fully developed flow in the test section.

2) The same blower was connected in series with the previous blower in order to extend the Reynolds Number range for measurements.

3) The movable pitot tube was set in the cross section at the distance 2.9 m from the entrance of the foreflow section, in order to measure the velocity field.

The experimental apparatus for water flow

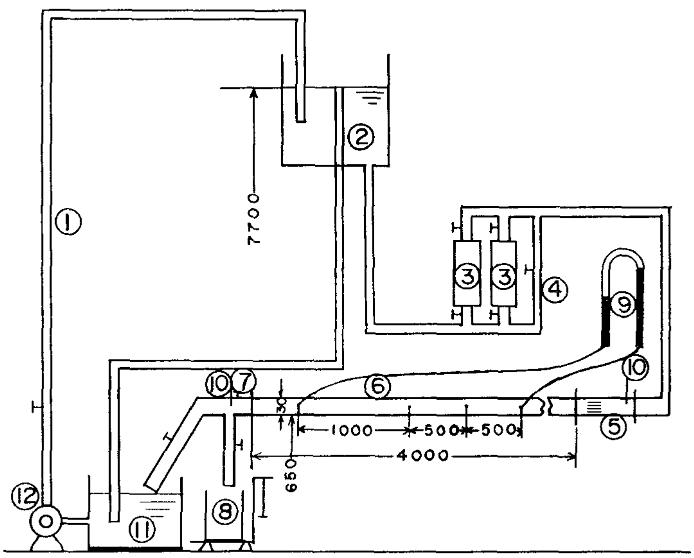

Fig. 1. The experimental apparatus for water flow

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is shown in Fig. 1. Water was drawn by the pump (12) from the reservoir (11) and sent to the overflow tank (2) through the vertical pipe (1). Then water flowed down through the rotameter (3), the foreflow section (5), the test section (6) and the valve (7) before discharging into the reservoir (11). The flow rate of water through the duct was measured either by the rotameter or by weighing the discharge.

The pressure drop was measured for sixteen spuare ducts. All square ducts, consisted of four hard vinyl chloride plates, were 4.0 m long and had 30 mm × 30 mm cross section. The rough walls were made using the method mentioned in previous report. Sixteen pressure taps were located at intervals of 25 cm along the length of each duct to measure the precise pressure gradient.

The pressure differences were measured by means of a inclined U-tube manometer.

The temparature of the water at the test section was measured with the alcohol-in-glass thermometers (10).

The foreflow section length necessary to establish fully developed flow could be directly determined from the knowledge of the pressure gradient along the ducts. At the lower flow rate, the measurement of pressure drop was made over the larger distance to insure accuracy by sufficient magnitude of manometer reading.

Table 1. Values k and $k/4m$

k (mm)		$k/4m$
for air	for water	
0.385	0.136	0.0043
0.905	0.324	0.01
1.840	0.650	0.02

The mean diameter k of the sand grain and the relative roughness $k/4m$ in both apparatuses are shown in Table 1. The arrangements of rough walls are shown in Fig. 2 as represented previously.

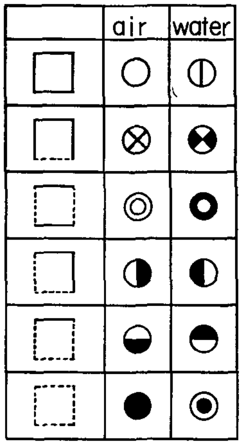


Fig. 2. The arrangements of rough walls
The dotted lines show the rough walls, the solid lines show the smooth walls and the circular symbols show the arrangcments of walls.

§ 3. Results and Discussion

Friction Coefficients for Sectionally Roughened Square Ducts

Figs. 3, 4 and 5 show the obtained friction coefficients for sectionally roughened, alloverly roughened and alloverly smooth square ducts versus Reynolds Number. In the figures, the measured values are shown by the circular symbols explained in Fig. 2. Assuming these values are represented by the function $\lambda = a\, Re^{b}$ in the Reynolds Number range from 4×10^4 to 4.5×10^5, Table 2 shows the values of a and b. The solid lines and the broken lines in the figures show the Kármán-Nikradse formula for rough circular pipe with same relative roughness and the Nikradse formula for smooth circular pipe respectively.

In Fig. 5, the disagreement between the measured values for water and air in the Reynolds Number range from 3.5×10^4 to 7×10^4 may be due to the errors in estimating the relative roughness for both cases. In all cases, for the

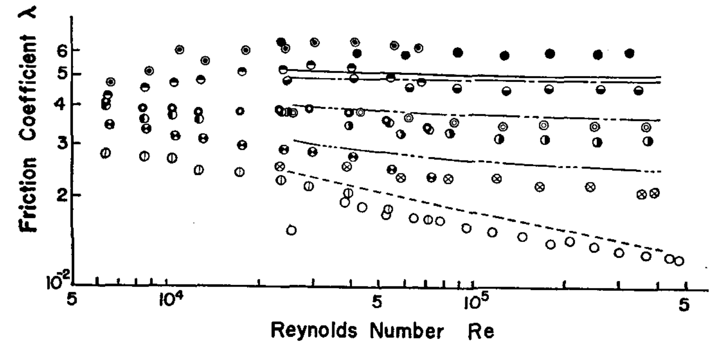


Fig. 3. Friction coeffcients for sectionally rough square ducts ($k/4m = 0.02$)

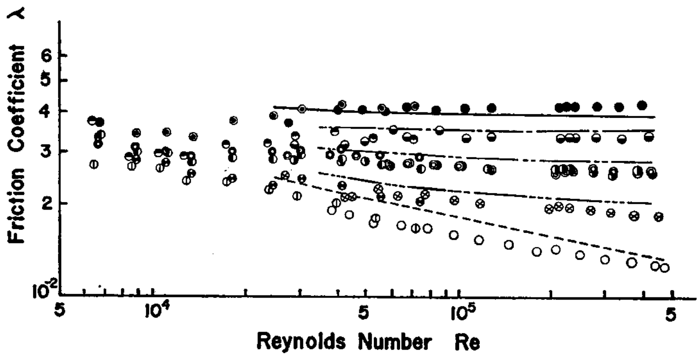
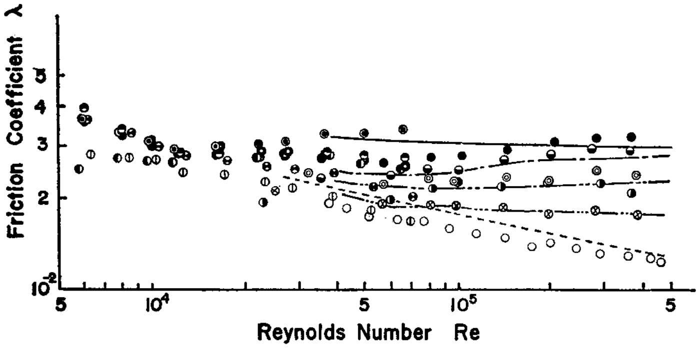


Fig. 4 Friction coefficients for sectionally rough square ducts (k/4m=0.01)



Pig. 5. Friction coefficients for sectionally rough square ducts ($k/4m$=0.0043)

relative roughness the further study must be atempted.

It is interest that in these experiments the friction coefficients for square duct with two opposite rough walls, tend to be a little larger than those with two adjacent rough walls.

Experimental Correction of the Equation predicting Friction Coefficients

The frction coefficients for sectionally roughened square ducts were previously predicted from

$$\lambda_n = [n\lambda_r + (4-n)\lambda_s]/4 \quad (1)$$

where λ_r and λ_s are the friction coefficients for alloverly roughened and smooth square ducts respectively, and n is number of rough walls. In Figs. 3, 4 and 5, the chain lines with one-dot, two-dots and three-dots show the equation (1) for $n=1$, 2 and 3 respectively. These lines do not predict the experimental values for corresponding cases with sufficient accuracy.

In this study, it is attempted to corect the eruation (1), as following procedure.

Fig. 6 gives an example of velocity distributions in the cross section of the duct with three rough walls. From the velocity distributions

Table 2. Values a and b

	$k/4m$	a	
four smooth walls	0	0.100	−0.161
one rough wall	0.02	0.0505	−0.0698
	0.01	0.0542	−0.0845
	0.0043	0.0383	−0.0620
two opposite rough walls	0.02	0.0546	−0.0397
	0.01	0.0468	−0.0492
	0.0043	0.0147	0.0360
two adjacent rough walls	0.02	0.0659	−0.0636
	0.01	0.0401	−0.0368
	0.0043	0.0378	−0.0466
three rough walls	0.02	0.0594	−0.0240
	0.01	0.0299	0.00697
	0.0043	0.00773	0.106
four rough walls	0.02	0.0522	0.00861
	0.01	0.0342	0.0141
	0.0043	0.00748	0.112

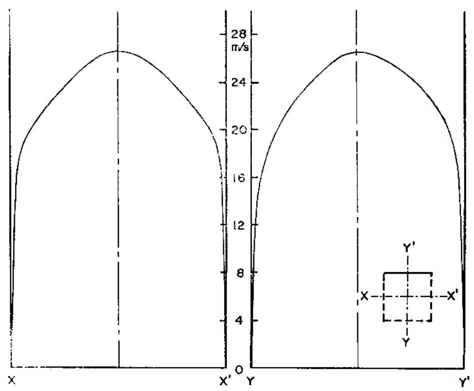


Fig. 6. The velocity distributions ($k/4m$=0.01, $Re=1.25\times10^5$)

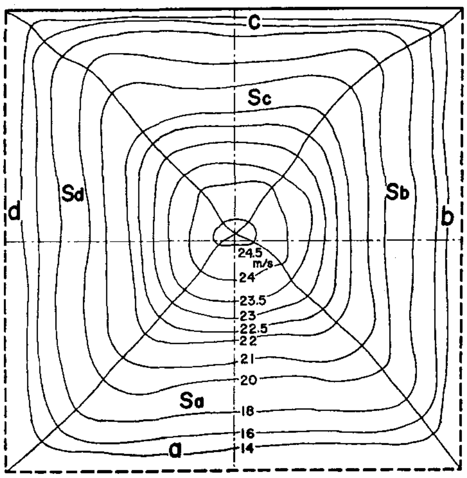


Fig. 7. The constant velocity lines

obtained, the constant velocity lines on the cross section are drawn, as shown in Fig. 7. The line drawn from the maximum velocity point to a corner is the velocity gradient line (normal to constant velocity line) on the cross section. The lines divide the cross section into four parts. Let S_a, S_b, S_c and S_d represent the area of each part respectively, and a, b, c and d represent four walls respectively. There are no shear forces acting on the velocity gradient lines, because the velocity gradient normal to these lines are zero.

Let τ_a, τ_b, τ_c and τ_d represent the average shear stresses acting on the walls a, b, c and d respectively, and let ΔP_3 represents the pressure gradient. From the balance between the shear stresses and the pressure gradient,

$$\left.\begin{aligned}&\tau_a+\tau_b+\tau_d=\Delta P_3(S_a+S_b+S_d)/h\\ &\text{and}\\ &\tau_c=\Delta P_3 S_c/h\end{aligned}\right\} \quad (2)$$

where h is the length of the side of square cross section.

Let ΔP_r and ΔP_s represent the pressure gradients, τ_r and τ_s represent the average shear stresses acting on the walls, for alloverly rough and smooth square ducts. Then

$$\tau_r=\Delta P_r S/4h \text{ and } \tau_s=\Delta P_s S/4h \quad (3)$$

where $S=S_a+S_b+S_c+S_a$.

From equations (2) and (3), next equations can be obtained.

$$(\tau_a+\tau_b+\tau_d)/\tau_r=4\Delta P_3(S_a+S_b+S_d)/\Delta P_r S \equiv \alpha \quad (4)$$

and

$$\tau_c/\tau_s=4\Delta P_3 S_c/\Delta P_s S \equiv \beta \quad (5)$$

The friction coefficient λ_3 for the square duct with three rough walls is given as follows,

$$\lambda_3=\Delta P_3\Big/\rho v^2/2h$$

where ρ is the density of the fluid and v is the mean velocity. Moreover $\lambda_r=\Delta P_r/\rho v^2/2h$ and $\lambda_s=\Delta P_s/\rho v^2/2h$, then from equations (4) and (5),

$$\lambda_3=(\alpha\lambda_r+\beta\lambda_s)/4 \quad (6)$$

The equation (4), (5) and (6) are adopted for the duct with three rough walls. The equations for the other square ducts are shown in Table 3.

The coefficients α and β obtained experimentally, for the relative roughness $k/4m=0.01$, are shown in Figs. 8 (a) and 8 (b). Assuming these values are represented by the functions $\alpha=aRe^p$ and $\beta=bRe^q$, Table 4 shows the values of a, p, b and q. Substituting α, β, λ_r and λ_s obtained from Tables 4 and 2 into the equation $\lambda_n=(\alpha\lambda_r+\beta\lambda_s)/4$, the values of λ_n are shown in Fig. 9 with the solid lines. In Fig. 9, the predicted values are in satisfactory

Table 3. α, β and λ_n

	one rough wall	two rough walls (opposite)	two rough walls (adjacent)	three rough walls
α	$\dfrac{4\ \Delta P_1\ S_a}{\Delta P_r S}$	$\dfrac{4\ \Delta P_2(S_b+S_d)}{\Delta P_r S}$	$\dfrac{4\ \Delta P_2(S_a+S_b)}{\Delta P_r S}$	$\dfrac{4\ \Delta P_3(S_a+S_b+S_d)}{\Delta P_r S}$
β	$\dfrac{4\ \Delta P_1(S_b+S_c+S_d)}{\Delta P_s S}$	$\dfrac{4\ \Delta P_2(S_a+S_c)}{\Delta P_s S}$	$\dfrac{4\ \Delta P_2(S_c+S_d)}{\Delta P_s S}$	$\dfrac{4\ \Delta P_3\ S_c}{\Delta P_s S}$
λ_n	$\lambda_n=(\alpha\lambda_r+\beta\lambda_s)/4$	$n=1,2$ and 3		

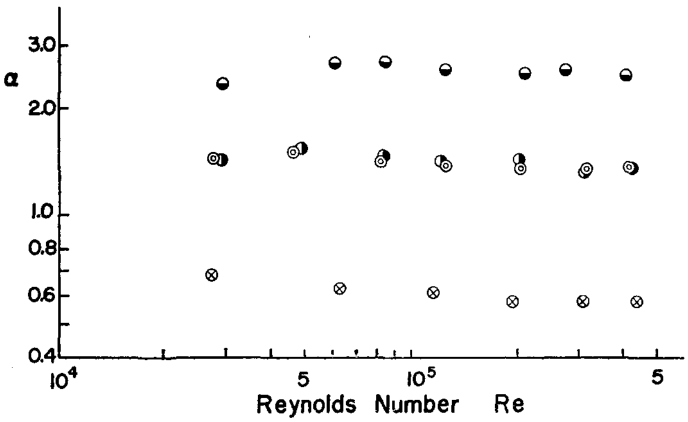


Fig. 8. (a) The coefficient α ($k/4m=0.01$)

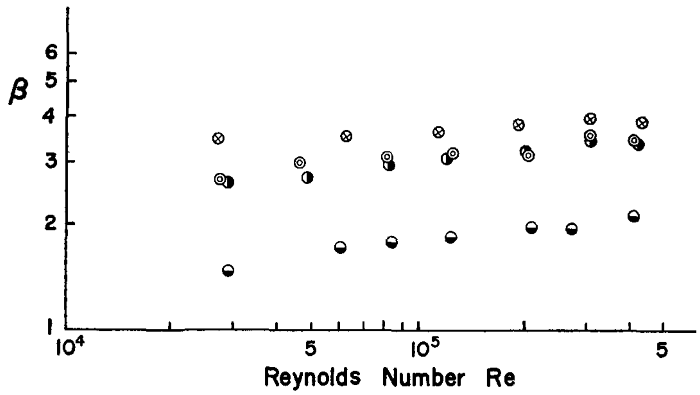


Fig. 8. (b) The coefficient β ($k/4m=0.01$)

Table 4. Values a, p, b and q

	a	p	b	q
one rough wall	1.279	−0.06344	2.012	0.05104
two opposite rough walls	2.171	−0.03715	1.128	0.08724
two adjacent rough walls	2.032	−0.03039	0.9209	0.1010
three rough walls	2.408	0.004748	0.4551	0.1182

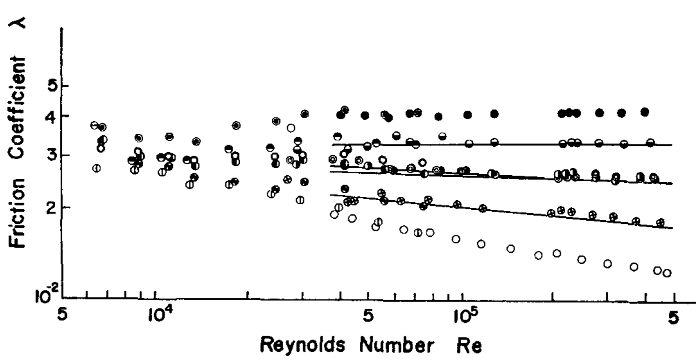


Fig. 9. Comparison of experimental and predicted friction coefficients for sectionally rough square ducts ($k/4m=0.01$)

agreement with the experimental values.

In further study, for other relative roughness the same analysis must be done, and it is desirable that α and β are represented as follows,

$$\alpha,\ \beta = F(Re,\ n,\ k/4m)$$

§ 4. Conclusion

1) The friction coefficients for sectionally roughened square ducts are measured in the Reynolds Number range from 3×10^4 to 4.5×10^5 for the relative roughnesses 0.0043, 0.01 and 0.02.

2) In the case of the relative roughness 0.01, the friction coefficients for sectionally roughened square ducts are predicted from the knowledge about the velocity fields on the cross section and coincide with the experimental values fairly well.

Reference

1) K. Hirose and Y. Asano : Memoires of Engineering, Okayama Univ. **3**, (1968) 13