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Determining Hydraulic Properties of Multilayered Aquifers from Pumping Test Data by Parameter Estimation

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SYNOPSIS

In this paper, a numerical procedure of determining hydraulic properties in multilayered aquifers are presented. From pumping test data in multilayered aquifers, the coefficient of permeability and specific storage for each aquifer are determined by using a combination of finite element analysis and nonlinear least-squares optimization technique. This study especially points out necessity of stress-flow coupling analysis to explain the behaviors of pressure head in multilayered aquifer during pumping test. As a example, practical pumping test data were evaluate and the coefficients of permeability and specific storage of aquifers and aquitard were obtained.

1. INTRODUCTION

Recently, the development and utilization of underground in urban area have increased. Lots of deep underground excavations are in construction and in plan. To draw up these plans, it is very important to predict the behavior of the groundwater in multilayered aquifers also aquitard. Numerical method, that is, finite element method is frequently used to calculate seepage flow in aquifer, and so this method becomes quite popular. But the methodology to determine the hydraulic properties of multilayered aquifers has not been established.

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For simple aquifer system, conventional pumping tests and their subsequent analysis are used to find hydraulic properties. Namely, two graphical methods by Theis[1] and Jacob[2] are commonly used to calculate hydraulic properties; coefficient of permeability and storage in engineering practice. It is, however, difficult to apply these methods to analyze the pumping test data in multilayered aquifers system and complicated situations. Because these theoretical methods are based on a set of simplifying assumptions for aquifer or pumping test conditions, which limit their use of certain problems. The basic assumptions made for the solution of these methods are as follows:

1. The aquifer is level, homogeneous, isotropic, constant in thickness, and infinite in horizontal extent.
2. The aquifer material is assumed to consist of porous media with laminar flow obeying Darcy's law.
3. There is only a single fully penetrating well with a constant pumping rate in the aquifer; the well diameter is assumed to be infinitesimally small.
4. Before pumping, the hydraulic head in the aquifer is horizontal; after pumping, the water is discharged instantaneously from aquifer storage with decline of the hydraulic head.
5. Water-level fluctuations caused by interference from nearby well, or other causes (e.g., tidal influences, rainfalls) are considered insignificant during the duration of the pumping test.

Strictly speaking, these assumptions are hardly met in the field. A large number of analytical or graphical methods for analyzing pumping test data has been investigated to remove the above restrictions in ground water literature for a long time. Some typical methods are presented in Table 1. Those methods require data plotting work and individual judgment during the curve-fitting procedures or complicated solutions. Therefore, they are cumbersome and time-consuming. When pumping tests are performed under the multilayered conditions, it is, however, difficult to analyze the data obtained from each aquifer under these conditions "analytically".

In this paper, a numerical procedure in which hydraulic properties affecting axisymmetric seepage flow in multilayered aquifers are back-analyzed from pumping test data is presented. By using a nonlinear least squares algorithm which incorporate finite element analysis of nonsteady seepage flow, this procedure allows to find unknown hydraulic properties which minimize the differences between calculated and measured drawdowns. The coefficient of permeability and specific storage coefficient are the hydraulic properties determined by this procedure. An application to practical pumping test data in multilayered aquifers provides the interesting results such as evaluation of the hydraulic properties both aquifers and aquitards.

Table 1. Some typical methods for analyzing pumping test data.

Pumping test condition	Author
Partially penetrated aquifers	Hantush[3]
A large diameter pumping well	Papadopulos & Cooper[4]
Unconfined aquifers with delayed yield	Boulton[5], Neuman[6]
Leakly Aquifers	Hantush[7], Neuman & Witherspoon[8]
Multilayered aquifer	Javandel & Witherspoon[9]

2. PARAMETER ESTIMATION PROCEDURE

Drawdown data obtained from pumping tests are simulated as the nonsteady radial ground water flow toward a well. The partial differential equation describing nonsteady radially symmetric flow in a confined aquifer can be written as

$$\frac{\partial}{\partial r}\left(k_r \frac{\partial h}{\partial r}\right) + \frac{k_r}{r}\left(\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial h}{\partial z}\right) = Ss \frac{\partial h}{\partial t} \qquad (1)$$

where k and Ss are the permeability and specific storage coefficient, respectively, and h is the piezometric head at the radial distance r from the pumping well at a time t since the start of pumping. k_r and k_z indicate anisotropic permeabilities.

Pumping well diameter cannot be considered infinitesimally small in practice. Hence, it is very important to take into account the storage capacity of the pumping well itself which is assumed to be negligible in the Theis' method. The water level in the pumping well with time is calculated by Equation (2).

$$h^{I+1}(t+\Delta t) = \alpha h^{I}(t+\Delta t)+(1-\alpha)[h(t)+\Delta t(QA^{I}(t+\Delta t)-QP)/\pi r_w^2 \qquad (2)$$

where QA is the discharge from the aquifer into the well; QP is the total discharge from the pump; r_w is the effective radius of the well; h(t) is the water level in the well; I is the number of iteration; α is the relaxation factor (α=0.8). Δt is the small increments in time.

The well losses are, however, negligible, i.e. the entrance resistance of the well is zero. Effective diameter of pumping well is calculated with considering the porosity and the width of filter. We solve Equation (1) with Equation (2) to simulate pumping test data in a confined aquifer. The solution of these equations were obtained by the Galerkin finite element method.

A set of drawdown measurements s^o at specific times t_i (i= 1,2,..N) and distance r from the pumping well can be obtained from pumping test. These $s^o(t_i,r)$ are employed as input data for back-analysis. And $s^*(t_i,r,\mathbf{b})$ is numerically calculated by Equation (1) with Equation (2) corresponding to a trial vector of unknown parameter values **b.** In parameter estimation problem for pumping test, it is assumed that geological and pumping test conditions have been measured respectively. Values of k and Ss are unknown parameters. The procedure formulates the problem of parameter estimation as the optimization problem to find an optimum combination parameters **b** which minimize the difference between calculated and measured drawdown.

Objective function:

$$\text{minimize } R = \sum_{i=1}^{n} W_i(s^*(t_i,r,\mathbf{b})-s^o(t_i,r))^2 \qquad (3)$$

where W_i is a weighting function taken as unity in the simulations considered here because measured errors were assumed to be constant. The formulated optimization problem is to be numerically solved by

Levenberg-Marquardt method in mathematical programming. This method represents an optimum combination of the method of steepest descent and the Gauss-Newton method, and is widely used for nonlinear least-squares optimization. The normal equation of this method can be expressed as follows.

$$(\mathbf{A}^T\mathbf{A} + \lambda \mathbf{I})\,\Delta\mathbf{b} = \mathbf{A}^T(s^* - s^o) \qquad (4)$$

where **A** is called the Jacobian or sensitivity matrix; $\mathbf{A}^T$ is the transpose of the matrix **A**; $\Delta\mathbf{b}$ is the improved vector of unknown parameters; s^o and s^* are vectors of observed and calculated drawdowns, respectively. λ is the convergence factor and **I** is the identity matrix. Initial values of λ are large and decrease towards zero as convergence is reached.

The normal equations (4) are solved for $\Delta\mathbf{b}$ and the new drawdowns are calculated by substituting the improved estimates (k,Ss) of the parameters in Equation (1). The error criterion is checked and if it is not satisfied, the process is repeated with the updated estimates of the parameters. In practice, excessively long computer runs are avoided by setting an upper limit to the number of iterations. If the solution fails to converge in the allowed number of iterations, we suggest that the solution process be started anew with different initial parameter values. The proposed procedure estimates the realistic vales of hydraulic properties based on a small number of drawdown data conveniently measured in pumping test.

3. THE HYDRAULIC RESPONSE OF MULTILAYERED AQUIFER UNDER THE INFLUENCE OF PUMPING

In multilayered aquifer systems, aquifers are separated by aquitards through which water flows from one aquifer to another. The hydraulic response of multilayered aquifers under the influence of pumping test is a problem of interest in the determination of hydraulic properties.

Let us consider pumping tests in the two confined aquifers separated by an aquitard, as in Figure 1. Each layer has its own hydraulic properties, is finite in thickness, and extends radially to infinite. T_i and S_i denote respectively transmissivity and storage coefficient in the layer i. As illustrated in Figure 1, partially penetrating well in the top layer of the system are considered to find hydraulic properties of each layer. The radius of well is finite. If the pumping rate Q is kept constant, we are interested in determining the value of drawdown at any point in the aquifer after pumping starts. Piezometers are used to monitor drawdown of each aquifer independently as shown in Figure 1. Piezometer is a device which requires sealing off a porous filter element so that the instrument responds only to groundwater pressure around the filter element.

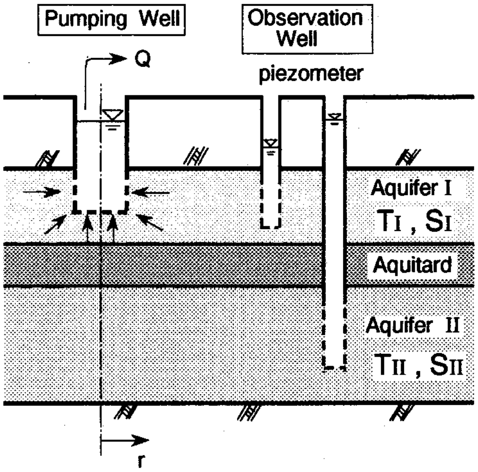


Fig.1 Schematic diagram of a two-aquifer system.

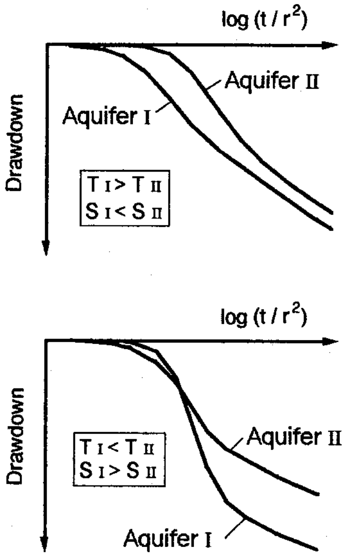


Fig.2 Typical pattern of time-drawdown.

Drawdowns in either aquifer are evaluated numerically to illustrate some typical cases. These results are presented in Figure 2.

The outline of the discussion about the behavior of drawdown in unpumped lower aquifer as follows;

When the well is pumped, the drawdown is measured in the lower unpumped aquifer simultaneously (Figure 2). There are two possible solutions to this phenomenon.

1) It is considered that the drawdown of unpumped aquifer is caused by the leakage from an unpumped aquifer to a pumped aquifer through the aquitard. In this approach, these drawdowns can be simulated by finite element analysis of conventional nonsteady seepage flow.

2) It is assumed that the drawdown of unpumped aquifer is caused by elastic deformation of confined aquifer. According to this solution, it is considered that a decrease in water pressure of the upper aquifer acts on the lower aquifer as a decrease in hydraulic load.

Elastic expansion of the lower aquifer is caused by a changes in this pressure of upper aquifer, and there is a decline in groundwater pressure of lower aquifer. This phenomenon is similar to the barometric efficiency of aquifer[2]. It is difficult to simulate such groundwater behavior only by conventional seepage flow analysis. To verify the mechanism of groundwater behavior in multilayered aquifer it is necessary to carry out coupled stress-flow analysis. For the analysis, a code of coupled stress-flow analysis, UNICOUP, is used. This code introduces an elastic-plastic constitutive model on the basis of Biot's theory of two-dimensional consolidation[10]. Based on these results, the proposed method is applied to the practical example of pumping test data in multilayered aquifer.

4. APPLICATION TO FIELD DATA

The following discussions give example calculations of this proposed method. The pumping test data were taken from a real multilayered aquifer at Nagoya City in Japan. The geological conditions and the construction of wells are shown in Figure 3. Two diluvial sand-gravel layers (Dg.2 & Dg.3) are revealed as a confined aquifers and aquitard (Dc.5) exist in the region. Pumping up was performed from only the upper aquifer (Dg.2) using average pumping rate of 0.3 m^3/min. Drawdowns in each aquifer were measured by pressure transducers which was installed to the piezometer. Air packers were used to seal. The drawdown curves which were obtained from each aquifer are shown in Figure 4.

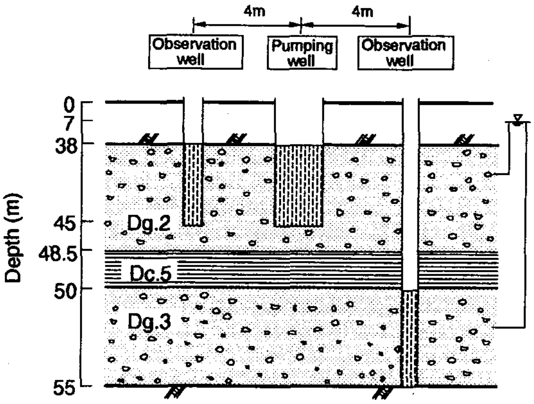


Fig.3 Geological conditions and construction of wells.

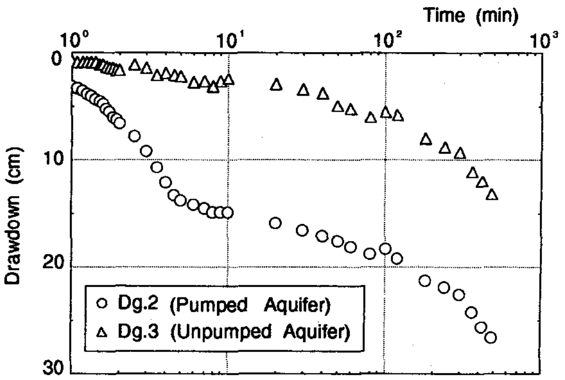


Fig.4 Drawdown curves obtained from each aquifer.

The drawdown in the unpumped lower aquifer (Dg.3) was caused simultaneously. There are alternative approaches to simulate numerically these behaviors of groundwater.

1) Method I:Conventional seepage flow analysis

In this Method, it can be considered that the drawdown of unpumped aquifer is caused by the leakage from an unpumped aquifer to a pumped aquifer through the aquitard. Coefficients of permeability in each aquifer and aquitard are sought by the back-analysis which incorporates a finite element analysis of conventional nonsteady seepage flow. We make the assumption that specific storage Ss of each aquifer is 1.0×10^{-6} cm^{-1}, and Ss of aquitard is 1.0×10^{-5} cm^{-1}. Figure 5(a) illustrates a numerical model and its boundary conditions for Method I.

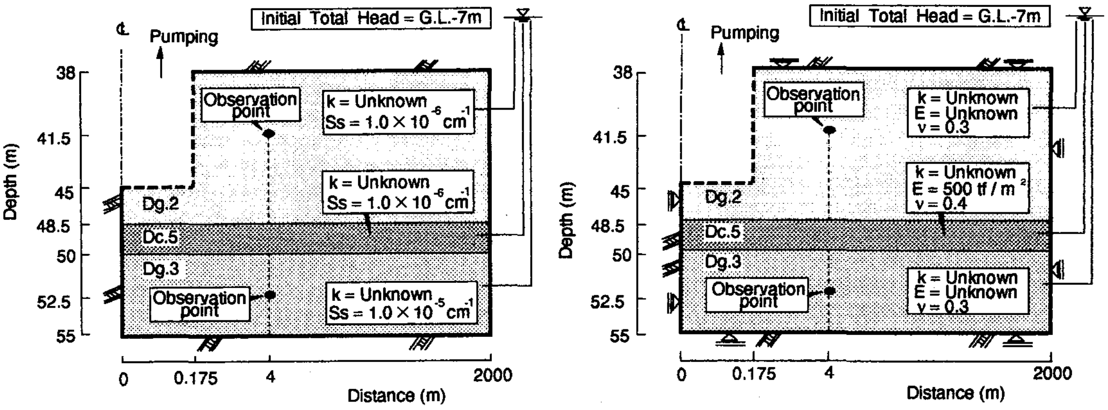
2) Method II: Coupled stress-flow analysis

In this approach, there are three unknown parameters to be identified in a aquifer, coefficient of permeability, modulus of elasticity E, and Poisson's ratio ν. However, Poisson's ratio was used for a fixed value because it is estimated for a pair of parameter with modulus of elasticity. We make the assumption that ν of each aquifer is 0.3, and ν of aquitard is 0.4. Moreover, we assumed that E value of aquitard is 500 tf/m^2.

In this method, the behavior of groundwater is governed by the proportion k/E. Therefore, parameter E plays a same part which is similar to Ss. Relationships between E, ν and Ss are as follows;

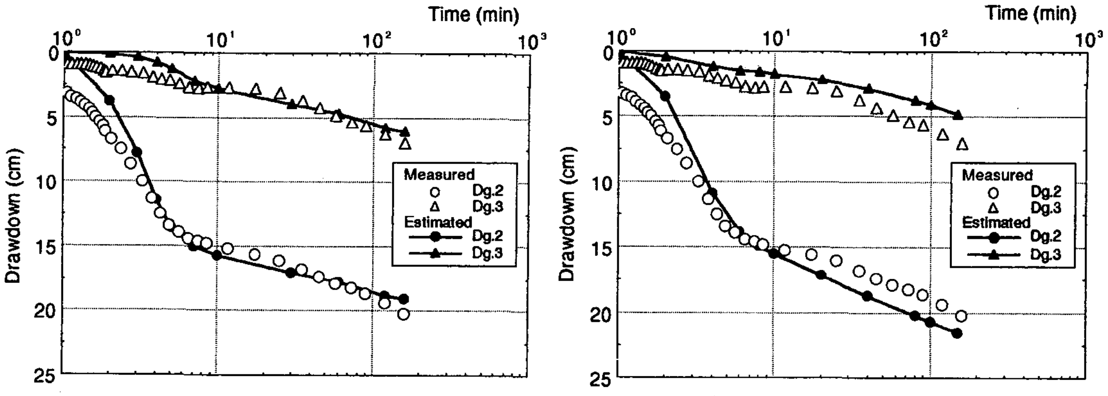
$$Ss = \frac{2(1+\nu)(1-2\nu)}{E} \tag{5}$$

Figure 5(b) illustrates a numerical model and its boundary condition for Method II. Parameter k and E in each aquifer and aquitard are sought by parameter estimation procedure which incorporates coupled stress-flow finite element analysis. Hydraulic properties which were back-analyzed by two kinds of methods are shown in Table 2. Figure 6 shows the comparison of a part of measured drawdown records with the drawdowns calculated using the back-analyzed hydraulic properties. Both drawdowns back-analyzed by the method I and II are agree well with the measured ones.



(a) seepage flow analysis (Method I)　(b) coupled stress-flow analysis (Method II)

Fig.5 Numerical model for back-analysis.



(a) Back-analysis by seepage flow analysis (Method I)　(b) Back-analysis by coupled stress-flow analysis (Method II)

Fig.6 Comparisons between measured and estimated drawdown.

According to the result of the method I, to simulate the behavior of drawdown in Dg.3 layer, high permeability must be given to aquitard, and the coefficient of permeability in Dc.5 layer is back-analyzed as 1.0×10^{-3} cm/s. In general, this is a too high value for diluvial clay layer. On the other hand, permeability of Dc.5 layer which was back-analyzed by method II is 1.0×10^{-5} cm/s. As the above back-analysis results show, it is considered that application of back-analysis must be selected by mechanism of groundwater behavior in multilayered aquifers.

Table 2. Estimated parameter from pumping test data in multilayered aquifers.

	Method I		Method II		
Layer	k(cm/s)	Ss(cm^{-1})	k(cm/s)	E(tf/m^2)	Ss*(cm^{-1})
Dg.2	1.15×10^{-5}	1.0×10^{-6}	1.52×10^{-1}	8000	1.30×10^{-6}
Dc.5	1.00×10^{-3}	1.0×10^{-5}	1.00×10^{-5}	500	1.12×10^{-5}
Dg.3	2.64×10^{-1}	1.0×10^{-6}	9.58×10^{-2}	80000	1.30×10^{-7}

Ss* is calculated by Equation (5)

5. CONCLUSIONS

In this paper, some new methods to estimate hydraulic properties from pumping test data in confined multilayered aquifers are presented.

The conclusions obtained in this paper as follows:

1. A numerical procedure in which hydraulic properties affecting axisymmetric seepage flow in multilayered aquifers are back-analyzed from pumping test data are presented. This procedure is able to seek the coefficient of permeability and specific storage coefficient which minimize the differences between calculated and measured drawdowns by using a combination of finite element analysis and a nonlinear least squares algorithm.

2. The hydraulic responses of multilayered aquifers under the influence of pumping test are evaluated numerically.

Two possible solutions to the behavior of drawdown in an unpumped aquifer are suggested. These solutions are proposed for interpretation of pumping test data in multilayered aquifers.

3. An application to practical pumping test data in multilayered aquifers provide the interesting results such as evaluation of the hydraulic properties both aquifers and aquitards.

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