

Optimal Pricing and Planning of Urban Expressway System

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SYNOPSIS

A transportation planning way of approach is applied to optimal pricing and planning of urban expressway network. Two kinds of network are investigated; four radials with one ring system and four radials with two rings system. The model is composed of three submodels; road system, trip generation-attraction and traffic diversion. Some implicative aspects are shown of the optimal solutions that maximize the aggregate number of the trips diverted to expressway under constraints; equilibrium of revenue and expenditure and traffic capacity constraint. (1) traffic flow has the peaks at junctions on radial expressway that are adjusted to be equal to the traffic capacity, (2) two rings system realizes the larger aggregate number of diverted trips by lower toll rate than one ring system does, (3) well-located second ring has a remarkable effect on improvement in accessibility to expressway and (4) some parameters in the model have curious effects on the optimal system.

1. INTRODUCTION

The authors are concerned with theoretical optimization of pricing

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and planning of urban expressway system, which is subject to equilibrium of revenue and expenditure. Pricing and planning are interdependent under the equilibrium condition because revenue is subjected to pricing and the number of trips diverted to expressway system, which is dependent on both pricing and planning the system, and expenditure is, it is assumed in this study, depends on the whole length of expressway.

Two approaches have been proposed to the optimization problem so far : an welfare economic approach and, so called, transportation planning way of approach. The former was tried by Yamada [1] and followed by Myojin et al. [2], [3], [4] and [5] through a close mathematical investigation. The latter was tried and developed by Sasaki et al. [6], [7] .

The present paper deals with some generalizations of the latter. Two points are revised for generalization : urban expressway system and origin-destination distribution of trips. That is,

- (1) urban expressway system : ring roads are added to the system that was assumed so far to consist of radial roads alone.
- (2) origin-destination distribution of trips : many-to-many distribution is given instead of many-to-one, where many and one are corresponding to all of the points and central point in urban area under study, respectively.

2. MODEL

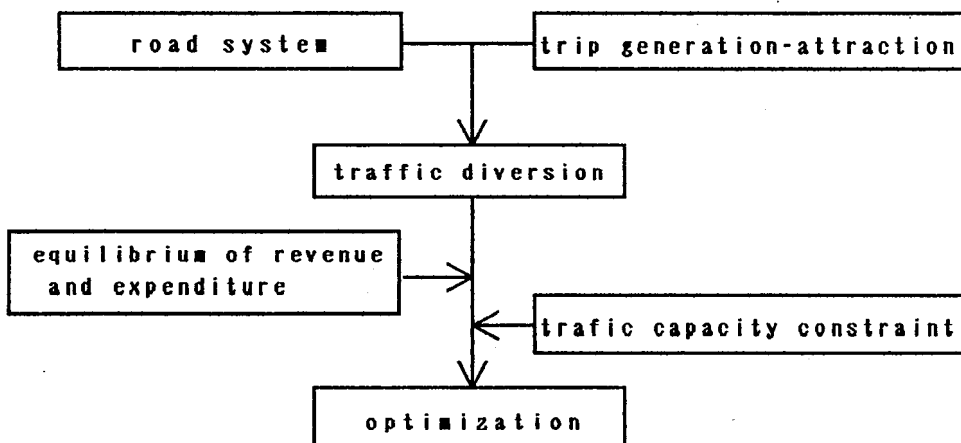


Fig.1 Model

As shown in Fig.1, the present model includes three submodels with two constraints. The submodels are called road system, trip generation-attraction and traffic diversion and the constraints are called equilibrium of revenue and expenditure and traffic capacity constraint hereafter in the paper.

What the authors have for their purpose of setting a model like this is to find characteristic behaviors of the system in its optimality that is defined later.

The following is the details of the submodels and the constraints.

(1) Road system (Fig.2)

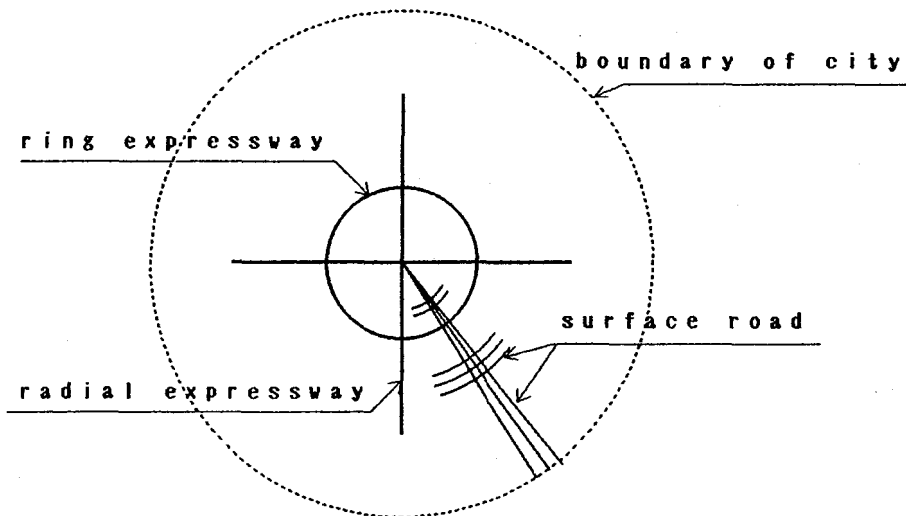


Fig.2 Road system

Road system consists of two classes : surface road and expressway. Surface roads are free to travel along and assumed to be supplied densely both in radial and circumferential directions. Expressway system has four radials with some rings. Radial expressways meet at right angles at the center of the city under study. Two premises are set to expressway system as follows : entrance and exit ramps are located along expressways as densely as surface roads are and expressways are toll road with flat rate. The first premise is rather an expedient for calculating traffic diversion and the second is existing condition in Japan.

(2) Trip generation-attraction

Each trip is generated and attracted in the city under study according to trip potential, whose function is assumed in common to generation and attraction as

$$f(r) = \mu e^{-\lambda r} \quad (1)$$

where

r ; straight line distance to the city center,
 μ and λ ; parameters.

The function (1) expresses trip-generating- and trip-attracting-rates as measured, for example, in the daily number of trip ends per unit area.

Gravity type is assumed to account for origin-destination distribution of trips.

$$q(r_1, \theta_1; r_2, \theta_2) = \frac{\alpha \{f(r_1)f(r_2)\}^\beta}{\{d(r_1, \theta_1; r_2, \theta_2)\}^\gamma} \quad (2)$$

where

q ; the number of trips between the points (r_1, θ_1) and (r_2, θ_2) ,
 $f(r_i)$; trip potential at distance r_i to the city center and $0 \leq \theta_i \leq 2\pi$, $i = 1, 2$,
 d ; straight line distance between the points,
 α , β and γ ; parameters.

No trip is assumed to travel in any other direction than in radial and circumferential directions, in which roads, whether surface or expressway, are supplied.

Clearly, integration of trip potential function in the possible regions of r and θ ($0 \leq \theta \leq 2\pi$) gives the daily number of trip generation in the city under study.

(3) Traffic diversion

This part is concerned with the diversion of trips from surface road to expressway. As shown in Fig. 3, four major steps are included in the diversion submodel. These are now called traffic speed, route search, calculation of diversion and convergence in traffic speed. The

following is detailed presentation of the steps.

1) Traffic speed

Traffic speed is in general a function of traffic flow. A linear function is assumed for the speed of traffic flow on expressway as follows ;

$$V = V_0 - k Q \quad (3)$$

where

V ; traffic speed in a section of expressway,

Q ; traffic flow in the section (the number of diverted trips passing through the section),

V_0 and k ; parameters.

Traffic speed is not uniform over expressway system clearly because traffic flow is not.

Traffic speed on surface is assumed as a function of distance to the city center. [7]

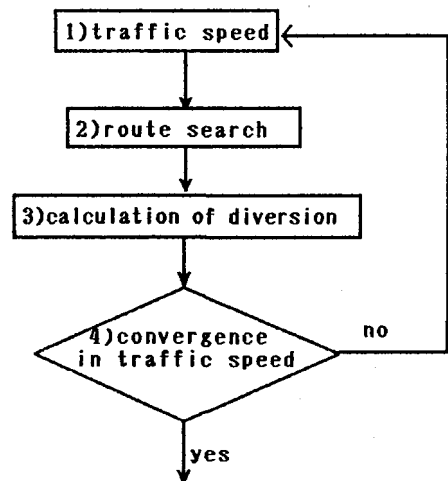
$$v = v_0 - a e^{-b r} \quad (4)$$

where

v ; traffic speed on surface road at distance r to the city center commonly in radial and circumferential directions,

v_0 , a and b ; parameters.

The above assumption on traffic speed on surface has no traffic engineering background which usually requires the speed function to include traffic flow. Eq. (4) is an expedient for saving computer time which should be incalculably long in case that traffic flow is also included in the speed function for surface road. What can be used for making an excuse for eq. (4) from the traffic engineering point of view is that traffic flow is much less active on traffic speed on surface than on the one on expressway because the former is in itself rather lower than the latter.



2) Route search

This step is concerned with what one is chosen by a trip from among innumerable number of routes between a pair of origin and destination. Two routes are selected for calculating traffic diversion from surface to expressway; one is the shortest time route on surface road network and the other is the shortest one by way of expressway.

The first one is easy to find assuming that it is one of the three typical routes R_1 , R_2 and R_3 as shown in Fig.4;

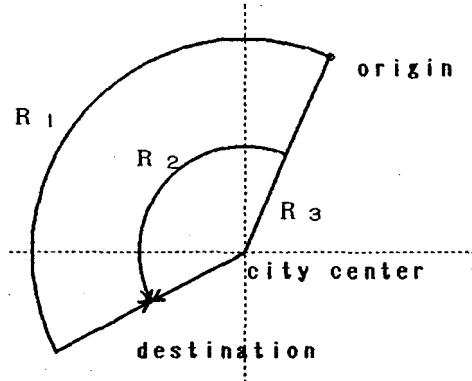


Fig.4 Surface routes

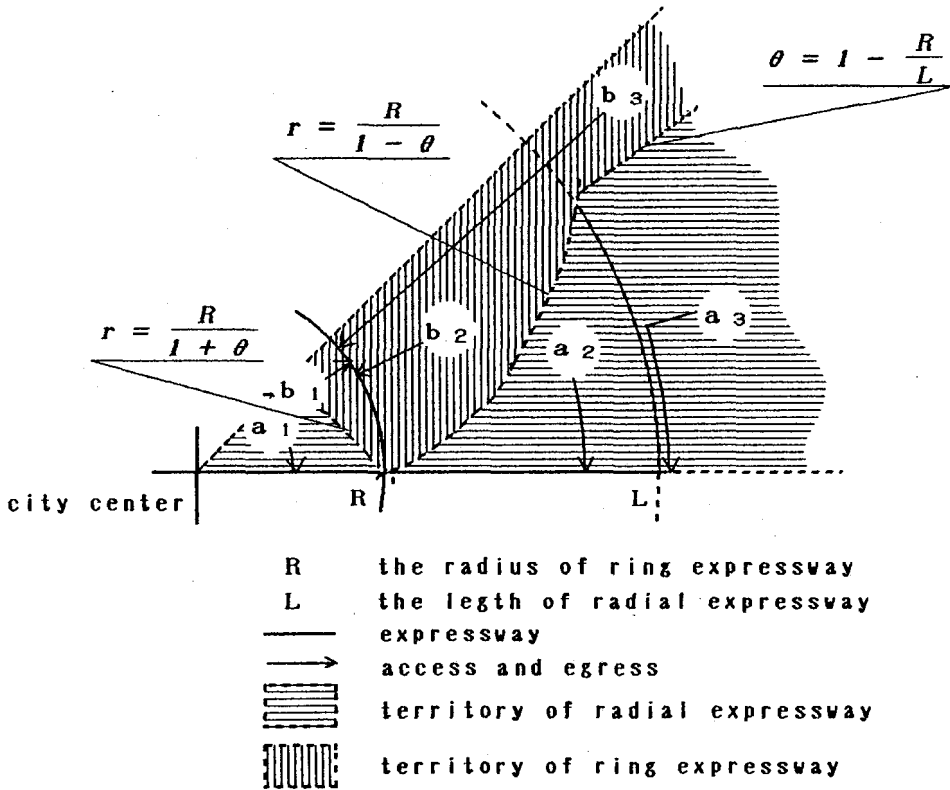


Fig.5 Territory, access and egress

R_1 ; A trip maker travels firstly along the ring road on which his origin lies until he meets the radial road on which his destination lies and turns to the radial.

R_2 ; He travels firstly along the radial road on which his origin lies until he gets to the ring road on which his destination lies and turns to the ring.

R_3 ; He travels along the radial roads alone by way of the city center until he gets to his destination.

It depends on the coordinates of the origin and destination and the values of parameters in eq.(4) which of the three is the shortest in travel time.

The second one is composed of surface access and egress linked by expressway of the shortest travel time. A trip maker is assumed to access to and egress out of his nearest expressways along the radial and/or ring surfaces. Accordingly the whole area under study is divided into, so called, expressways' territories. Each territory has its nearest expressway within itself. Fig.5 shows territories in part together with surface accesses and egresses.

There are three kinds of accesses and egresses ; radial surface, ring surface and radial-ring surface. The first is found within the territory of ring expressway, the second and the third are within the one of radial expressway. In detail, the second and the third are found within the radial expressways' territories inside and outside, respectively, of the circle where a flat rate of toll is imposed on expressway users. In this connection, the radius of the circle is equivalent to the length of radial expressway from the junction at the city center to the marginal end.

Fig.6 shows the possible routes, except for apparent detours, between two points on expressway with two rings. The route of the shortest travel time is chosen from among the possible ones. It is found every time when the whole traffic diversion has been calculated.

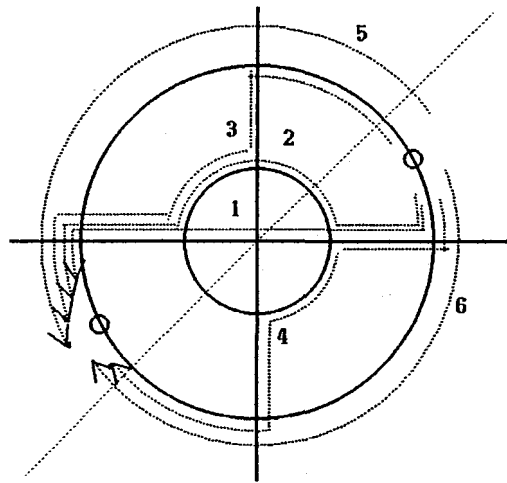


Fig. 6 Routes on expressway system with two rings

3) Calculation of diversion

The number of diverted trips is calculated for every pair of origin-destination by multiplying the number of origin-destination trips that is given by eq.(2) by the diversion ratio, whose function is assumed by

$$p = \frac{1}{1 + T^6} - 0.05 \quad (5)$$

where

p ; diversion ratio that is defined as the ratios of the number of diverted trips of a certain OD pair to the number of trips of the pair,

T ; travel time ratio that is defined as the ratios of the time necessary to travel by way of expressway to the one by surface route.

Eq.(5) is one of so called diversion ratio function by travel time ratio that has been often applied to urban expressway system in Japan. [8]

In the present study, travel time ratio is defined by

$$T = \frac{T_e + F / \delta}{T_s} \quad (6)$$

where

T_e ; time necessary to travel between origin and destination along the route including surface access and egress linked by expressway of the shortest time,

T_s ; time necessary to travel between the same origin and destination along surface route,

F ; toll rate that is imposed on those diverted to expressway and

δ ; time value.

The second term in the numerator on the right hand side of eq.(6) is the time equivalent to toll rate, which is flat here.

4) Convergence in traffic speed

Traffic speed in every section of expressway is calculated by eq.(3), where traffic flow Q is given by the number of diverted

trips passing through the section. The time for passing through the section is given by the inverse of the speed, whose integration along a route on expressway system gives the travel time by the route. The shortest travel time between a given pair of points on expressway system is found from among those obtained by the integration along the possible routes. It gives the time T_e in eq. (6) added to the surface access and egress time. The time T_s in eq. (6) is easy to find because it is assumed as uninfluenced by traffic flow at any surface section.

Given fixed values of F and δ , travel time ratio is calculated. It gives new diversion ratio, which revises traffic speed in the sections of expressway through changing the value of Q . The preceding calculation is repeated hereafter until every section is found converged in speed.

(4) Equilibrium of revenue and expenditure

Assuming that the expenditure, composed of construction and maintenance cost and so on, is proportionate simply to the length of expressway, we have

$$C = 4cL + 2\pi c \Sigma R \quad (7)$$

where

C ; the expenditure,

L ; the length of radial expressway from the junction at the city center to the marginal end,

$\Sigma R = R_1$ in case of one ring expressway and $R_1 + R_2$ in case of two rings, where R_1 and R_2 are the radii of the smaller and the larger, respectively,

c ; proportional constant.

L is also called the radius of the circle of flat rate. The first and the second terms are the expenditures for radial and ring expressways, respectively.

Equilibrium condition requires that the expenditure should be equal to revenue, which is given by the aggregate number of diverted trips multiplied by toll rate.

(5) Traffic capacity constraint

This constraint requires that traffic flow Q in eq. (3) must not be over the traffic capacity in any section of expressway. In this study a standard capacity is given commonly to all of the sections.

Maximization of the aggregate number of diverted trips is adoptable as a measure of optimality of the present system because it is equivalent to maximum revenue which may bring the maximal spanning of expressway network under constraints. By the measure an optimal solution is picked out of the feasible region of toll rate, the radius of ring expressway and the radius of the circle of flat rate.

3. INVESTIGATION

As stated previously, the authors are concerned not in the optimal solution itself but in the characteristic states of the optimal system.

Two kinds of expressway system were put under static study ; four radials with one ring and those with two rings.

(1) Four radials with one ring expressway system

The values of the parameters used are shown in Table 1, many of which were taken from those used by Sasaki et al. [7].

Two peaks of traffic flow are found on each radial and another two peaks are on each quadrant ring. The formers appear just outside of the junction of radial and ring and at the junction of city center. The latters do at the both ends of quadrant ring. Generally in the feasible region, the formers are of the different height while the latters are always of the same. The formers, however, are of the same in the optimal state of the system.

Fig.7 is an illustration of the traffic flow distributions on radial and quadrant ring. Two peaks on radial are of nearly the same height which implies that the system stands near the optimal state. The arrows ① and ② show the directions of traffic flow on the peak spots on radial.

Fig.8 shows the variations in traffic flow distribution on radial caused by changing the value of the radius of ring keeping the others unchanged. As ring grows, the traffic flow peak on radial gets higher at the central junction while, at the other junction, it begins to get lower after having been kept equal to the traffic capacity. This is explained by the fact that, as ring grows, the territory of radial inside of ring increases in area while the one outside of ring decreases. It is worthy to pay attention to the fact that, in the process of reciprocal changes in height of the traffic flow peaks, there will be a balance of height which gives the optimal system.

Table 1 Parameters

function including parameter	parameter	value	remarks
trip potential function	μ λ	12808.9 0.16	trips/km ² /day 1/km
origin-destination distribution of gravity type	α β γ	7.88×10^{-8} 1.0 { 0.0 1.0	one ring two rings
speed function on expressway	V_0 k	70.0 0.3×10^{-2}	km/hr km/trip
speed function on surface road	v_0 a b	30.0 10.0 0.15	km/hr km/hr 1/km
travel time ratio	δ	25.8	yen/trip·min
expenditure	c	135.6×10^4	yen/km/day
traffic capacity constraint on expressway		8.75×10^4	trips/day
the aggregate number of daily trips	$\frac{2\pi\mu}{\lambda^2}$	316×10^4	trips/day integration of trip potential function by r and θ

the values were taken from ref.[7]
except for $\alpha, \beta, \gamma, \delta$ and c .

It is because of both of the traffic capacity constraint and the principle of maximizing the aggregate number of diverted trips that the balance of height should be kept on radial in the optimal state of the system. Suppose the unbalanced peaks, one of which is over the traffic

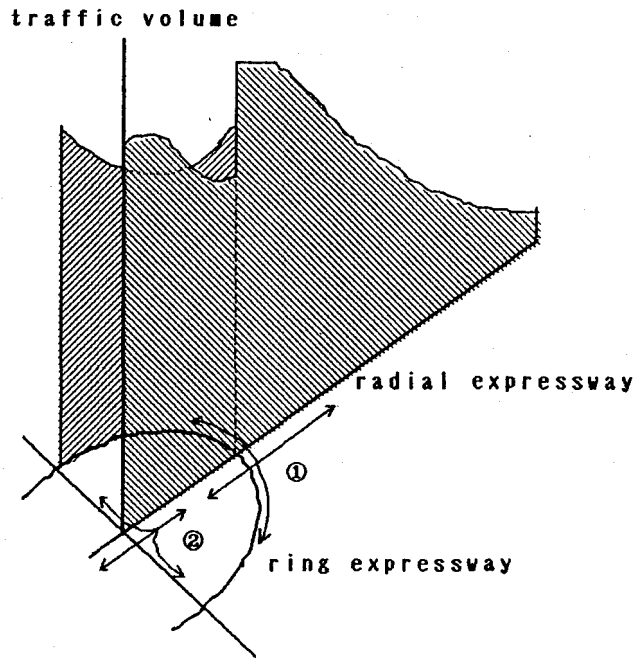


Fig.7 Distribution of traffic flow on expressways

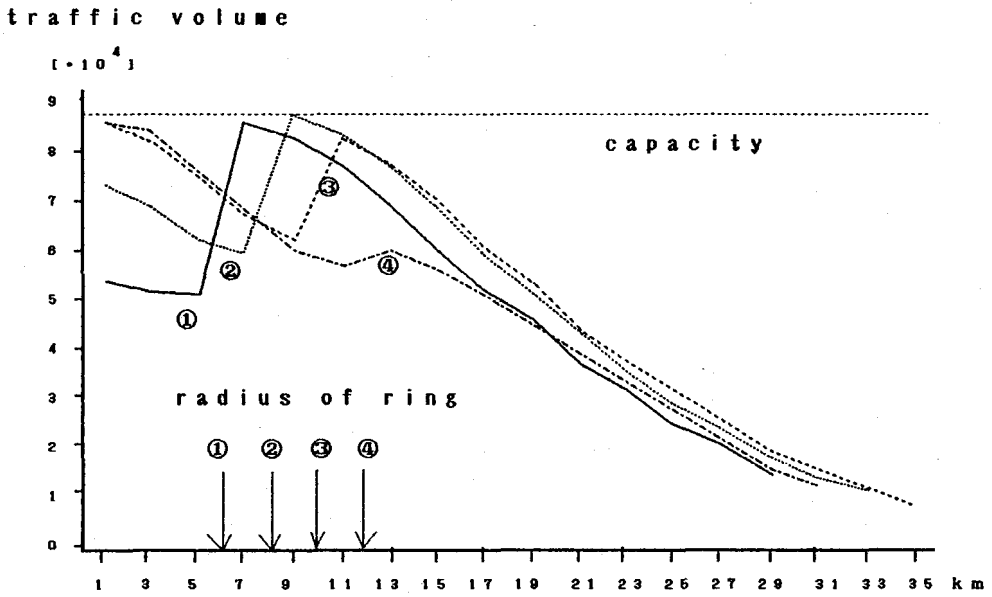


Fig.8 Distribution of traffic flow on radial vs. ring radius

capacity and the other is under the capacity, the traffic capacity constraint requires of the former not to be over the capacity while the maximization principle does of the latter to get higher toward the capacity. There are three variables for balancing the peaks ; the radius of ring, the radius of the circle of flat rate and toll rate. Those are interdependently changeable in the model.

(2) Four radials with two rings expressway system

The same values of the parameters as above are also used except for the value of γ that is included in eq. (2). The value 1.0 of γ is used instead of 0.0.

Fig.9 shows the aggregate number of diverted trips vs. the radii of two rings. The maximum aggregate number of diverted trips are found on the radii of 6 and 10 km, which give the optimal system in the case of $\gamma=1.0$. The authors, however, have concern for the system behaviors.

A characteristic behavior is observed in Fig.9, where the local maximum of the aggregate number of diverted trips are shown to exist for each radius of smaller ring. As larger ring grows keeping the smaller as it is, the expressway territories suffer changes in area which causes the local peak of the aggregate number.

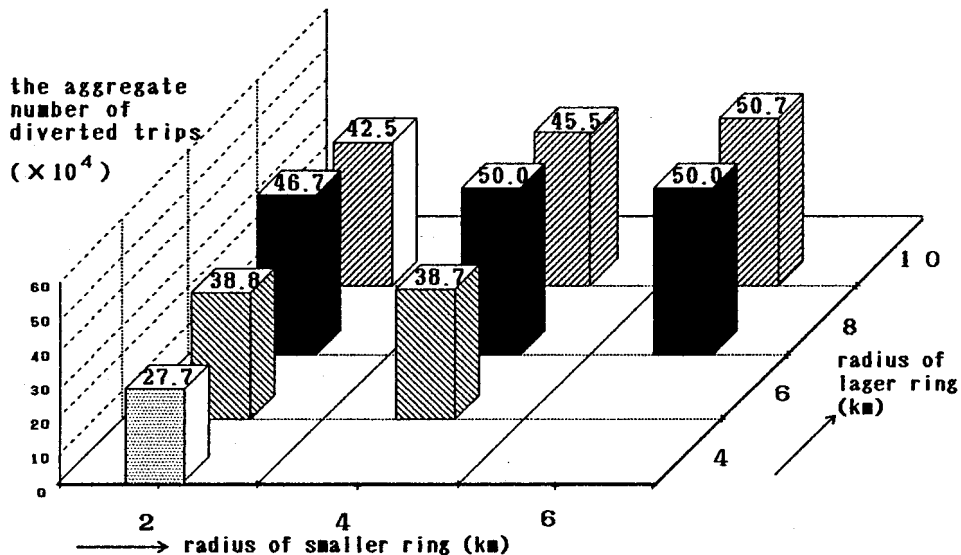


Fig.9 The aggregate number of diverted trips vs. the radii of two rings

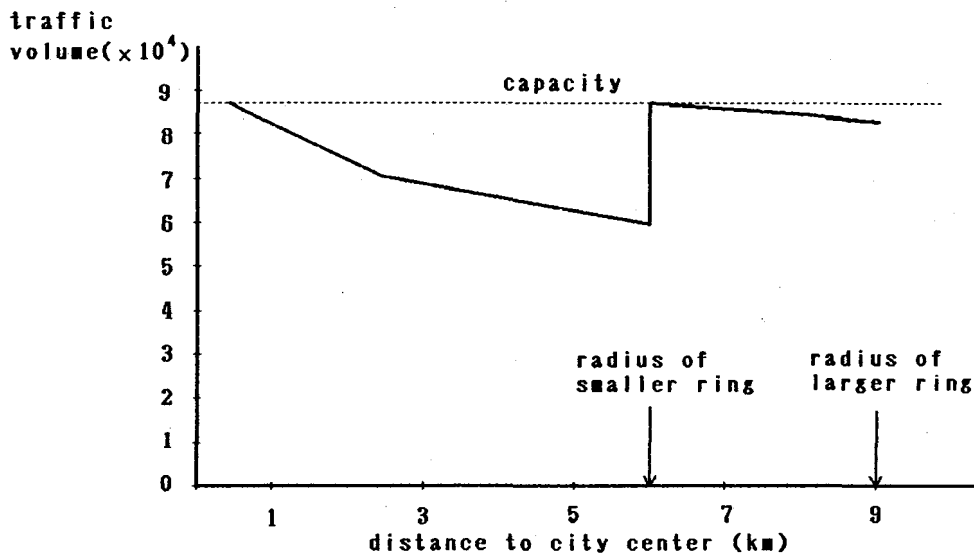


Fig.10 Distribution of traffic flow on radial with two rings

The traffic flow peaks on radial appear at the central junction and just outside of each of two other junctions. The three peaks are adjusted to be of the same height in the optimal state as was seen in one ring system.

Fig.10 shows the traffic flow distribution on radial. Though the right half where larger ring is surely located is neglected, the distribution implies that the system is getting closer to the optimal state because the peaks are of nearly the same height.

(3) Comparison of one ring system with two rings system

Table 2 shows the optimal solutions of one ring and two rings systems. The values of the parameters used are the same as those used above for two rings.

Toll rate is worthy of attention. The rate is lower for two rings than for one ring, though the whole length of expressway is a little longer for the former than for the latter. This, though seemingly paradoxical, is interpreted by two kinds of the effects of the well-located second ring; improvement in accessibility to expressway and distributing traffic flow on expressway.

It is a matter of course that collective accessibility to expressway system is much improved by the second ring. Also as a matter of course, the second ring makes it possible to find new route of shorter travel time bypassing the bottle-necks that would have appeared in case of one ring

Table 2 Solutions of the one and two ring systems

system	two rings	one ring
the length of radial expressway (km)	10	18
the radius of ring expressway (km)	smaller 6 larger 10	8
toll rate (yen/trip)	380	400
the aggregate number of diverted trips (trips/day)	50.7×10^4	43.3×10^4
the whole length of expressway (km)	140	122

system, which is so called the distributing effect of ring road. These effects join to attract much more trips to expressway. Incremental expenditure on two rings system is surely supported by the increasing number of diverted trips who pay lower toll rate.

(4) Sensitivity

The parameter γ is well known to have a great influence upon origin-destination distribution of trips, in other word, distribution of trip length. Accordingly the optimal solutions are expected to be considerably sensitive to the parameter. Though as is expected, the solutions are affected by γ in a curious way. That is, both the optimal length of radial expressway (the radius of the circle of flat rate) and the optimal toll rate are powerfully affected while the optimal radius of ring and the maximum aggregate number of diverted trips are not so much affected. As the value of γ is larger, mean trip length is shorter, which causes relatively more centralized distribution of trips. The centralization of trip distribution leads to shorter radial expressway. It is preferably required to lower toll rate so as to attract the increasing number of shorter trips to expressway. These are rough interpretation of the curious effect of γ on the solutions. Table 3 is an example of the solutions vs. the values of γ .

Table 3 Effects of γ on solutions

the value of γ	0.0	1.0
the length of radial expressway (km)	36	18
the radius of ring expressway (km)	10	8
toll rate (yen/trip)	690	400
the aggregate number of diverted trips (trips/day)	41.4×10^4	43.3×10^4

4. CONCLUDING REMARKS

The traffic flow peaks on radial expressway are balanced in height when the aggregate number of diverted trips is maximized under traffic capacity constraint. As a matter of course, the length of radial expressway, the radii of ring expressways and toll rate are decided so as to keep the peaks balanced in height despite how many rings are introduced.

Though two rings system is a little longer than one ring system in the whole length, it gives the larger aggregate number of diverted trips and lower toll rate than one ring system does.

The parameter γ has a curious effect on the solution that it affects more powerfully on radial expressway and the aggregate number of diverted trips. It is interpretable by the influence of γ upon trip length. The following is some of the problems to be studied another time.

Improvement of the speed functions either for expressway and surface road. Improvement of the speed function for surface may be possible by introducing so called segment speed function which will include, for example, traffic flow density and surface road density as variables. Here segment speed is, so called, a representative speed to travel through. As for the speed on expressway, various kinds are presumable; a function including two kinds of linear functions of traffic flow which meet or are discontinuous at a certain point, a negative exponential function of traffic flow and so on. Improvement of the surface speed

function seems essential.

Investigation in what is the implication of the maximization of the aggregate number of diverted trips that is adopted as a measure of optimality in the present study. The implication of the measure will not be revealed until an improved speed function, especially for surface road, is introduced.

Extension of the present study to the dynamic.

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