

# *Plasma Oscillation in Semiconductor Superlattice Structure*

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## Abstract

The statistical properties of two-dimensional systems of charges in semiconductor superlattices are analyzed and the dispersion relation of the plasma oscillation is calculated. The possibility to excite these oscillations by applying the electric field parallel to the structure is discussed.

## 1. Introduction

The layered structure of semiconductors with thickness of the order of  $10^{-6}$  cm or less is called semiconductor superlattice. The superlattice was first proposed by Esaki and Tsu [1] as a structure which has a Brillouin zone of reduced size and therefore allows to apply the negative mass part of the band structure to electronic devices through conduction of carriers perpendicular to the structure.

The superlattice has been realized by subsequent developments of technologies such as molecular beam epitaxy (MBE) and metal-organic chemical vapor deposition (MOCVD) in fabricating controlled fine structures. At the same time, many interesting and useful physical phenomena related to the parallel conduction have also been revealed in addition to the parallel conduction.

From the view point of application to devices, the enhancement of the carrier mobility due to separation of channels from ionized impurities may be one of the most important progresses. Some high speed devices are based on this technique. The superlattice structure is now being applied to a very wide variety of electronic devices including optical ones.

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Among numerous applications of high mobility carriers, there has been reported an experiment of radiation from the plasma oscillation of these carriers [2]: By applying the electric field along the two-dimensional system of electrons at the interface of heterojunction, the far infrared radiation has been obtained presumably from the two-dimensional plasmons.

The plasma oscillation of two-dimensional charges has a unique property in comparison with the one in three dimensions: The frequency strongly depends of the wave number. In fact, it is proportional to the square root of the wave number in the limit of long wavelength. This gives another possibility to control the frequency of the plasma oscillation along with changing the areal carrier density by doping: Typical values of the latter range from  $10^{10}\text{cm}^{-2}$  to  $10^{12}\text{cm}^{-2}$ . In the aforementioned experiment, the grating of a fixed pitch is placed near the two-dimensional electron system and served to determine the frequency.

In the superlattice structures, we have systems of carriers which are spatially separated but coupled via the electromagnetic field. We thus expect that the relative drift between systems of carriers may easily result from the electric field along the layers, especially in the type II superlattice where we have alternate layers of the electrons and holes.

In the analyses of carriers in superlattice structures, the electrons are usually assumed to be degenerate. As is shown in 2, however, we have classical or half-degenerate carriers at the temperature higher than that of liquid nitrogen. We also expect that the dispersion relation of the oscillation will be changed by the drifting motion of carriers. The purpose of this work is to analyze the plasma oscillations of two-dimensional charges in superlattice structures, to compare the results with those in three dimensions and to obtain the conditions for unstable oscillations which may possibly be used in the active device.

## 2. Physical Parameters of Two-Dimensional Charge System

The Fermi energy of electrons is given by

$$E_F = \hbar^2 K_F^2 / 2m. \quad (2.1)$$

Here  $\hbar$  is the Planck constant,  $m$  is the mass, and the two-dimensional Fermi wave number  $K_F$  is related to the surface number density  $n$  as

$$K_F = (2\pi n)^{1/2}. \quad (2.2)$$

In Fig.1 we plot the relation  $k_B T = E_F$  ( $k_B$  being the Boltzmann constant) for systems of electrons and heavy holes in GaAs. We see that for moderate densities these two-dimensional systems can be regarded as classical at the room temperature and classical or half-degenerate at the temperature of liquid nitrogen, 77K. We assume that the charges are classical in what follows.

The effect of Coulomb interaction in two-dimensional classical systems is characterized by the parameter  $\Gamma$  defined by

$$\Gamma = (\pi n)^{1/2} e^2 / \epsilon k_B T \quad (2.3)$$

where  $e$  is the electronic charge,  $T$  the temperature, and  $\epsilon$  the relative dielectric constant of the medium which surrounds the system. According to the condition  $\Gamma \lesssim 1$  or  $\Gamma \gtrsim 1$ , the two-dimensional charge system is weakly or strongly coupled. [4]

The relation  $\Gamma=1$  is also shown in Fig.1. We see that for typical value of the density  $10^{11} \text{cm}^{-2}$ ,  $\Gamma \lesssim 1$  is satisfied at the temperatures higher than 77K.

In the case of weak coupling, the screening and the plasma oscillation in the two-dimensional system of charges are characterized by the Debye wave number  $K_D$  and the frequency  $\omega_p$  defined respectively by [3]

$$K_D = 2\pi n e^2 / \epsilon k_B T \quad (2.4)$$

$$\begin{aligned} \omega_p &= [(2\pi n e^2 K / \epsilon m)^{1/2}]_{K=K_D} \\ &= 2\pi n e^2 / \epsilon (m k_B T)^{1/2} \end{aligned} \quad (2.5)$$

Values of  $K_D$  and  $\omega_p$  calculated from the parameters of electrons and heavy holes in GaAs [5] are shown in Figs.2 and 3. We observe that the inverse of the Debye wave number or the Debye length is comparable with the spacing between layers in the superlattice structure and the typical frequency of the plasma oscillation is around  $10^{13} \text{rad/s}$ .

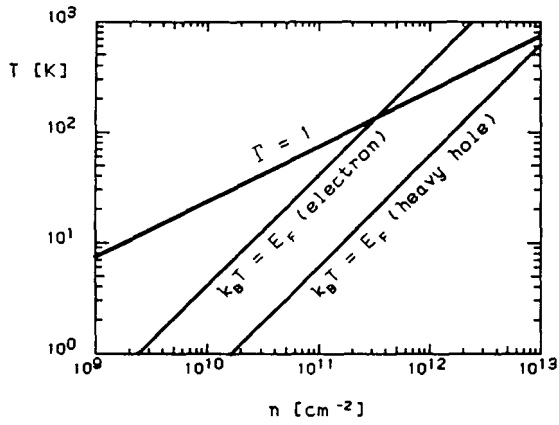


Fig. 1. Fermi energy and the Coulomb coupling parameter for electron and heavy hole in GaAs.

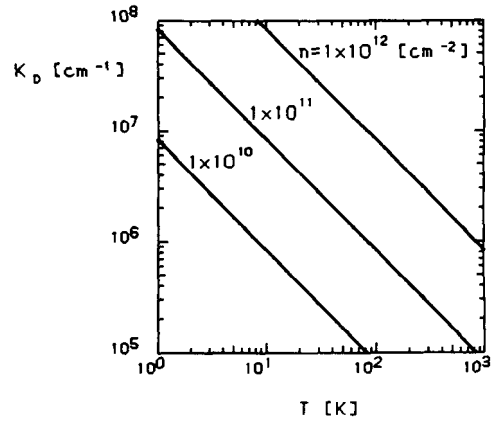


Fig. 2. Two-dimensional Debye wave number in GaAs.

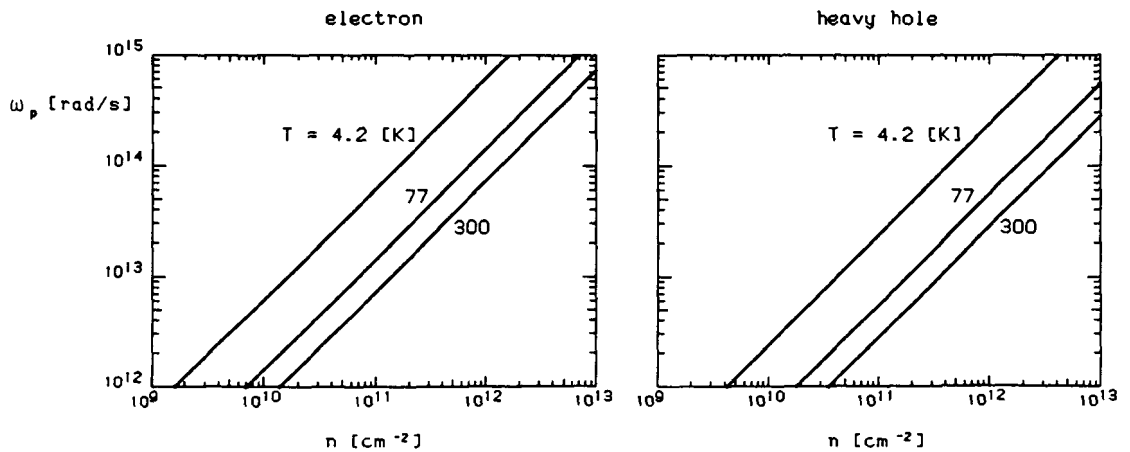


Fig. 3. Characteristic frequency of two-dimensional plasma oscillation in GaAs.

### 3. Plasma Oscillation in Superlattice Structures

#### 3.1 Electrostatic formulation

Our system is composed of infinitely thin parallel layers of two-dimensional charges. We take the  $z$ -axis perpendicular to these layers and denote three-dimensional vectors like  $\vec{r}$  and  $\vec{k}$  as  $\vec{r}=(\vec{R}, z)$  and  $\vec{k}=(\vec{K}, k_z)$  using capital letters for  $x$ - and  $y$ - components. Since our system is stationary and translationally invariant in two dimensions, we consider the Fourier component of related quantities proportional to  $\exp(i\vec{K}\cdot\vec{R}-i\omega t)$ .

We denote the charge and current densities of the layer  $i$  at  $z=z_i$  by  $\rho_i(\vec{K}, \omega)\delta(z-z_i)$  and  $(\vec{J}_i(\vec{K}, \omega)\delta(z-z_i), 0)$ , respectively. From the Poisson equation, the electrostatic potential  $\phi(\vec{R}, z)$  is calculated as

$$\phi(\vec{R}, z) = \frac{2\pi}{K} \exp(i\vec{K}\cdot\vec{R}-i\omega t) \sum_j \rho_j \exp(-K|z-z_j|). \quad (3.1)$$

The current density is related to  $\rho_i$  by

$$\vec{J}_i(\vec{K}, \omega) = -\overleftrightarrow{\sigma}_i(\vec{K}, \omega) \cdot \frac{\partial}{\partial \vec{R}} \phi(\vec{R}, z_i), \quad (3.2)$$

where  $\overleftrightarrow{\sigma}(\vec{K}, \omega)$  is the conductivity tensor of the two-dimensional charges on the layer  $i$ . From (3.1), (3.2) and the equation of continuity

$$\omega \rho_i(K, \omega) = \vec{K} \cdot \vec{J}_i(\vec{K}, \omega), \quad (3.3)$$

we obtain

$$\sum_j [\delta_{ij} + i \frac{2\pi}{K\omega} \vec{K} \cdot \overleftrightarrow{\sigma}_i \cdot \vec{K} \exp(-K|z_i-z_j|)] \rho_j = 0. \quad (3.4)$$

Thus the dispersion relation of the charge density oscillations in our system is determined by

$$\det(\delta_{ij} + i \frac{2\pi}{K\omega} \vec{K} \cdot \overleftrightarrow{\sigma}_i \cdot \vec{K} e^{-K|z_i-z_j|}) = 0 \quad (3.5)$$

in the electrostatic approximation.

In the case without magnetic fields, the conductivity tensor is diagonal and Eq. (3.5) takes a simpler form

$$\det(\delta_{ij} + i \frac{2\pi K}{\omega} \sigma_i e^{-K|z_i - z_j|}) = 0. \quad (3.6)$$

### 3.2 Random phase approximation

When the effect of Coulomb interaction is relatively small, the dynamical properties of our system may be described by the random phase approximation (RPA). It has been pointed out [3] that due to reduced dimensionality, the validity of RPA is not guaranteed in some respects even in the case of weak coupling. In this work, however, we are interested in the general behavior of the plasma oscillation and the RPA may be used as a first step.

In the RPA, the conductivity of the classical two-dimensional charges with Maxwellian velocity distribution is given by [3]

$$\sigma(K, \omega) = -i \frac{ne^2 \omega}{k_B T K^2} W \left[ \frac{\omega - \vec{K} \cdot \vec{V}_d}{K} \left( \frac{m}{k_B T} \right)^{1/2} \right], \quad (3.7)$$

with

$$W(z) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dx \, x \exp(-x^2/2) / (x - z - i0). \quad (3.8)$$

Here 0 denotes the positive infinitesimal and  $\vec{V}_d$  the average drift velocity.

The electrons or holes confined in a single layer in vacuum may be regarded as the simplest case of our layered charges. The dispersion relation of the plasma oscillation in this system is given by [3]

$$1 + (K_D/K) W[(\omega/K)(m/k_B T)^{1/2}] = 0. \quad (3.9)$$

The real and imaginary parts of the frequency for long wavelengths are calculated as

$$\begin{aligned}\omega(\mathbf{K}) &= \omega^{(r)}(\mathbf{K}) + i\omega^{(i)}(\mathbf{K}), \\ \omega^{(r)}/\omega_p &\simeq (K/K_D)^{1/2} [1 + (3/2)(K/K_D)], \\ \omega^{(i)}/\omega^{(r)} &\simeq -(\pi/8)^{1/2} (K_D/K)^{3/2} \exp[-K_D/2K - 3/2].\end{aligned}\quad (3.10)$$

In this case, the average drift of charges only affects the real part of the frequency through the Doppler shift.

Since our charge layers are embedded in the bulk material of the dielectric constant  $\epsilon$ , the conductivity of the  $i$ -th layer is written as

$$\begin{aligned}\sigma_i(\mathbf{K}, \omega) &= -i \frac{1}{2\pi} \frac{K_i}{K} \frac{\omega/\omega_p}{K/K_i} \left( \frac{k_B T}{m_i} \right)^{1/2} \\ &\quad \times W[(\omega/\omega_i)/(K/K_i) - (\vec{\mathbf{K}} \cdot \vec{\mathbf{v}}_d/K)(m_i/k_B T_i)^{1/2}].\end{aligned}\quad (3.11)$$

Here

$$K_i = 2\pi n_i q_i^2 / \epsilon k_B T, \quad (3.12)$$

$$\omega_i = 2\pi n_i q_i^2 / \epsilon (m_i k_B T)^{1/2}, \quad (3.13)$$

and  $n_i$ ,  $q_i$  and  $m_i$  are the surface number density, the charge and the effective mass of particles in the layer  $i$ .

It is well known in plasma physics that the relative drift between groups of charges has a destabilizing effect on the charge density oscillation. The drift of electrons against ions and the existence of high velocity beams are typical examples and the dependence of the growth rate on characteristic parameters has been thoroughly investigated.

In the case of superlattice structures in semiconductors, we have an extra parameter, the spacing of layers, which has not been seriously considered in the analysis of plasma oscillation in three dimensions. In the latter, this parameter is either irrelevant or uncontrollable. Naturally we expect that the increase of the spacing has a stabilizing effect on the dispersion.

#### 4. Examples

##### 4.1 Two layers with relative drift velocity

As a first simple example, we here consider two layers ( $i=1, 2$ ) of electrons (or holes) with the number density  $n_i$  and the effective mass  $m_i$  drifting with the relative velocity  $V_d$ . From (3.6), we have

$$\left(1 + \frac{K_1}{K} W_1\right) \left(1 + \frac{K_2}{K} W_2\right) - \frac{K_1}{K} \frac{K_2}{K} W_1 W_2 e^{-2K\delta} = 0, \quad (4.1)$$

where  $\delta$  is the distance between two layers and

$$W_i = W \left[ \frac{\omega - \vec{K} \cdot \vec{V}_{di}}{K} \left( \frac{m_i}{k_B T} \right)^{1/2} \right]. \quad (4.2)$$

When  $\delta=0$ , (4.1) reduces to

$$1 + \sum_i \frac{K_i}{K} W_i = 0. \quad (4.3)$$

This equation is the two-dimensional version of the dispersion relation for counterstreaming plasmas

$$1 + \sum_i \frac{k_i^2}{k^2} W_i \left[ \frac{\omega - \vec{k} \cdot \vec{V}_{di}}{k} \left( \frac{m_i}{k_B T} \right)^{1/2} \right] = 0, \quad (4.4)$$

where  $k_i = (4\pi n_i e^2 / k_B T)^{1/2}$  and  $\omega_i = (4\pi n_i e^2 / m_i)^{1/2}$  are the Debye wave number and the plasma frequency in three dimensions.

An example of the dispersion relation obtained from (4.1) is plotted in Fig 4: The ratios of the number densities and the effective masses are taken to be  $n_1/n_2=1$  and  $m_1/m_2=0.5$ , respectively. We see that the plasma oscillation becomes unstable for small values of the wave number.

The change of the domain of unstable wave number with the increase of the relative drift velocity is shown in Fig 5. The increase of  $\delta$  reduces the unstable domain of the wave number in accordance with naive expectation.



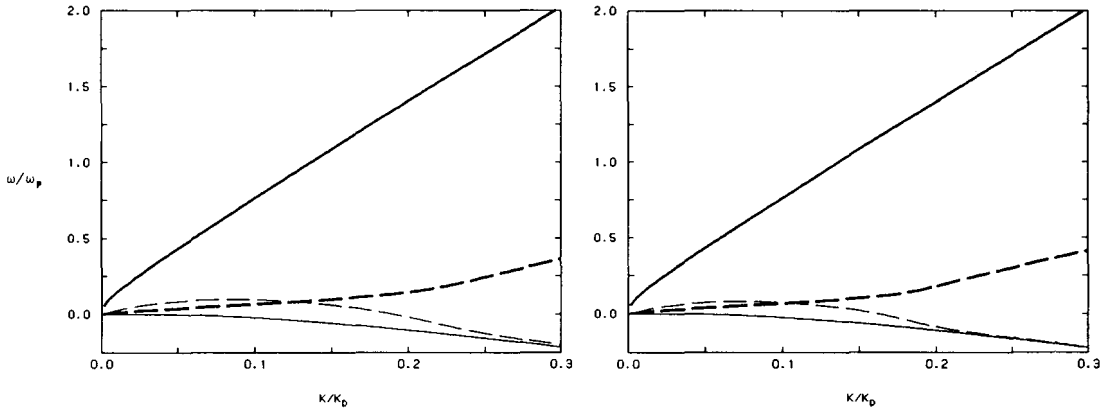


Fig.4. Dispersion relation of oscillation of parallel charge layers ( $n_1/n_2=1$ ,  $m_1/m_2=0.5$ ) separated by  $\delta=0$  (left) and  $\delta=2/K_D$  (right), where  $K_D=K_1+K_2$ . Pairs of thick and thin lines of the same kind show the real and imaginary parts of two branches. In this case,  $\vec{k} \cdot \vec{v}_d / K(k_B T / m_1)^{1/2} = 5$ .

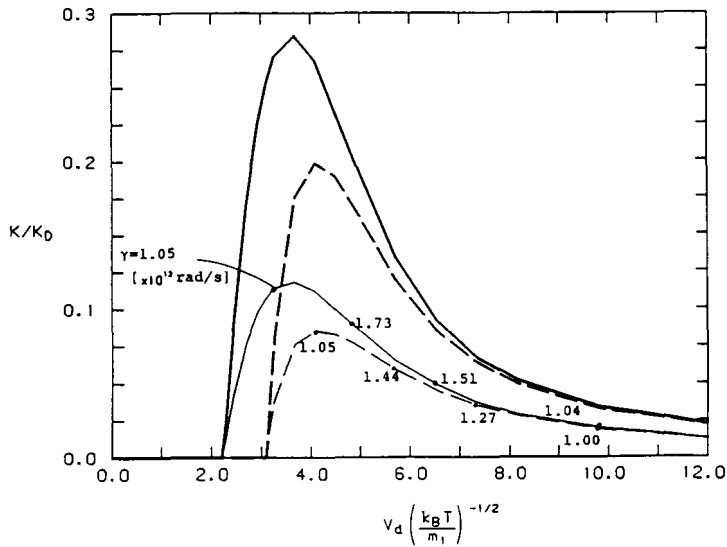


Fig.5. Domain of instability. Plasma oscillation is unstable for wave numbers smaller than solid or broken lines when  $\delta=0$  or  $2/K_D$ . Other parameters are the same as in Fig.4. Thin lines with numbers show the most unstable wave number with the growth rate in the unit of  $10^{12}$  rad/s for the case of  $T=77K$ ,  $n_1=n_2=10^{11} \text{ cm}^{-2}$ , and  $m_1=0.067m_{\text{electron}}$ .

#### 4.2 Multiple layers

The dispersion relation of the two-dimensional plasma oscillation in the system composed of a series of two alternate layers is shown in Fig.6 for the case of two, four, and six layers: The parameters are  $n_1/n_2=1$ ,  $m_1/m_2=0.5$ ,  $\delta=2/(K_1+K_2)$ , and  $V_d=0$ . With the increase of the number of layers, there appear increased number of branches in the dispersion relation and we have a band of plasma oscillation for layered structures repeated infinitely in space.

For large values of the wave number, the real part of the frequency approaches to two branches corresponding to independent oscillation of each layer: With the increase of the wave number, the coupling between layers becomes small as is clear from the nondiagonal elements  $\exp(-K\delta)$  in (3.5).

When the adjacent layers have finite relative drift velocity, some of these branches become unstable as in the case shown in Fig.4. An example is shown in Fig.7. From the most unstable branch plotted there, we may extrapolate the one in the band of the plasma oscillation.

In Figs.4-7, the ratio of the effective masses  $m_1/m_2$  is taken to be 0.5. We show the effect of the decrease of this ratio in Fig.8 where  $m_1/m_2=0.149$  (the ratio of the effective mass of electron to that of heavy hole in GaAs). We see that the high frequency branches and the low frequency branches are separated more clearly. The overall behavior of the dispersion, however, is similar to the case of  $m_1/m_2=0.5$ .

#### 5. Competition with Relaxation Processes

Electrons and holes suffer collisions which serve to damp oscillations of these carriers. In order to excite the plasma oscillation in semiconductor superlattices, the growth rate of the oscillation should exceed the damping rate due to such relaxation processes.

The relaxation time  $\tau$  for collisions may be roughly estimated by the value of the mobility  $\mu$  based on the simple Drude formula

$$\mu = e\tau/m^* \quad (5.1)$$

where  $m^*$  is the effective mass determined by the band structure.

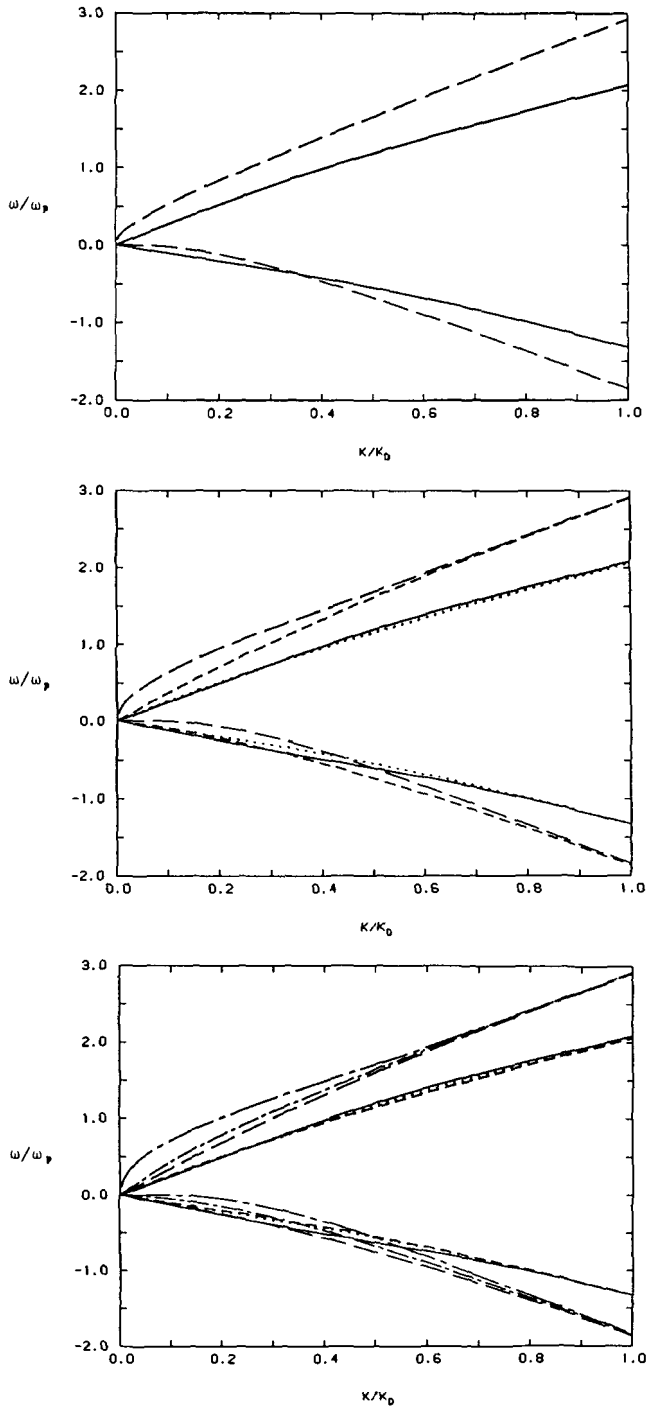


Fig.6. Formation of band of two-dimensional plasma oscillation:  $n_1/n_2=1$ ,  $m_1/m_2=0.5$ ,  $\delta=2/(K_1+K_2)$ , and  $V_d=0$ . Lines of the same kind show the real (upper curves) and the imaginary (lower curves) parts of a branch.

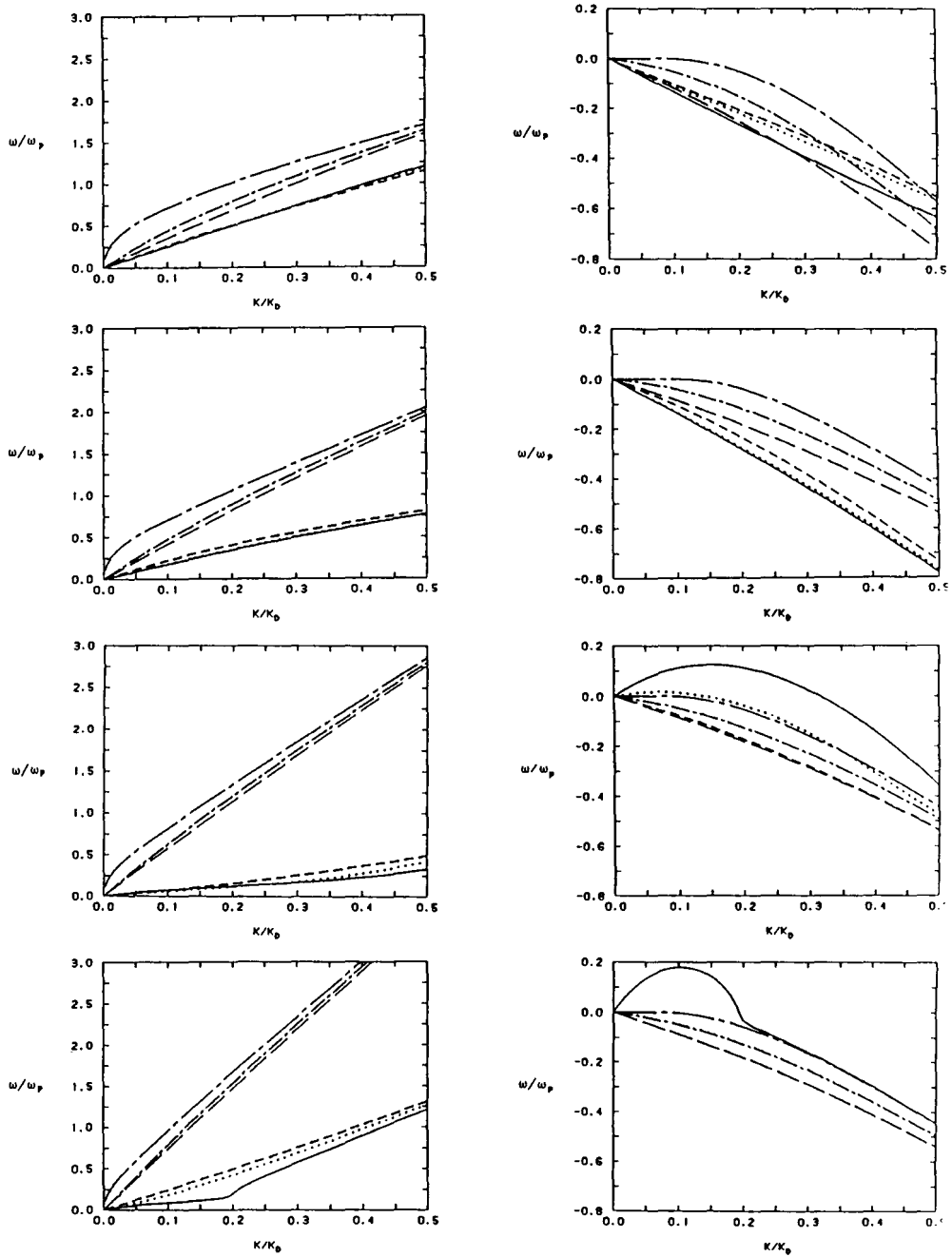


Fig.7. Change of the dispersion relation with increase of relative drift velocity. From the top,  $\vec{k} \cdot \vec{V}_d / K(k_B T / m_1)^{1/2} = 0, 2, 4,$  and  $6$ . Figures on the left and the right are the real and imaginary parts, respectively.

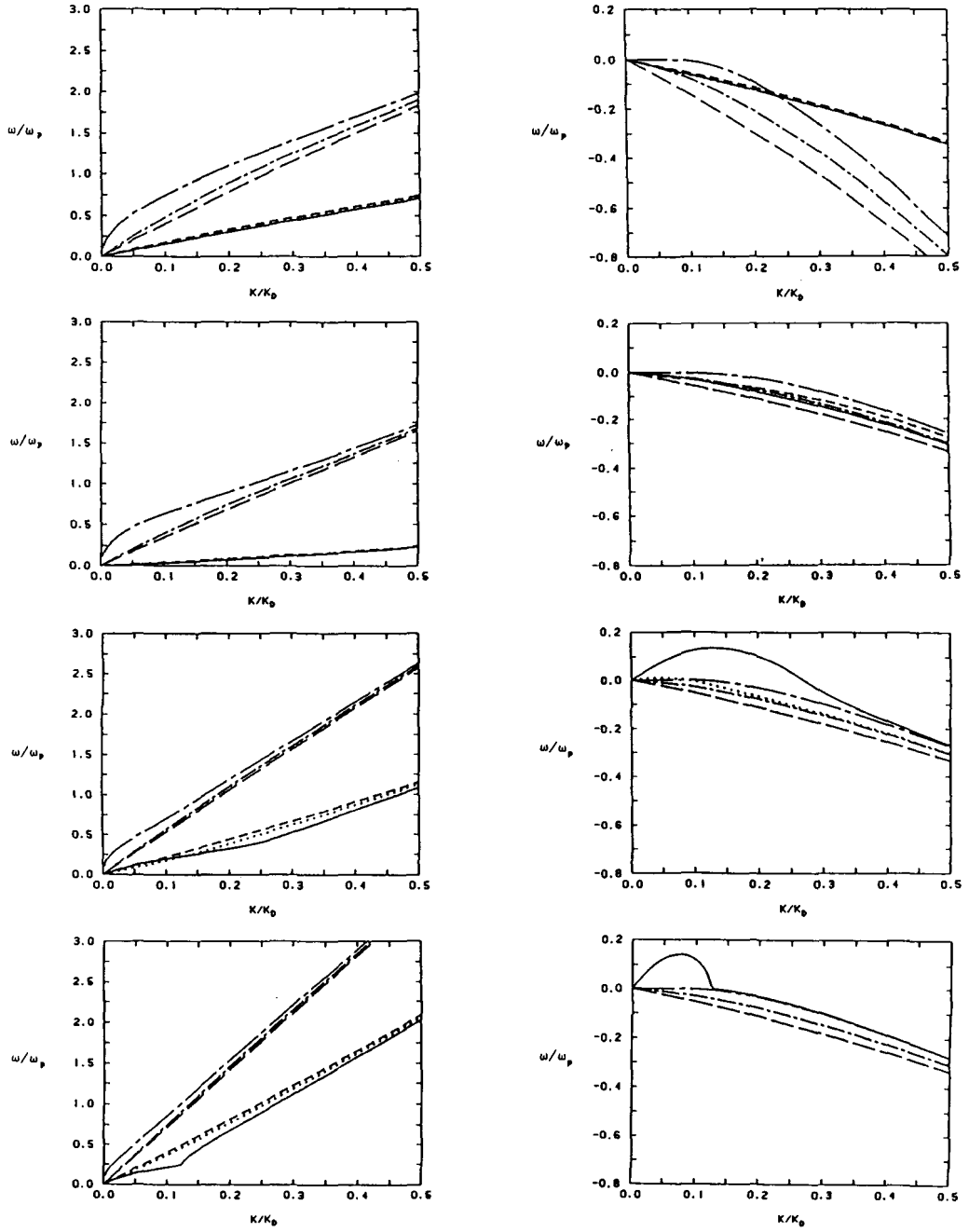


Fig.8. The same as Fig.7 with  $m_1/m_2=0.149$ .

We tentatively take GaAs at 77K as an example. It is known that the electron and hole have the mobilities  $\mu_n \sim 2 \cdot 10^5 \text{ cm}^2/\text{Vs}$  and  $\mu_h \sim 6 \cdot 10^3 \text{ cm}^2/\text{Vs}$ , respectively, for small electric fields and the mobilities decrease with the increase of the field intensity. Weak field mobilities give  $\tau_n \sim 8 \cdot 10^{-12} \text{ s}$  and  $\tau_h \sim 2 \cdot 10^{-12} \text{ s}$  for relaxation times of electrons and holes, respectively. Expecting the electric field of a few times  $10^2 \text{ V/cm}$ , we may here adopt  $\tau \sim 10^{-12} \text{ s}$  as a typical relaxation time and require the condition

$$\omega^{(i)} \tau \gtrsim 1 \quad (5.2)$$

for instability in realistic situations.

In order to check whether the above condition can be satisfied, we plot in Fig.5 the values of the wave number of the most unstable oscillation together with the growth rate: The surface number density is assumed to be  $10^{11} \text{ cm}^{-2}$ . We see that there exists a domain where the two-dimensional plasma oscillation grows in spite of collisional damping. The relative velocity around  $5(k_B T/m^*)^{1/2}$  is sufficient for this condition. The magnitude of the electric field corresponding to this drift velocity is calculated to be  $6 \cdot 10^2 \text{ V/cm}$ . We also observe that the condition (5.2) is satisfied only marginally and we need more detailed investigation on the effect of relaxation processes.

We expect that the above values may apply as rough estimates also for the case of other materials such as InAs-GaSb which form Type II superlattices and plasma instabilities in these structures may be used as the source of active functions.

#### References

1. L. Esaki and R. Tsu, (BM J. Res. and Dev. 14, 61(1970).
2. N. Okisu, Y. Sambe, and T. Kobayashi, Appl. Phys. Lett. 48, 776(1986).
3. H. Totsuji, J. Phys. Soc. Japan 39, 253(1975).
4. H. Totsuji, Phys. Rev. A17, 399(1978); H. Totsuji and H. Kakeya, Phys. Rev. A22, 1220(1980).
5. For example, S. M. Sze, Physics of Semiconductor Devices, 2nd ed., John Wiley and Sons, New York, 1981.