

An Experimental Study on Levee Failure Caused by Seepage and Preventive Measures

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SYNOPSIS

There are two classifications of the mechanism of levee failure caused by floods; local seepage failure and progressive failure. The fundamental causes of levee failure produced by piping and erosion were studied and the safety of river levees during floods evaluated in terms of soil mechanics.

The critical hydraulic gradient and the process of progressive failure were obtained from one- and two-dimensional model experiments for piping and erosion.

Problems inherent in and preventive measures against levee failure are discussed. In particular, effects of the Tsukinowa method, the most representative Japanese flood fighting method, were studied experimentally and improvements proposed.

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1. INTRODUCTION

During flooding, river levees may be destroyed by seepage and piping. The mechanisms of levee failure caused during floods can be classified (1) as local seepage or sliding failure in which the water pressure or seepage forces exceed the resisting forces of the levee, and (2) progressive failure in which internal erosion within the levee is enlarged by seepage flow. To make clear the mechanisms of river levee failure caused by seepage and leakage during floods, we used a soil mechanics approach to develop a method of levee failure prediction. Existing prevention methods are evaluated, and suggestions for improving them made.

Several studies have been done in order to clarify the mechanisms of river levee failure.[1]-[5] Our study reported here includes discussion of the progressive erosion and failure of a levee due to piping based on experimental and theoretical results.

In section 2, the fundamental types and mechanisms of seepage failure are classified as 1) failure that takes place when seepage forces exceed the resistance forces of a levee, and 2) failure caused by progressive erosion.

In section 3, one- and two-dimensional sand model experiments on erosion are described that show the mechanism that operates during progressive failure of a levee owing to seepage,

Lastly, in section 4 existing preventative measures against levee failure are evaluated based on the results reported here, and new preventative measures are proposed.

2. LEVEE FAILURE OWING TO SEEPAGE OR LEAKAGE

2.1 Shearing Failure Produced by Water Pressure

When there is a sudden rise in the water level in a river, or along an impervious slope on the riverside of a levee, shearing failure of that levee is easily produced by water pressure, especially in levees built on soft foundations. The most dangerous stage is when the water level has reached the crown.

2.2 Failure Caused by Increasing Pore Pressure and Water Content

With an increase in pore pressure the shear strength decreases

because the effective stress is lessened. In the unsaturated condition, the effective stress increases because of capillary suction, which is a negative pore pressure. But, with increasing water content suction forces are decreased, and the shear stress becomes small. It is well documented that as the water content increases, cohesion decreases.[6]

The resisting forces of a levee against water pressure, seepage and erosion become weak because of the above process and the degree of danger of levee failure increases markedly. Failure, which often appears as sliding failure on the levee's slope, usually has been studied by the circular slip surface method. Recently, evaluations of levee safety and failure have been improved by the use of finite element analysis.

2.3 Slope Failure of a Levee Caused by Seepage

The seepage force (j) is defined as

$$j = r_w * i \quad \dots\dots\dots(1)$$

in which i is the hydraulic gradient,
and r_w is the unit weight of the water.

Seepage force is applied to the unit volume of the soil mass because of seepage flow. This force works in the same direction as the seepage flow in an isotropic aquifer. For convenience, seepage force often is treated as body force in finite element analysis. This treatment is the same as for gravity; but, assuming that a large soil mass is a rigid body and that a balance of forces is considered in the circular slip surface method, it is more convenient that seepage force be treated as the water pressure present at each boundary.[7],[8]

While the water level in a river remains high, the wetting front rapidly progresses into the levee. Until water infiltrates the levee; that is, until there is no seepage from its slope, the levee generally can be considered more stable than if there were an actual leak. This is important in the design of a levee.

2.4 Failure Caused by Erosion

Local shear failure occurs at locations where the seepage force

is high and the confining force or strength of the soil is low, e.g., on the surface of the slope or a channel. In particular, seepage water accumulates in channels in the soil, and the seepage force becomes very strong because the hydraulic gradient becomes high. As a result, because of local failure the channel in the soil is enlarged and extended into the heart of the levee or is spread two- or three-dimensionally causing general failure of the levee.

The types of progressive failure are

(1) Hydraulic Fracturing

If the surface of the levee is exposed to high water pressure, cracks will appear at weak and highly permeable areas of the levee body. These cracks eventually cut deeply into the levee and become wide erosion channels. This phenomenon, called hydraulic fracturing, is dependent on heterogeneous permeability, levee strength, deformation characteristics and the existence of initial small cracks. It is very difficult to evaluate quantitatively as well as to predict the behavior of hydraulic fracturing.

Hydraulic fracturing usually progresses from the upstream to the downstream side, a phenomenon opposite to regular piping. Hydraulic fracturing has been reported to be the cause of failures of earthen dams; e.g., the Teton Dam in the U.S.A.. The effects of such fracturing are readily observable in laboratory sand model experiments.[9]

(2) Piping and Roofing

The phenomenon in which soil is washed away by seepage forces and the channel advance from downstream to upstream in the form of a pipe is called piping. Channels also may develop two-dimensionally or spherically. There are many kinds of piping owing to various seepage characteristics and soil strength distributions.

Whenever rigid structural elements are placed over a semirigid or erodible foundation, seepage which causes movement of particles from the foundation generally is called 'roofing'.

(3) Soil erosion caused by discontinuity of the soil structure

When water flows through a fine soil layer to a coarse soil region, as in a blind drainage conduit, the fine-grained soil is washed into the pores of the coarse layer by the high seepage force. As a result, channels or loose regions appear along the boundary between the two regions. This phenomenon produces a very strong possibility of piping occurring. Therefore, care must be taken in levee design to ensure that there is no great difference in soil

particle diameter (within reasonable limits) at soil layer boundaries [10],[11]. When a fine soil levee is constructed on a highly permeable coarse layer, there is danger of failure conditions being produced.

3. PROGRESSIVE FAILURES OF RIVER LEVEES CAUSED BY SOIL EROSION

3.1 One-Dimensional Model Experiment of Progressive Erosion

The equation for the critical hydraulic gradient suggested by Terzaghi generally has been used to evaluate the safety of piping, boiling and quick sand. His equation is applicable only to cases in which no shear stress exists and there is only vertical seepage flow in sand. Also, deformation of soil is not considered. These deficiencies have been shown experimentally in many reports.[12]-[19] Because, at present, there is no standard method for establishing the size of a specimen, it is difficult to establish the relation that corresponds to the field. Similarly, much research on the critical hydraulic gradient has been published [16],[19],[20], but it is difficult to obtain a clear index with which to quantitatively evaluate piping and boiling. Therefore, we did experiments, which used two kinds of soil, sand and granite, (grain size distribution is shown in Fig. 4), for various specimen sizes. The permeability coefficient of the sand was 2.31×10^{-2} cm/s and of the granite soil 1.50×10^{-3} cm/s.

(a) Experimental Method

A cylindrical mold with an inside diameter of 19.4 cm and a length of 36 cm was used. A filter which had an open region was placed at the top of this mold. The pressure head at the bottom of the mold was raised at a constant rate to increase the hydraulic gradient. The critical hydraulic gradient (i_c) was the point at which the volume of the outflow from the specimen suddenly became large. The relation of

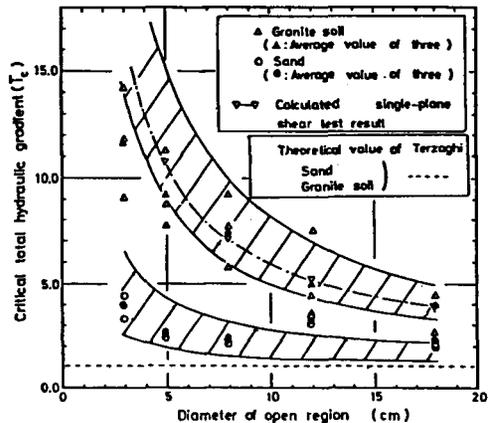


Fig.1 Relation of the diameter of the open region to the critical hydraulic gradient. (sample length 12cm)

the diameter of the open region to the critical hydraulic gradient is shown in Fig.1 for a specimen 12 cm long, and is compared to theoretical values by Terzaghi's method. For sand, i_c has a constant value that is independent of the diameter of the open region, but for granite soil i_c is reduced as the diameter of the open region increases. The relation of the critical hydraulic gradient to specimen length when the diameter of the open region was 8cm is shown in Fig. 2. Clearly, the length of the experimental specimen must be more than 12cm because at a length less than 12cm experimental results were effected by the open region, and the specimen usually failed owing to the seepage force at the critical hydraulic gradient value.

To study changes in the shearing resistance produced by the void ratio, we ran experiments at various void rates and obtained the relation of the void ratio to the critical hydraulic gradient as shown in Fig.3.

(b) Considerations of the experimental results

For the vertical one-dimensional experiment for piping, the critical hydraulic gradient (i_c) can be expressed by the equilibrium condition of the submerged unit weight of the soil and the shearing resistance;

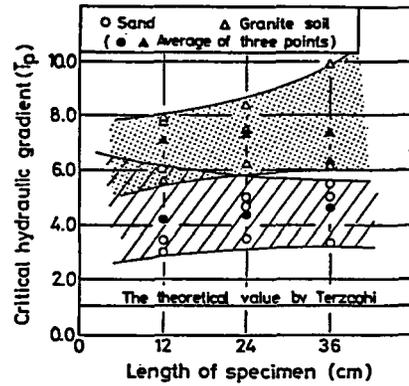


Fig. 2 Relation of the length of the sample to the critical hydraulic gradient.

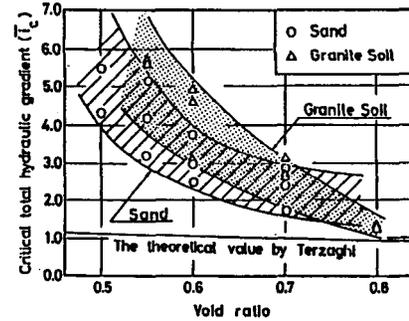


Fig. 3 Relation of the void ratio to the critical hydraulic gradient.

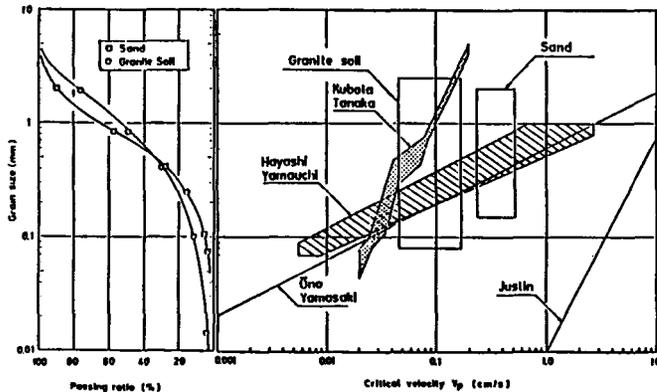


Fig. 4 Relation of the critical velocity to grain size.

$$\bar{i}_c = \frac{\gamma_{sub}}{\gamma_w} + \frac{2c}{\gamma_w \cdot r} = \frac{G_s - 1}{1 + e} + \frac{2c}{\gamma_w \cdot r} \quad \dots (2)$$

in which

- γ_{sub} is the submerged unit weight of the soil,
- γ_w is the unit weight of the water,
- c is the cohesion of the soil,
- r is the radius of the failure region,
- e is the void ratio,
- and G_s is the specific gravity.

When Eq. (2) is compared with experimental results for which the value of the cohesion obtained from the shear test ($c=0$ for river sand and $c=10$ gf/cm² for granite soil) was used (Fig. 1), the calculated values for \bar{i}_c agree well with the experimental values found for granite soil.

Previous results obtained for the relation of grain size to critical velocity are shown in Fig. 4. Piping occurred with liquefaction of grains when the grain size was less than $D_{25} - D_{35}$ as reported by Ohno [20] (D_{25} is the grain size at which 25%, by weight, of the grains are smaller; similarly for D_{35} , D_{60} , etc.). This figure shows that experimentally it was possible to move the fine-grained soil, which explains why boiling may appear intermittently under conditions in which the quick sand phenomenon does not.

3.2 Two-Dimensional Model Experiments of the Progressive Erosion

Horizontal and vertical two-dimensional experiments, used the same river sand and granite soil utilized in the one-dimensional experiment done to study the mechanics of soil erosion caused by seepage. The form of failure and the distribution of pore water pressures were measured, the mechanisms of failure being determined from models made of the leakage from the levee itself or from its foundation.

(a) Horizontal two-dimensional experiment

The apparatus in Fig. 5 shows that piping grows after the appearance of the seepage face. It was made of intensified glass 2 cm thick with a 10 cm open zone on one vertical side of the apparatus. The pressure head on the upstream side was increased at a

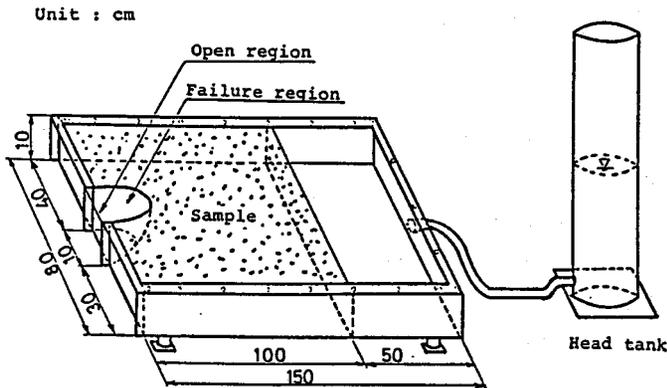


Fig. 5 Experimental apparatus used to measure horizontal two-dimensional erosion.

constant rate of flow from the head tank and the advance of piping measured. Results are shown in Table 1, in which, the average hydraulic gradient is the value for the head of the upstream side divided by the length of the specimen before the experiment was begun. Representative cases of failure progression are shown in Fig. 6 and 7. We observed that the granite soil specimens failed in the form of pipes because of the effect of cohesion.

(b) Vertical two-dimensional experiment

The apparatus for the vertical experiment was the same as for the horizontal experiment soil (Fig. 8). Results are shown in Table 2. As to the type of soil failure, local failure first occurred in one part of the seepage face, the soil from that part being washed away. This local soil failure was repeated, causing piping to progress inward on the upstream side. The channel of sand soil maintained its shape by arch action, but this force was not very strong, and the soils forming the ceiling of the channel readily collapsed, after which the failure face advanced inward on the upstream side (Fig. 9).

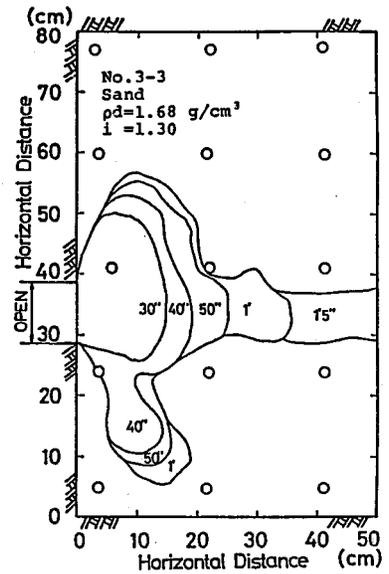


Fig. 6 Horizontal erosion. (Sand)

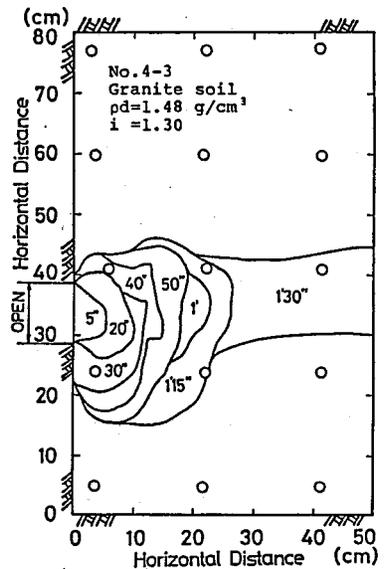


Fig. 7 Horizontal erosion. (Granite soil)

(c) Erosion Experiment with the Slope Model

Three types of model levee experiments were done to study progressive failure. The models used were a homogeneous levee, a three-layer levee and leakage from a levee foundation (Table 2).

(1) The homogeneous levee (CASES H-1 - H-4)

The experimental model of a homogeneous levee was made with the apparatus used in the vertical two-dimensional experiments. The water level at the upstream boundary was raised slowly. A partial soil slide, which took place on the seepage face, advanced progressively upstream. When it reached the crown of the levee, overflow occurred after which the levee failed (Fig. 10). In the slope model experiment, in which the gradient was 25 degrees (nearly equal to the angle of repose), seepage water flowed down the slope, but the model remained stable.

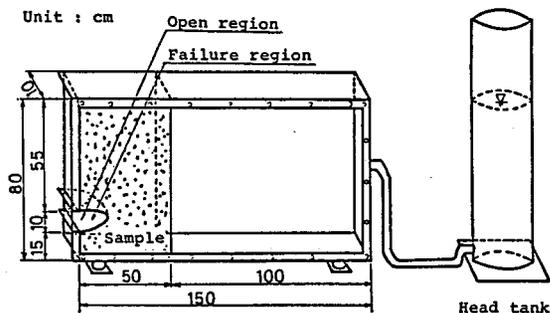


Fig. 8 Experimental apparatus used to measure vertical two-dimensional erosion.

Table 1 Experimental results for horizontal two-dimensional erosion.

Case (Sample)	Dry density (g/cm ³)	Total hydraulic gradient (i)	Time of complete failure	Form of failure *
(sand)				
3-1	1.55	1.60	15"	b
3-2	1.68	1.60	30"	-
3-3	1.68	1.30	1'00"	b
3-4	1.66	0.70	1'45"	b
3-5	1.52	0.40	-	d
3-6	1.68	0.40	-	d
3-7	1.56	0.80	1'15"	b
3-8	1.67	0.80	3'00"	b
3-9	1.69	0.80	10'00"	b
3-10	1.61	0.31	-	d
(Granite sand)				
4-1	1.69	2.60	6'45"	b
4-2	1.69	1.30	-	d
4-3	1.48	1.30	1'20"	b
4-4	1.48	0.50	-	d
4-5	1.54	1.30	-	d
4-6	1.50	1.30	-	c
4-7	1.45	1.30	-	a
4-8	1.44	1.30	-	a
4-9	1.50	0.80	-	d
4-10	1.47	0.80	-	c
4-11	1.38	0.80	1'00"	b
4-12	1.34	0.35	3'00"	-

- * a: Sudden total failure.
- b: Local failure spread like a fan and became total failure.
- c: Local failure advanced upstream like pipes.
- d: Local failure did not advance.

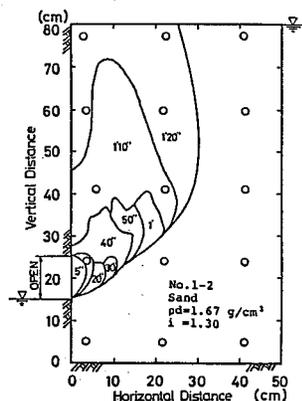


Fig. 9 Vertical erosion. (Sand)

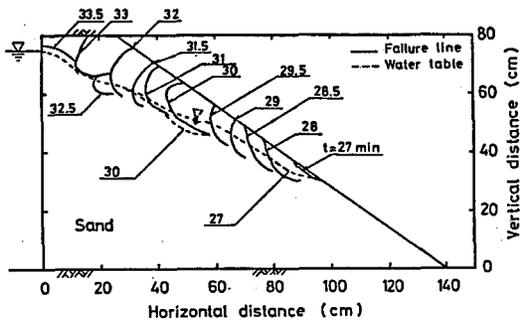


Fig. 10 Results of the sand slope experiment.

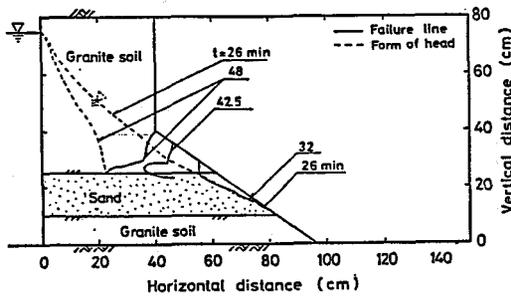


Fig. 11 Model of the layered levee experiment.

Table 2 Experimental results for vertical two-dimensional erosion.

Case (Sample)	Dry density (g/cm ³)	Total hydraulic gradient (i)	Time of complete failure	* Form of failure
(sand)				
1-1	1.56	1.30	20"	a
1-2	1.67	1.30	1'20"	a
1-3	1.74	1.30	2'00"	b
1-4	1.68	0.50	14'20"	b
1-5	1.70	0.25	—	c
1-6	1.55	0.25	—	c
(Granite soil)				
2-1	1.35	1.30	30"	a
2-2	1.40	1.30	40"	a
2-3	1.50	1.30	1'45"	a
2-4	1.57	1.30	8'45"	b
2-5	1.60	1.30	5'00"	b
2-6	1.62	1.30	—	c
2-7	1.68	1.30	—	c

* a:Sudden total failure.

b:Local failure advanced, and piping appeared.

c:Local failure did not advance.

Table 3 Results of the seepage experiment.

Case	Gradient of slop (°)	Dry density (t/m ³)	Water-level (cm)	Wide (cm)	Length (cm)	Initial water content (%)	Note
H-1	35	1.66	75			7.7	—
H-2	35	1.63	75			9.7	Tsukinowa
H-3	25	1.68	50			9.2	—
H-4	25	1.68	50			6.6	Tsukinowa
L-1	35	1.63	50	15	62	10.6	—
L-2	35	1.63	50	15	62	7.0	Tsukinowa
L-3	25	1.63	45	10	75.5	9.2	—
L-4	25	1.63	99	10	75.5	8.5	—
L-5	25	1.63	140	10	75.5	6.7	—
B-1	35	1.63	55	20	54.3	9.2	—
B-2	35	1.63	55	20	111.4	6.8	—

H:Homogeneous levee

L:Three-layer levee

B:Leakage from the levee foundation

(2) The three-layered levee (CASES L-1 - L-5)

A three-layered levee model was constructed of sand and granite soil as shown in Fig. 11. In L-1 and L-5, the levee failed because of piping in the sand layer, but in L-3 and L-4 it did not fail. Fig. 11 shows that after the appearance of the seepage face, the sand layer failed up to the angle of saturated repose (24-25 degrees) and roofing took place along the boundary face between the sand and granite soil layers. The levee failed successively because of the outflow of the granite soil layer. Failure advanced along the upstream side of seepage just like piping. When the critical hydraulic gradient was reached the levee failed because boiling occurred intermittently along the boundary face between the sand and granite soil layers.

3.3 Theoretical Considerations

(a) Evaluation of the experiments results

In the experimental model, in which the direction of the seepage flow was horizontal, failure occurred at the critical hydraulic gradient reported by Terzaghi. In the model, in which the direction of seepage flow was vertical, failure took place at nearly the critical hydraulic gradient of Terzaghi. The critical hydraulic gradient for horizontal seepage flow has yet to be definitively determined. At a value greater than the critical velocity proposed by Ohno, failure can be considered to progress rapidly because of the outflow of soil. Clayey soil such as granite soil fails when the critical velocity of the clay and silt fraction is low, and such fine-granite fractions are easily washed out at a low velocity. When the pores on the downstream side of the seepage become filled as the fine-granite fraction flows out, the water pressure increases greatly along the seepage face, and intermittent block failure occurs. This is why levee failure takes place.

(b) Considerations based on the seepage theory

Taking into account the two-dimensional half circular seepage region in Fig.12, and assuming that the seepage point is enlarged to a half circular channel, the hydraulic gradient (i_0) is

$$i_0 = (H - h_0) / \{r_0 \ln(R/r_0)\} \dots\dots (3)$$

in which $r=r_0$, $h=h_0$ on the surface of the channel, and $r=R$, $h=H$ at the source point.

Assuming water flow in the three-dimensional half spherical seepage region shown in Fig. 12 , the following equation is obtained;

$$i_0 = (H - h_0) / (r_0 - r^2/R) \dots\dots (4)$$

These two equations are shown in Fig. 13. As the seepage force is proportional to the hydraulic gradient, special features can be seen from Fig. 13. When circular or spherical soil erosion increases in the half circular, or half spherical, seepage region, the seepage force decreases with the enlargement of the region of erosion. As erosion advances around the center point of the distance between the seepage and source points, the seepage force is minimal. But, as erosion advances further into the levee, the seepage force again increases and leads to levee failure.

In the two-dimensional seepage region shown in Fig. 14, when linear piping reached point S(AB) in the region of L width, the results shown in Fig. 15 were obtained by calculating the seepage discharge (Q) and the average hydraulic gradient (i_0) of AB. These results show that the phenomenon of piping has the following special features: There is rapid advancement in the initial phase, after

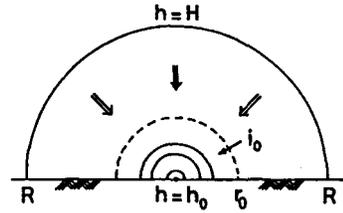


Fig. 12 Erosion enlargement.

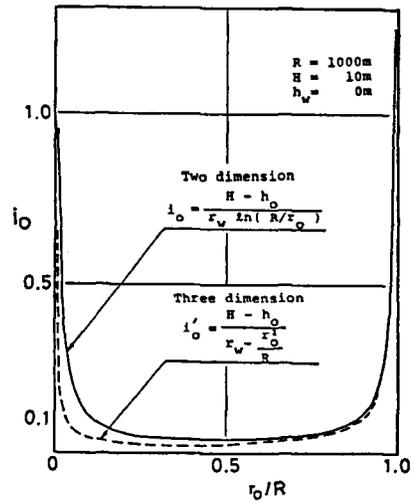


Fig. 13 Change in the hydraulic gradient as failure progresses.

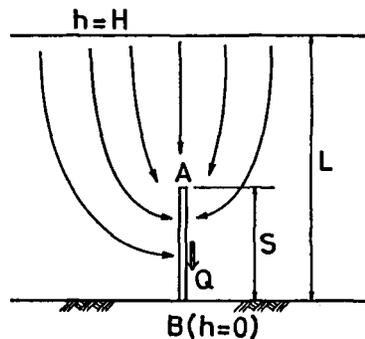


Fig. 14 Piping model of the two-dimensional seepage region.

which the speed of piping becomes reduced. Piping speed again increases and reaches the upstream side of the seepage. This characteristic of piping presents many problems for quantitative determinations because the distribution of the rate of seepage discharge depends on the initial distribution of the pressure head before the soil is eroded, as well as on the change in that distribution caused by the advancement of soil erosion. The above explanation is understood to give the qualitative index.

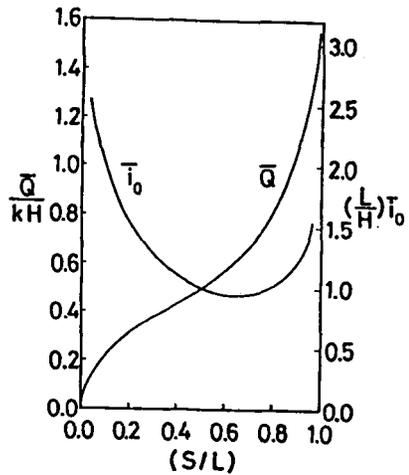


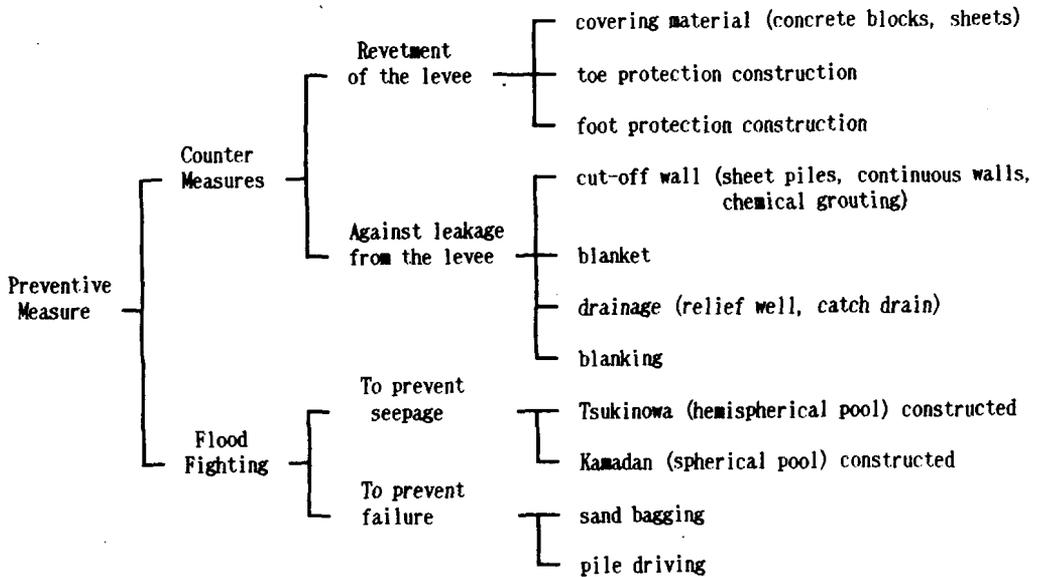
Fig. 15 Relation of (S/L) to \bar{Q} , \bar{I}_0 .

4. MEASURES TO PREVENT LEVEE FAILURE CAUSED BY SEEPAGE AND LEAKAGE

4.1 Preventive Measures and Their Characteristics

Current preventive measures are listed in Table 4. These are broadly divisible into two types; counter measures taken in advance at dangerous places before floods occur and emergency flood-fighting measures taken to prevent a levee from cracking or from boiling after flooding. For the preventative measures, the basic premise is to protect against river seepage and to drain off seepage water before it reaches the land side. Therefore, the river side slope generally should be covered with an impermeable material such as asphalt or cement blocks; whereas, the land side slope should be covered with permeable material. But, because the ground water level sometimes reaches a dangerous level owing to infiltration of rain or overflow water, it has been suggested that the entire surface of the levee should be covered with soil of low permeability or with asphalt. But if this is done, once seepage water flows into the levee the water pressure will greatly increase and failure occur. Moreover, should the water level in the river fall suddenly, the levee will fail because of residual pore water pressure. Therefore, before adopting prevention measures, it is necessary to examine the behavior of seepage flow due to flooding.

Table 4 Preventive measures against levee failure caused by seepage and leakage.



4.2 Counter Measures

(a) Cut-off wall method

There are several ways to prevent leakage through the levee's foundation (Table 4). For example, sheet piles can be driven in to cut off seepage, or an impervious or semi-impervious membrane (called a blanket) can be used on the river side to make the seepage path a long one. It is most important to determine the most suitable depth of penetration of the sheet piles or the length of the blanket when investigating the permeability of a levee.

(b) Drainage ditch method

At the land side toe, a drainage ditch, relief well and catch drain can be constructed to decrease the water pressure in a levee. These are very effective ways to prevent the boiling and piping caused by leakage from a levee foundation. But when using these measures, we must consider the capacity of the drainage pump and construct a filter layer to prevent soil erosion. Furthermore, constant attention must be paid to maintenance of the drainage facilities.

4.3 Flood Fighting Methods

Representative current flood fighting methods are the Tsukinowa

(half circular pool) and Kamadan (circular pool) methods. In terms of basic engineering, these methods serve the same purpose, which is to decrease the seepage face on the land side of a levee. These methods have been used many times, but few studies have been done that explain and verify their effectiveness; therefore, we studied the Tsukinowa method and its application experimentally.

(a) Experimental study of the Tsukinowa method

(1) Homogeneous levee model

For the failed experimental case in Table 3, we established a Tsukinowa to prevent the type of failure shown in Fig. 16. The model failed, however, because the head in the levee became high and the sand under the Tsukinowa was washed away. We concluded that the efficacy of the Tsukinowa is not as great as expected for a homogeneous type of levee.

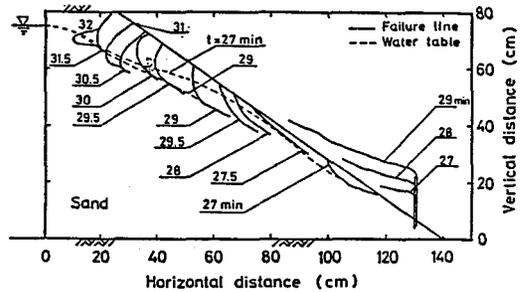


Fig. 16 Experimental results; the Tsukinowa method. (Homogeneous levee model)

(2) Three-layer levee model

For the failed model in Fig. 11, we established a Tsukinowa against the type of failure shown in Fig. 17. As the water level rose in the Tsukinowa, the rate of seepage and the hydraulic gradient decreased. Even when there was a highly permeable layer, the seepage velocity became large and local failure occurred, the soil was not washed away because of the presence of the Tsukinowa. Therefore, construction of a Tsukinowa is an effective preventive measure against failure of a layered levee.

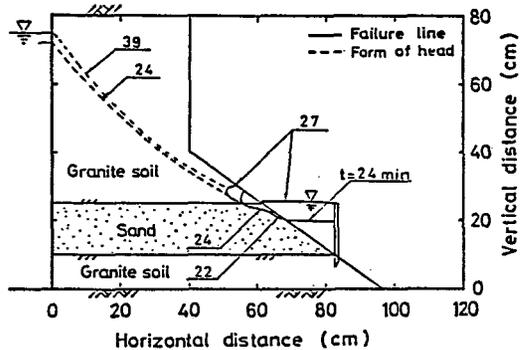


Fig. 17 Experimental results; the Tsukinowa method. (Layered levee model)

(b) Evaluation

The Tsukinowa method usually is adopted for a steep slope and

the Kamadan method for a gentle slope, both being set at the point of leakage caused by flooding, but a significant decrease in the seepage force can not always be obtained with these measures. Their effects are

(1) to decrease the hydraulic gradient by raising the pressure head on the downstream side of seepage

(2) to prevent the concentration of seepage flow at the local failure point

(3) to inhibit the outflow of soil

To prevent piping failure from spreading, permeable heavy materials should be placed (e.g., gravel in Fig. 18) in the Tsukinowa.

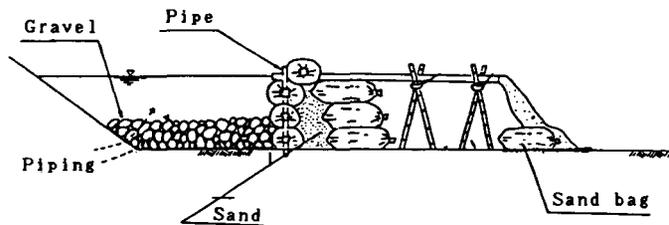


Fig. 18 A Tsukinowa constructed with gravel.

5. CONCLUSIONS

We have here reported the prediction and prevention of river levee failures caused by seepage and leakage due to flooding. We concluded that are

1) in determining whether there is progressive failure of a levee, the effect of cohesion at the critical hydraulic gradient and the process of progressive failure can be shown by one- and two-dimensional model experiments of piping and erosion. Our experimental results agreed qualitatively with results obtained by the seepage theory.

2) results of a vertical one-dimensional experiment showed that the length of the experimental specimen must be more than 12cm, because at less than 12cm, our experimental results were effected by the open region and specimen failed at a seepage force below the critical hydraulic gradient.

3) results of our horizontal two-dimensional experiment showed that cohesive soil specimens composed of granite soil failed in the form of 'pipes'.

4) based on the type of failure seen in the vertical two-dimensional experiment, local soil failure in one part of the seepage face was repeated and piping advances inward on the upstream side.

5) the results of the homogeneous levee model experiment showed that at a gradient of a levee less than 25 degrees (nearly equal to the angle of repose) a levee remains stable, and seepage water flows down the slope.

6) our three-layer levee model made of sand and granite soil showed that once the seepage face appears the sand layer will fail up to the angle of saturated repose after which roofing will take place along the boundary face between sand and granite soil layers.

7) the Tsukinowa method, the most common Japanese flood-fighting measure does not always prevent the outflow of soil from the levee. The need for heavy permeable materials placed in the tsukinowa is indicated.

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