

# *Simultaneous Optimization of Production and Transportation Planning by using Logic Cut Algorithm*

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Various kind of productions are made in semiconductor factories, where it employs the production system with multiprocess and multiple Automated Guided Vehicles(AGVs) for transportation. It is difficult to optimize planning of production and transportation simultaneously because of the complicated flow of semifinished products. This paper describes the formulations of production scheduling and transportation routing, and algorithm for simultaneous optimization of plannings by using logic cuts. The entire problem is decomposed to the master problem and the sub problem. If it derives the infeasible solutions, new constraints are added to the master problem to eliminate the solution area including infeasible solutions. The results of about optimality and computation time by using CPLEX solver are shown compared with conventional decomposition method to check up effectivity of proposed method in small size problem, and about optimality and computaion time for large scale problem.

## 1 Introduction

In semiconductor factories, where severe competitions between other makers exist, decreasing of operation energy and keeping environment are required from society running its prduction line effectively.

In the semiconductor factory, Automated Guided Vehicles(AGVs) transport the semifinished products between the processes. The method, Constraint-Based Genetic Algorithm(CBGA) to handle a complex variety of variables and constraints in a typical FMS-loading problem, is proposed [1]. A hybrid algorithm based on tabu search and simulated annealing is employed to solve the FMS-loading problem [2]. To utilize the bottleneck machines at the maximum level, vehicle dispatches are decided using by a vehicle dispatching procedure based on the theory of constraints [3]. Ant colony optimisation-based software system is proposed to solve FMS scheduling in a job-shop environment with routing flexibility, sequence-dependent setup and transportation time [4]. Realtime scheduling method by using genetic machine learning and reactive

scheduling method are proposed to optimize production scheduling and transportation routing simultaneously [5]. To attain optimum transportation route, an autonomous distributed route planning method [6] [7] is proposed. This method is that each AGV plans its own route, communicate each other and plan no conflict route. To attain optimum dispatching and transportation routing simultaneously, the hybrid method by using constraint programming mixed integer linear programming and cutting method if infeasible solution solved, is proposed [8]. In the decomposition method, production system, tranportation system and handling plan its own schedule and exchange their plans, is proposed [9]. In the past, the simultaneous optimization problem of production schedule and transportation routing was rarely treated because there are many complex variables of FMS needed to decide. In this paper, a method that can solve the approximate solutions for short calculation time is proposed. Using the proposed method, is the entire problem, with production schedule problem and transportation routing problem, is divided to master and sub problems. Then by generating cuts to the area including infeasible so-

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lution of the entire problem, it can derive a feasible solution. Generating cuts is that the constraint to add the master problem based on the logics preliminariy. This method using logic cuts is expected to derive the solutions efficiently because of eliminating the area including infeasible solutions.

## 2 The Problem of Production and Transportation Planning

This chapter indicates production and transportation planning problem description and formulation as an integer linear problem.

### 2.1 Problem Description

The object of this research is FMS that is the construction of number of production processes and two-dimensional transportation system. In the following, production process will be stated. The number of Jobs, processes and AGVs are decided previously. It is impossible to transport until process finished after the process machine begin to start. It is impossible to transport more than 2 products by one AGV. Process span includes setup span of processing. Production's setup time is disregard. Sequence of productions process is given beforehand. Fig.1 indicates transportation system. Node is the place that AGV can stop or turn, and Edge, AGV's route, is connecting route between nodes.

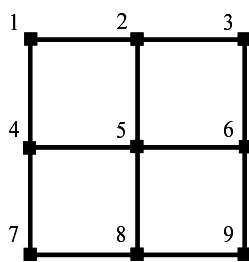


Fig. 1: Transportation Model

To digitize, transportation routing problem is defined below. All of the length of edges are assumed to be equal. AGV's velocity are set to be equal. AGV can stop or turn only on nodes. Transportation requests rise on nodes. And to avoid collusions, following constraints are added.

- Two AGVs cannot exist on a node in the same time period.(Constraint for collision avoidance on node)
- Two AGVs cannot travel on an edge at the same time period.(Constraint for collision avoidance on edge)

And each AGV must synchronize with production processing, so AGV must reach the reserved node until the schedule time is over. Fig.2 is the example of gantt chart. In Fig.2, the brack borders is the situation of the production process and AGV. The internal number of the brack borders is the products number. Horizontal axis is time axis. Production scheduling problem

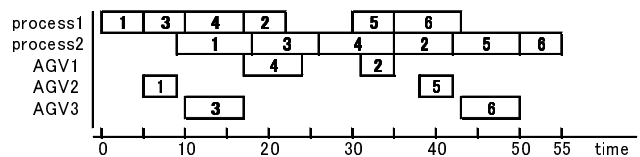


Fig. 2: Example of Gantt chart

determine the process turn and start time of processing product in the each processes. It may happen late in reach to reserved node if there isn't enough time to transport even if production is optimal. And if there is much time to transport, it may extend the makespan because of waiting time is longer. Transportation problem determine the assignment of request to AGVs and their routing. Depending to AGV routing, it may extend the assumed time to transport. So it may influence the production schedule. And depending on the assignment of request to AGV, it may prolong the time to transport. So it is needed to determine good assignment. In this research, the problem of simultaneous optimal production and transportation plannings is to decide the production scheduling and transportation routing to minimize the makespan.

### 2.2 Problem Formulation

In this section, problem formulation will be described.

#### 2.2.1 Constants and Variables

It indicates the constants and variables used in this paper.

- constant

- $R$  : set of jobs
- $P$  : set of production processes
- $M$  : large number
- $L_i^l$  : span of processing product  $\#i$   
in process  $\#l$
- $A$  : span to transport next request
- $N$  : set of nodes
- $N_i$  : set of nodes which directly  
connect to node  $i$
- $V$  : set of AGV
- $R_T$  : set of transportation request
- $H$  : time horizon
- $S_k$  : initial node of AGV  $\#k$
- $S_r$  : loading node of transport request  $\#r$
- $F_r$  : unloading node of transport request  $\#r$

- variable

- $s_i^l$  : the start time of job  $\#i$  in process  $\#l$
- $T_i$  : span of transport request  $\#i$
- $V_r$  : AGV number to transport request  $\#r$
- $t_{S_r}$  : start time of transportation request  $\#r$
- $t_{F_r}$  : finish time of transportation request  $\#r$

The span  $t$  defines from the time  $(t - 1)$  to the time  $t$ . The explanation of binary variables is below.

The definition of binary variable  $\epsilon_{i,j}^l$  that indicate precedence relation about job  $\#i$  and job  $\#j$  in the process  $\#l$ , is below.

$$\epsilon_{i,j}^l = \begin{cases} 1 & (\text{In process } \#l, \text{ job } \#i \text{ precedes job } \#j) \\ 0 & (\text{other}) \end{cases}$$

The binary variable  $\gamma_{i,j,k}$  that indicate precedence relation about transportation request  $\#i$  and  $\#j$  to assigned AGV  $\#k$  is defined below.

$$\gamma_{i,j,k} = \begin{cases} 1 & (\text{At AGV } \#k, \text{ request } \#i \text{ precedes } \#j) \\ 0 & (\text{other}) \end{cases}$$

The binary variable  $\delta_{i,k}$  that indicate transportation request  $\#i$  assign to AGV  $\#k$ , is defined below.

$$\delta_{i,k} = \begin{cases} 1 & (\text{when transportation request } \#i \\ & \text{assign to AGV } \#k) \\ 0 & (\text{other}) \end{cases}$$

The variable  $x_{i,j,t}^k$ , that indicate the condition of AGV's movement, is defined below.

$$x_{i,j,t}^k = \begin{cases} 1 & (\text{AGV } \#k \text{ moves from node } i \text{ to } j \\ & \text{in span } t) \\ 0 & (\text{other}) \end{cases}$$

The variable  $\eta_{r,t}^k$ , that indicate the condition of transportation request  $\#r$ , is defined below.

$$\eta_{r,t}^k = \begin{cases} 1 & (\text{When AGV } \#k \text{ transports request } \#r \\ & \text{in span } t) \\ 0 & (\text{other}) \end{cases}$$

### 2.2.2 Problem Constraints

When job  $\#i$  finishing processing in the process  $\#l$  is transported to the reserved process, the start time of transportation  $t_{S_i}$  must be later than the completion of process in the process  $\#l$ . The finish processing time is sum of  $s_i^l$ , that is the variable of the start time of process in the process  $\#l$ , and  $L_i^l$  that is the variable of the span. So the relation of  $s_i^l$  and  $L_i^l$  is given formulation(1).

$$t_{S_i} \geq s_i^l + L_i^l \quad (\forall i \in R; \forall l \in P) \quad (1)$$

When job  $\#i$  is processed in the next process  $\#(l + 1)$ , the variable of start time of process  $s_i^{l+1}$  must be later than the finish time of transportation. Because the finish time of transportation is related to the start time of transportation and the span, the start time of process in the next process  $s_i^{l+1}$ , the start time of transportation  $t_{S_i}$  and the span  $T_i$  are given formulation(2).

$$s_i^{l+1} \geq t_{S_i} + T_i \quad (\forall i \in R; \forall l \in P) \quad (2)$$

The relation between the finish time of transportation of job  $\#i$ ,  $t_{F_i}$  and the start time of process in the process  $s_i^{l+1}$  are given formulation(3).

$$s_i^{l+1} \geq t_{F_i} \quad (\forall i \in R; \forall l \in P) \quad (3)$$

The decision variable  $\epsilon_{j,i}^l$  indicate the process precedence about job  $\#i$  and  $\#j$  in the process  $\#l$ . The one is decided, the other is decided simultaneously, so the constraint is given formulation(4).

$$\epsilon_{i,j}^l + \epsilon_{j,i}^l = 1 \quad (\forall i, \forall j \in R; \forall l \in P) \quad (4)$$

Job  $\#i$  is transported to the reserved process after finishing processing. So to transport by AGV, they must

be assigned. The constraint formulation(5) is added below.

$$\sum_{k \in V} \delta_{i,k} = 1 \quad (\forall i \in R; \forall k \in P) \quad (5)$$

If the transportation request  $\#i$  and  $\#j$  aren't assigned to AGV  $\#k$ , AGV  $\#k$  need not to regard them. So the variable  $\gamma_{i,j,k}$  that indicate the precedence of process sequence about request  $\#i$  and  $\#j$ , are assigned to AGV  $\#k$ , and variable  $\delta_{i,k}$  indicated the situation about the assignment to AGV  $\#k$  are given formulation(6), (7).

$$\gamma_{i,j,k} \leq \delta_{i,k} \quad (\forall i, \forall j \in R; \forall k \in P) \quad (6)$$

$$\gamma_{i,j,k} \leq \delta_{j,k} \quad (\forall i, \forall j \in R; \forall k \in P) \quad (7)$$

When the variable  $\gamma_{i,j,k}$ , that indicate the precedence about transportation request  $\#i$  and  $\#j$ , is decided, the variable of start time of transportation,  $t_{S_i}$  and  $t_{S_j}$ , are decided. The relation of  $\gamma_{i,j,k}$ ,  $t_{S_i}$  and  $t_{S_j}$  is given below.

$$\gamma_{i,j,k} = \begin{cases} 1 & (t_{S_j} \geq t_{S_i} + T_i + A) \\ 0 & (t_{S_i} \geq t_{S_j} + T_j + A) \end{cases}$$

The constraint about route planning of AGV  $\#k$  is given below.

$$\sum_{j \notin N_i} x_{i,j,t}^k = 0 \quad (k \in V, i \in N; t = 1, \dots, H) \quad (8)$$

$$\sum_{j \in N_i} x_{i,j,t}^k \leq 1 \quad (k \in V, i \in N; t = 1, \dots, H) \quad (9)$$

$$\sum_{j \in N_i} x_{j,i,t}^k = \sum_{n \in N_i} x_{i,n,t+1}^k \quad (10)$$

$$(k \in V, i \in N; t = 1, \dots, H - 1)$$

$$\sum_{j \in N_{S_k}} x_{S_k,j,0}^k = 1 \quad (k \in V) \quad (11)$$

The formulation(8) indicates that AGV  $\#k$  cannot travel from node  $i$  to node  $j$  which is not directly connected to node  $i$ . The formulation(9) indicates that AGV  $\#k$  can take only one edge in a same time span. The formulation(10) indicates the time continuity constraints of the movement of AGVs. On span  $t$ , it indicates that AGV  $\#k$  in node  $i$ , can move only the node connected node  $i$ . The formulation (11) indicates the initial condition of the place of AGV.

The constraint of AGV not to collide is given below.

$$\sum_{k \in V} \sum_{j \in N} x_{j,i,t}^k \leq 1 \quad (i \in N; t = 1, \dots, H) \quad (12)$$

$$\sum_{k \in V} (x_{i,j,t}^k + x_{j,i,t}^k) \leq 1 \quad (13)$$

$$(i \in N, j \in N_i; t = 1, \dots, H)$$

The formulation(12) indicates that more than one AGV cannot travel from a node to the connected node in a same time span, The formulation(13) indicates that more than one AGV cannot travel on an edge in a same time span. The constraints about the variables of start time of transportation request  $\#r$   $t_{S_r}$ , the finish time  $t_{F_r}$ , the start node of transportation node  $S_r$  and the goal node of transportation node  $F_r$  are the formulation (14) and (15).

$$\sum_{j \in N_{S_r}} x_{S_r,j,t_{S_r}}^{V_r} = 1 \quad (14)$$

$$\sum_{i \in N_{F_r}} x_{i,F_r,t_{F_r}}^{V_r} = 1 \quad (15)$$

The formulation(14) indicates that AGV  $\#V_r$  assigned the request  $\#r$  must reach the start node  $S_r$  at the start time  $t_{S_r}$ . The formulation(15) indicates that AGV  $\#V_r$  assigned the request  $\#r$  must reach the goal node  $F_r$  at the goal time  $t_{F_r}$ . The constraint of each AGVs are limited products to transport at the same time are given formulation(16).

$$\sum_r \eta_{r,t}^k \leq 1 \quad (16)$$

The formulation(16) indicates the constraint which AGV  $\#k$  can transport only one request on the span  $t$ .

### 2.2.3 Objective function

In this paper, the objective function aims to minimize the makespan that imply all requests finish their processing.

$$\min Q \quad (17)$$

$$Q = \max\{s_i^l + L_i^l\} \quad \forall i \in R, \forall l \in P \quad (18)$$

## 3 Simultaneous Optimization of Production and Transportation planning using Logic Cut

In this chapter, the entire problem, consisting of production schedule and transportation routing, is divided into the master problem and the sub problem. This paper indicates the algorithm using the their decomposition method by logic cut.

### 3.1 Logic Cut

Today CPLEX solver is using the algorithm of branch and bound approach. But calculation time is increasing by using this algorithm. So the current CPLEX is using the Gomory cut to short. Briefly, it indicates the explanation of the Gomory cut. It gets the lower bounds from the linear problem that is approximated the Mixed Integer Linear Problem(MILP). Almost of the case, this lower bound is indicated by real number not integer. So, to get integer solution in Fig.3, using the lower bound, that is real number, it generates the Gomory cut to cut the area including the lower bound and gets the optimal solutions. The basis of logic cut proposed in this paper can get the feasible solution by cutting the area including the infeasible solution likely gomory cut. It gets the solution by solving the master problem, and check it which the solution is feasible or infeasible by solving the sub problem. If it is infeasible, to eliminate the area including it, it can get the feasible solution by generating the cut based on the logic set preliminarily. Logic is defined from the method to solve the various problem that happen actually. For example, the one is that extending transportation span to enable to transport. In this research, logic to solve the problem, that may happen actually, are defined preliminarily. In the Fig.4, it indicates the flow of solving the feasible solution by logic cut.

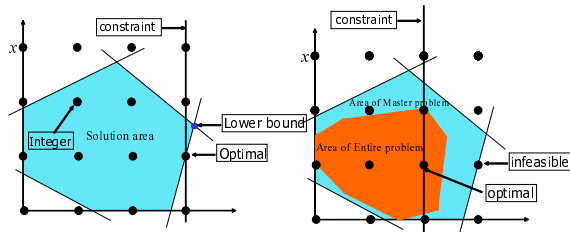


Fig. 3: The image of Gomory cut

Fig. 4: The image of logic cut

### 3.2 Decompose to Master Problem and Sub Problem

Production schedule and transportation routing problem are described in the following.

- Production Schedule Problem
  - Decision of production turn in the each processes

- Decision of the start/finish time of production process
- Transportation Routing Problem
  - Decision of the start/finish time of transportation request
  - Decision of the assignment of request to AGV
  - Decision of the transportation routing

In this paper, the entire problem including production schedule and transportation routing is divided into master and sub problems. These indicates below.

- Master Problem
  - Decision of production turn in the each process
  - Decision of The start/finish time of production process
  - Decision of the start/finish time of transportation request
  - Decision of the assignment of request to AGV
- Sub Problem
  - Verdict of feasible of transportation routing

It is needed to set the production schedule to decide the transportation routing which AGV transport the products and where process. So the master problem decide the production schedule, the assignment of transportation request and the start/finish time of transportation request simultaneously. Sub problem check the possibility of transportation based on the production plan decided in the master problem.

### 3.3 Simultaneous Optimization of Production and Transportation Planning using Logic Cut Algorithm

The flow chart of the simultaneous optimal algorithm of production schedule and transportation routing using logic cut is shown in Fig.5. Steps in Fig.5 are described in the following.

#### Step1 Master Problem: Production Schedule and Assignment of Request to AGV

To minimize the makespan, it decides the production schedule and assignment of request and the start/finish time of transportation based on the required products,

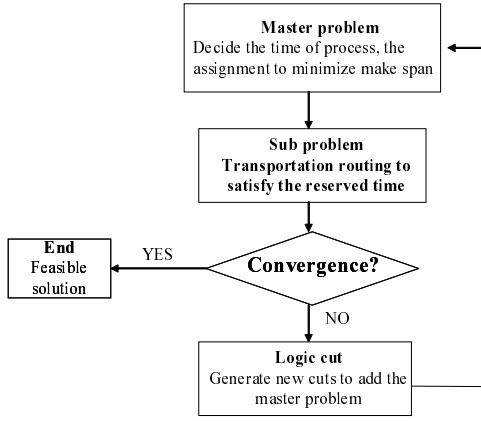


Fig. 5: The algorithm using logic cut

the span of products to process and the span to transport. The span to transport equal to the minimum span to move the goal node from start node. The master problem is formulated as Mixed Integer Linear Programming, and solve the optimal solution. In this research, it takes one span to move neighbor node and the minimum span to transport is defined by the formulation(19).

$$\min T_r = \sum_t x_{S_r, F_r, t}^k \quad (19)$$

### Step2 Sub Problem:Transportation Routing

Based on them decided by master problem, it decides the transportation route to reach the reserved node without late from the reserved time. It uses the autonomous decomposition optimal distributed routing method[6] regarded the start/finish time to transport. This method is that it add the penalty, calculated from the delay span between the reserved time  $T_k$  and the now time  $T_{due}^k$ , to objective function using transportation routing if AGV don't reach the reserved node till the reserved time.

$$D_k = \max\{0, T_k - T_{due}^k\} \quad (20)$$

### Step3 Convergence

If the transportation routing in Step2 can be decided the transportation routing without delay, calculation is finished. Even if it can not plan the delay routing, go to Step4.

### Step4 Cut

To eliminate the area including the infeasible solution,

it generates the cut about extension of transportation span, the change of assignment of transportation request and change of turn process.

### 3.3.1 Master Problem

The master problem decides the start/finish time of products in the processes, the assignment of request to AGV and the start/finish time of transportation. In this paper, it solves the master problem by using the formulation about production schedule and transportation routing that are formulated in Chapter 2.

At first, the formulation about the constraint of production schedule and assignment of transportation. The formulation are described from formulation (21) to (29) as follows.

$$t_{S_i} \geq s_i^l + L_i^l \quad (\forall i \in R; \forall l \in P) \quad (21)$$

$$s_i^{l+1} \geq t_{S_i} + T_i \quad (\forall i \in R; \forall l \in P) \quad (22)$$

$$s_i^{l+1} \geq t_{F_i} \quad (\forall i \in R; \forall l \in P) \quad (23)$$

$$\epsilon_{i,j}^l + \epsilon_{j,i}^l = 1 \quad (\forall i, \forall j \in R; \forall l \in P) \quad (24)$$

$$\sum_{k \in V} \delta_{i,k} = 1 \quad (\forall i \in R; \forall k \in P) \quad (25)$$

$$\gamma_{i,j,k} \leq \delta_{i,k} \quad (\forall i, \forall j \in R; \forall k \in P) \quad (26)$$

$$\gamma_{i,j,k} \leq \delta_{j,k} \quad (\forall i, \forall j \in R; \forall k \in P) \quad (27)$$

$$t_{S_j} \geq t_{S_i} + T_i + A \quad (28)$$

$$\text{when } i \prec j \quad (\forall i, \forall j \in R)$$

$$t_{S_i} \geq t_{S_j} + T_j + A \quad (29)$$

$$\text{when } j \prec i \quad (\forall i, \forall j \in R)$$

Following formulations are rewritten become because there isn't regarded assignment of request  $\#i$  and  $\#j$  to AGV  $\#k$  about the constraint formulation(28) and (29) that precedence  $\#i$  and  $\#j$  that assigned AGV  $\#k$ .

$$t_{S_j} + M(1 - \gamma_{i,j,k}) + M(2 - \delta_{i,k} - \delta_{j,k}) \geq t_{S_i} + T_i + A \quad \text{when } i \prec j \quad (30)$$

$$t_{S_i} + M\gamma_{i,j,k} + M(2 - \delta_{i,k} - \delta_{j,k}) \geq t_{S_j} + T_j + A \quad \text{when } j \prec i \quad (31)$$

### 3.3.2 Sub Problem

At first, the formulations of sub problem are indicated from (32) to (39).

$$\sum_{j \notin N_i} x_{i,j,t}^k = 0 \quad (k \in V, i \in N; t = 1, \dots, H) \quad (32)$$

$$\sum_{j \in N_i} x_{i,j,t}^k \leq 1 \quad (k \in V, i \in N; t = 1, \dots, H) \quad (33)$$

$$\sum_{j \in N_i} x_{j,i,t}^k = \sum_{n \in N_i} x_{i,n,t+1}^k \quad (34)$$

$$(k \in V, i \in N; t = 1, \dots, H-1)$$

$$\sum_{j \in N_{S_k}} x_{S_k,j,0}^k = 1 \quad (k \in V) \quad (35)$$

$$\sum_{k \in V} \sum_{j \in N} x_{j,i,t}^k \leq 1 \quad (i \in N; t = 1, \dots, H) \quad (36)$$

$$\sum_{k \in V} (x_{i,j,t}^k + x_{j,i,t}^k) \leq 1 \quad (37)$$

$$(i \in N, j \in N_i; t = 1, \dots, H)$$

$$\sum_{j \in N_{S_r}} x_{S_r,j,t}^{V_r} = 1 \quad (38)$$

$$\sum_{i \in N_{F_r}} x_{i,F_r,t}^{V_r} = 1 \quad (39)$$

The sub problem, dealing with transportation routing, decides using the autonomous distributed route planning method regarded the delay penalty. The explain of autonomous distributed route planning method regarded the delay penalty is given in below and the flow chart indicates Fig.6.

#### Step1 Initial Routing

Each AGV plans own most optimal route without thinking other AGVs.

#### Step2 Exchange Information

It exchanges with the transportation route  $x_{i,j}^l$  of AGV $\#l$ .

#### Step3 Convergence

This step judges convergence based on route gotten from Step2. If it is convergence, the calculation is finished. Judgement of convergence is that all AGV's route don't update in rerouting without collision.

#### Step4 Skip

Not to plan route having collision at interval, skip rerouting and go to Step6 by certain odds.

#### Step5 Rerouting

Based on the information from the Step2, it plans rerouting. To minimize the objective function indicates the formulation(40), it plans the routing. This problem is applied the Dijkstra algorithm because of the shortest route problem of each AGV added the penalty of collision and delay as cost.

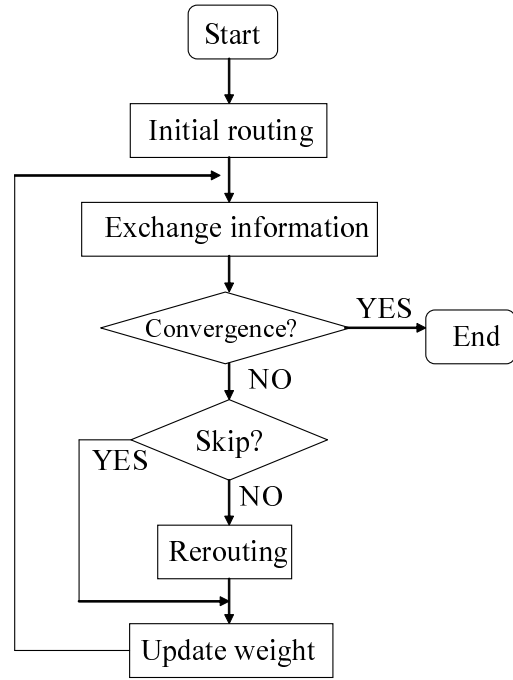


Fig. 6: The algorithm of route planning

$$I_k = \sum_t \pi_{k,t} + \sum_{l \in V, l \neq k} \alpha_{k,l}(r)(C_{k,l}^1 + C_{k,l}^2) + \beta D_k \quad (40)$$

The variable  $\pi_{k,t}$  is that, when AGV $\#k$  reach the reserved node in span  $t$ , equal 0, the other, equal 1. This variable's constraint is given below.

$$\sum_{i \in N_{G_k}} x_{i,G_k,t}^k \leq (1 - \pi_{k,t}) \quad (41)$$

$$(k \in V; t = 1, \dots, H)$$

$$\sum_{i \in N_{G_k}} x_{i,G_k,t}^k \geq 1 - \pi_{k,t} \quad (42)$$

$$(k \in V; t = 1, \dots, H)$$

At once, because AGV reaching the reserved node stop there, the added constraint is given below.

$$-\pi_{k,t} + \pi_{k,t+1} \leq 0 \quad (43)$$

$$(k \in V; t = 1, \dots, H-1)$$

The variable  $C_{k,l}^1, C_{k,l}^2$  indicate the times of collision of AGV $\#k$  and AGV $\#l$ .

#### Step6 Update of Weighting Factor of Penalty Function

If the route from rerouting is infeasible, for AGV collision according to the formulation(44), update of weighting factor of penalty function  $\alpha_{k,l}(r+1)$  and go back to Step2. The variable  $r$  indicates the

times of rerouting.

$$\alpha_{k,l}(r+1) = \alpha_{k,l}(r) + \Delta\alpha \sum_{l \neq k} (C_{k,l}^1 + C_{k,l}^2) \quad (44)$$

### 3.4 The Constraints of Logic Cut and Formulation

When the master problem solution find infeasible assessed from the sub problem, by adding the constraint generating from logic cut, it is method to eliminate the area including infeasible solution and to solve efficiency. This section explains the logic to eliminate the solution area. When logic cut do is that when it is impossible to plan the route to satisfy the start or finish time of transportation request. So it is thought that the condition about transportation decided by the master problem is bad. So, by doing the logic cut, the constraints about sub problem are relaxed and the constraint of the master problem constrain is added. By those constraint, logic is defined to change the factor of master and sub problem the assumption that noted previously. In this paper, following the three types of logic are checked.

- extension of transportation span
- change in assignment of request to AGV
- change of processing turn

The explain of formulation of each logic is given below.

#### Extension of transportation span

The extension of span of transportation request  $T_r$  is add more one span.  $T_r^*$  is the previous span of transportation request  $\#r$ . The variable  $\zeta_r$ , indicate the previous span of request  $\#r$  is changed as following.

$$\zeta_r = \begin{cases} 1 & T_r = T_r^* + 1 \\ 0 & T_r = T_r^* \end{cases}$$

The variable  $\zeta_r$ , about the follow constraint is formulated describe the formulation(45) to (48).

$$T_r + M(1 - \zeta_r) \geq T_r^* + 1 \quad (45)$$

$$T_r \leq M\zeta_r + T_r^* \quad (46)$$

$$T_r + M\zeta_r \geq T_r^* \quad (47)$$

$$T_r \leq T_r^* + 1 + M(1 - \zeta_r) \quad (48)$$

#### Change in Assignment of Request to AGV

The explain of change assignment in transportation request is given below. The binary variable  $\delta_{r,V_r}^{*(k)}$  that

indicate the assignment of request is changed define below. Here  $\#V_r$  indicate the previous assignment of request  $\#r$  to AGV.

$$\delta_{r,V_r}^{*(k)} = \begin{cases} 1 & \text{(Iteration } k \text{ of request } \#r \\ & \text{assigned to AGV } \#V_r) \\ 0 & \text{(other)} \end{cases}$$

$$\sum_{r \in R_T} \delta_{r,V_r}^{*(k)} \leq |R_T| - 1 \quad (49)$$

$$\text{when } \delta_{r,V_r}^{*(k)} = 1 \quad (\forall i \in R; \forall l \in P)$$

#### Change of Processing Turn

It describes that the constraint of change turn of process. The binary variable  $\epsilon_{i,j}^{*(k),l}$  that indicate the turn process is changed as follows.

$$\epsilon_{i,j}^{*(k),l} = \begin{cases} 1 & \text{(At iteration } k, \#i \prec \#j \text{ in the process } \#l) \\ 0 & \text{(other)} \end{cases}$$

$$\sum_{i \in R} \sum_{j \in R} \epsilon_{i,j}^{*(k),l} \leq |R^2| - |R| - 1 \quad (50)$$

$$\text{when } \epsilon_{i,j}^{*(k),l} = 1$$

Formulation(50) is introduced to derive solution different from that obtained before.

Change in assignment of transportation request to AGV and Change of processing turn are integer cut. So formulation(49) and (50) are transformed into formulation (51).

$$\sum_{r \in R_T} \delta_{r,V_r}^{*(k)} + \sum_{i \in R} \sum_{j \in R} \epsilon_{i,j}^{*(k),l} \leq |R^2| - |R| + |R_T| - 1$$

$$\text{when } \delta_{r,V_r}^{*(k)}, V_r = 1, \quad \epsilon_{i,j}^{*(k),l} = 1 \quad (51)$$

## 4 Numerical Experiments

In this chapter, it indicates the way to do the method and the effective of the proposed method indicated the previous chapter .

### 4.1 Example Problem

The layout using the simulation is given in Fig.7. The place where take out products from process  $\#1$  is node 1 and node 4, the place bring in to process  $\#2$  is node 17 and node 20. Node 1 and node 4 are called input buffer, node 17 and node 20 are called output buffer. It indicates the result, that when the number



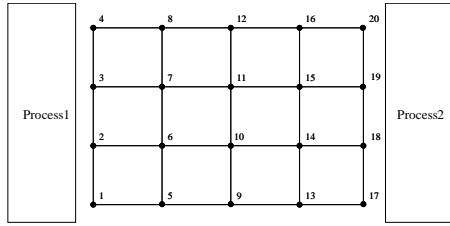


Fig. 7: The FMS model

Table. 1: The input data of exmple problem

Product ID	pro.1	pro.2	start node	goal node
1	6	5	1	20
2	9	6	1	20
3	6	6	4	20
4	7	5	1	17
5	9	9	1	20
6	5	8	4	17

of AGV is 3 and request is 6, in Table 1. In addition, it calculated using the parameter skip ratio is 30 %, the increment of penalty function is 0.8, the increment of delay penalty is 30.

### 4.2 The Result of Exmple Problem

Here, the result until feasible solution when the exmple problem is solved. It indicates that the infeasible solutions until getting the feasible solution by logic cut in Fig.8 to 10.

The gantt chart of Fig.8 is infeasible solution because of happened delay from the start time of transportation of job #6. So by doing logic cut, the assignment of transportation request #4 and #5 is changed, and the turn of process is changed. But it is infeasible in Fig.9. By doing twice logic cut, the transportation request assigned to AGV are changed. The solution in Fig.10 is feasible one.

The feasible result of transportation routing indicate Fig.11 ~ 14. The calculation time is 31.4 seconds.

Table. 2: Initial node of AGV

AGV ID	initial node
1	2
2	17
3	13

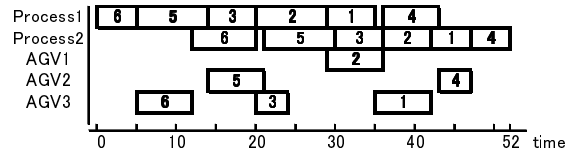


Fig. 8: The initial solution(no cut)

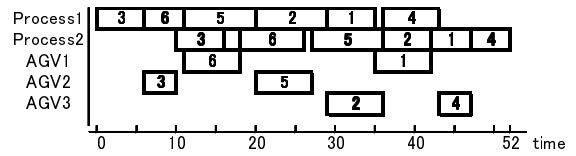


Fig. 9: The solution(one cut)

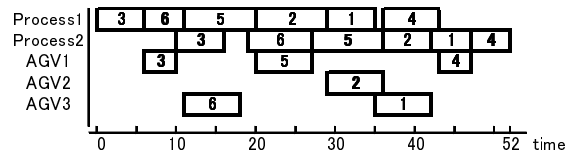


Fig. 10: The solution(two cut)

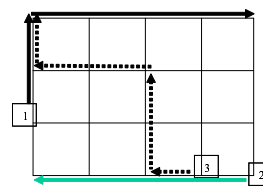


Fig. 11: span0 ~ 10

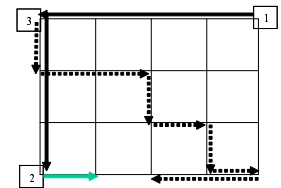


Fig. 12: span11 ~ 20

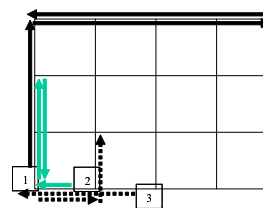


Fig. 13: span21 ~ 30

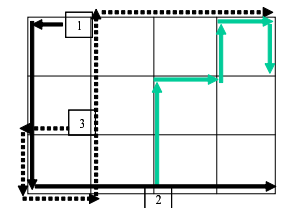


Fig. 14: span31 ~

### 4.3 The Result of Simulation of the Small Problem

The simulation in this section is carried out using the layout of the production process as is Fig.7. Input buffer are node 1 and node 4, and output buffer are node 17 and node 20. The initial node of each AGV and the span of process of each request are decided randomly. It indicates the each 15 pattern results by using

proposed method and CPLEX solver for the 2 AGVs and 2 or 3 requests scale problem. Proposed method

Table. 3: The result of 2 requests

Case	CPLEX		Proposed method		
	Obj.[-]	Time[s]	Obj.[-]	Time[s]	Cut
1	27	27.9	27	0.328	2
2	25	25.7	25	0.641	6
3	26	12.3	26	0.172	1
4	26	36.9	26	0.422	4
5	27	103.2	27	0.625	4
6	30	827.8	30	1.266	10
7	28	42.8	28	0.172	1
8	25	37.5	26	0.562	5
9	22	46.2	23	0.484	4
10	25	15.7	25	0.360	3
11	26	787.3	28	1.000	10
12	24	28.0	26	0.750	7
13	23	29.0	24	0.578	4
14	29	6367.5	29	0.375	3
15	29	95.1	29	0.281	2
Ave.	26.1	565.5	26.6	0.534	-

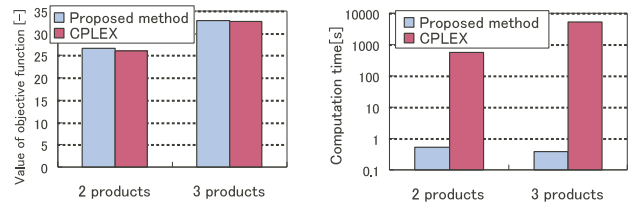


Fig. 15: The comparison of objective function calculation time

problem divide into the master and sub problems, so the weight of CPLEX solver is decrease. The objective function of the proposed method equals to that of CPLEX solver in the almost same problem pattern. But sometimes there occurs bad results. Where the objective function of the solution is worse when cut is done many times. Because the extension of transportation span is used as logic. It is thought that if this logic is done many times, transportation time of each request is extended, and the makespan as objective function is tend to worse.

#### 4.4 The result in the large scale problem

Table. 4: The result of 3 requests

Case	CPLEX		Proposed method		
	Obj.[-]	Time[s]	Obj.[-]	Time[s]	Cut
1	30	8043	30	0.328	1
2	31	1962	31	0.282	1
3	34	877	34	0.328	1
4	33	1024	33	0.250	1
5	32	1488	32	0.281	1
6	35	7588	35	0.687	2
7	34	485	34	0.406	2
8	35	7261	35	0.266	1
9	34	1062	34	0.281	1
10	33	9628	34	0.890	5
11	35	6199	35	0.313	1
12	30	8956	30	0.547	3
13	34	9305	34	0.296	1
14	30	13940	30	0.329	1
15	30	425	30	0.422	2
Ave.	32.82	5216	32.76	0.394	-

In this section, the result of comparing the conventional method and proposed method by solving the 10 pattern problems using the layout Fig.7 are described. As for the solver, conventional autonomous distributed method is used. The results indicated from Table.5 to 9. The part of autonomous distributed method is using SA method. The parameter is that max temperature is 100, freeze temperature is 0.01, annealing ratio is 0.99 and loop time in same temperature is 100.

It is possible to see that the objective function

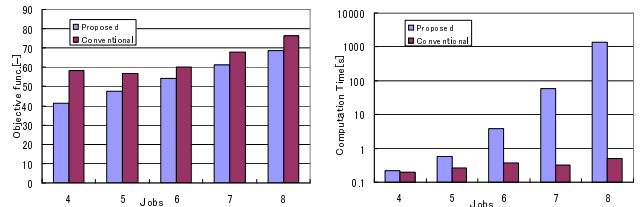


Fig. 17: The comparison of objective function calculation time

can solve the small scale problem faster than CPLEX solver. Because transportation routing is solved by the autonomous distributed route planning method that can solve the problem in short time. And the entire

of peoposed method is better than that of the conventional method and the proposed method is longer than heretofore method to derive the feasible solution.

Table. 5: The result of 4 request

Case	Proposed			Conventional	
	Obj.[-]	Time[s]	Cut	Obj.[-]	Time[s]
1	47	0.17	0	60	0.19
2	47	0.28	1	58	0.17
3	42	0.17	0	50	0.17
4	44	0.31	1	59	0.18
5	44	0.23	0	58	0.17
6	42	0.35	1	55	0.53
7	44	0.21	0	54	0.18
8	42	0.32	1	58	0.15
9	44	0.32	1	56	0.17
10	41	0.25	0	50	0.17
Ave.	41.2	0.22	-	58.1	0.20

Table. 6: The result of 5 request

Case	Proposed			Conventional	
	Obj.[-]	Time[s]	Cut	Obj.[-]	Time[s]
1	52	0.25	0	61	0.20
2	47	0.36	0	49	0.21
3	47	0.37	0	50	0.20
4	50	0.23	0	53	0.20
5	51	0.26	0	55	0.82
6	45	3.79	1	54	0.42
7	45	0.65	1	52	0.20
8	49	0.71	1	56	0.20
9	45	1.65	4	60	0.35
10	48	0.68	1	49	0.25
Ave.	47.4	0.58	-	56.7	0.26

Table. 7: The result of 6 request

Case	Proposed			Conventional	
	Obj.[-]	Time[s]	Cut	Obj.[-]	Time[s]
1	53	1.14	0	60	0.21
2	60	3.70	1	63	0.21
3	54	1.15	0	56	0.21
4	55	2.23	1	59	0.23
5	54	1.60	0	57	0.23
6	55	11.1	0	57	0.21
7	52	5.76	0	56	0.21
8	52	4.76	4	58	0.23
9	55	31.3	3	58	0.21
10	57	2.45	1	66	0.81
Ave.	54.2	3.86	-	60.0	0.36

Table. 8: The result of 7 request

Case	Proposed			Conventional	
	Obj.[-]	Time[s]	Cut	Obj.[-]	Time[s]
1	61	13.5	0	64	0.35
2	59	154	8	66	0.23
3	62	41.6	0	66	0.31
4	62	10.7	0	66	0.25
5	58	33.2	0	67	1.04
6	65	41.5	1	68	0.31
7	63	296	1	68	0.23
8	63	73.8	1	65	0.51
9	64	46.2	1	70	0.53
10	60	81.7	1	66	0.23
Ave.	61.3	58.3	-	67.7	0.32

Table. 9: The result of 8 request

Case	Proposed			Conventional	
	Obj.[-]	Time[s]	Cut	Obj.[-]	Time[s]
1	72	544.6	0	76	0.265
2	68	1748	1	71	0.266
3	68	1275	1	73	0.266
4	67	2174	5	73	0.265
5	65	6219	3	75	0.266
6	71	2792	0	77	0.250
7	73	1375	2	78	0.344
8	73	4293	1	73	2.375
9	68	1290	1	68	0.266
10	72	1067	2	77	0.265
Ave.	68.7	1388	-	76.23	0.49

The conventional method, using SA method in production schedule, can reduce calculation time but it derives an approximate solution. The calculation time of proposed method increases at an exponential rate from Fig.18 because the master problem is solved by CPLEX. From these reasons, it is difficult to solve very large scale problem because calculation time is very long by using the proposed method.

## 5 Conclusion

In this paper, the simultaneous optimal problem and the formulation about the FMS's production schedule and transportation routing is treated. The entire problem is divided into the master problem and sub problem. Then the method that is able to derive the approximate solution with efficiency is proposed. When it occurs infeasible solution, by generating the logic cut, it eliminates the area of the master problem including the infeasible solutions until getting the feasible solution. In the proposed method, the master problem is solved by CPLEX and the sub problem is solved by the autonomous distributed routing planning considering the penalty late from the schedule time. Where, the logics, used in logic cuts, are the extension of transportation span, the change of assignment request to AGV, and the change in precedence of processing jobs in the processes. Also, the simulation result of makespan that compares the result of the proposed method by using the logic cut and the one by using CPLEX in the small scale problem is stated. It becomes clear that, comparing by makespan, the result by using the proposed method is worse than the one by using CPLEX. And the calculation time of proposed method is faster than that of CPLEX. The future work is targeted to obtain practicable solution by the proposed method in the production system with iteration processes. To derive better approximate solution, another logic are to be found and formulated. To derive feasible solution faster than now, the master problem is to be solved by meta heuristic method instead of CPLEX, and master problem is more divided into small problems.

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