

Transfer of Quantum Information by Spin Chains: Effect of Noise

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(Received December 12, 2007)

In order to transfer quantum information, the use of spin chains has been proposed and their transfer characteristics have been analyzed. As one of the latter, the maximum of the transfer probability over sufficiently long time period is often considered important and some examples with high maximum values have been given. In these examples, the coupling between neighboring spins are tuned so as to attain high efficiency. In this article, we discuss the effect of noise on the values of coupling constant. As a result, we propose a system where the efficiency is high and also the effect of noise is small.

I. INTRODUCTION

In many cases, the information stored in a quantum system reduces to the combination of states in two level systems. A typical example may be the electron spin which is either parallel or antiparallel to some chosen axis in space. As a method to transfer the information stored in the spin state of electrons, there have been proposed to utilize the chains of coupled electronic spins with the input and output of information placed on the end of the chains[1].

If the couplings between spins are of ferromagnetic nature, the ground state of the spin chain is the state where all spins are aligned. When the spin on one end is flipped, this information is transferred via the spin chain and finally detected on the other end. The result of observation of the output spin is given by a probability and its maximum over the past since the flip of the input spin is one of the target of investigation. Usually the maximum is not of the order of unity within reasonable time duration but some kind of chain transfer the information with the probability unity.

In these investigations, the couplings between chains are tuned so as to give high efficiency in the information transfer. Though such a tuning might be possible by applying semiconductor manufacturing processes in mesoscopic or microscopic scale, we always have some source of disturbance which gives perturbations to those tuned couplings. The effects of these disturbance or noise has not been analyzed

seriously. The purpose of this paper is to analyze the latter effect mainly by numerical simulations.

II. SPIN CHAIN MODELS

We assume that we have a chain of n spins with the input and output gate on each end. We here consider three kinds of spin chains with the coupling given by the xy -model of the spin system and denote the spin component at the site j by $(\sigma_x^j, \sigma_y^j, \sigma_z^j)$.

Model I[2]

$$H = \frac{a}{2}(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 + \sigma_x^{n-1}\sigma_x^n + \sigma_y^{n-1}\sigma_y^n) + \frac{1}{2} \sum_{j=2, \dots, n-2} (\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1}) \quad (1)$$

Here $a < 1$ is an adjustable parameter and, when appropriate value of a is chosen, the fidelity is shown to be comparable to unity.

Model II[3, 4]

$$H = \sum_{j=1, \dots, n-1} J_j(\sigma_x^j\sigma_x^{j+1} + \sigma_y^j\sigma_y^{j+1}). \quad (2)$$

Here J_j is tuned so that

$$J_j = [j(n-j)]^{1/2}. \quad (3)$$

This chain has the property of the perfect transfer.

Model III

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$$H = a(\sigma_x^1 \sigma_x^2 + \sigma_y^1 \sigma_y^2 + \sigma_x^{n-1} \sigma_x^n + \sigma_y^{n-1} \sigma_y^n) + \sum_{j=2, \dots, n-2} J_j (\sigma_x^j \sigma_x^{j+1} + \sigma_y^j \sigma_y^{j+1}). \quad (4)$$

In this case, *Model II* is modified so as to take the characteristics of Model I into account.

The state space for these Hamiltonians is spanned by the product of eigenstates of each spin, $\{|0\rangle_j, |1\rangle_j\}_{j=1, \dots, n}$. Since the sum of z -components

$$\sum_j \sigma_j^z \quad (5)$$

commutes with these Hamiltonians, the initial state develops in the subspace where only one of spins is flipped. When we denote such a state as

$$|\mathbf{j}\rangle \equiv |1\rangle_j \times \prod_{k(\neq j)} |0\rangle_k, \quad (6)$$

the initial state is given by

$$|\Psi(t=0)\rangle = |\mathbf{1}\rangle \quad (7)$$

and the time development of the spin chain is expressed as

$$|\Psi(t)\rangle = \sum_j a_j(t) |\mathbf{j}\rangle. \quad (8)$$

When the spin on one end at $j = 1$ is reversed at the time $t = 0$, this information is transferred via the coupling between the spins in the chain and finally observed on the other end at $j = n$. The aim of this kind of investigation is to maximize the expectation value at $j = n$ or

$$P_{n,max}(t) = \max_{0 < t' < t} \{P_n(t')\}, \quad (9)$$

where

$$P_n(t) = |\langle \mathbf{n} | \Psi(t) \rangle|^2 = |a_n(t)|^2. \quad (10)$$

We here consider the effects of noise which is expressed as the fluctuation in the coupling constant. The Hamiltonian is then expressed as

$$H = H^{(0)} + H^{(1)}, \quad (11)$$

where $H^{(0)}$ is given by the above expressions and $H^{(1)}$ is the noise in the coupling constant. For the Hamiltonian to be hermitian, the matrix elements of $H^{(1)}$ is assumed to be real and symmetric.

In simulations, we introduce random numbers $-0.5 < \delta_i < 0.5$ and modify the matrix element as

$$H_{ij} = H_{ji} = H_{ij}^{(0)} (1 + \epsilon \delta_i). \quad (12)$$

Here ϵ is the strength of perturbations.

III. RESULTS OF SIMULATIONS AND DISCUSSIONS

We have performed simulations of the time development for Model I, II, and III with $n \leq 500$. We note that, since the performance is closely related to the structure of the eigenvalue around 0, the parity of the Hamiltonian matrix has a significant effect. We find that generally the case of even parity ($n = \text{even}$) gives better results. In this article we present the results for $n = 32$.

In *Model I*, the fidelity never attains unity. As shown in Ref.[2], the maximum is controlled by a and is obtained when $a \sim 0.6$ as is shown in Fig.1. The effect of noise is shown in Fig.2.

In *Model II*, $P_{n,max}(t)$ take on exactly unity when t is sufficiently large (perfect transfer). This is due to the structure of eigenvalues for this Hamiltonian: They are exactly of equal spacing. When we expand the initial state into eigenstates, the expansion coefficient has a broad spectrum. Since the perfect transfer is achieved by the superposition of eigenstates within the broad spectrum, the noise in the tuned coupling constants has a large effect on the transfer[5].

In *Model III*, the coupling constant a on both sides is regarded as a control parameter. We observe that the value of $P_{n,max}$ is close to unity as shown in Fig.3 and, at the same time, the effect of noise is sufficiently small as shown in Fig.4. We thus propose to use *Model III* for quantum information transfer.

The effect of noise is small when the initial state has a narrow spectrum. In this case, the maximum of $P_n(t)$ is mainly determined by the beat of two eigenfrequencies and it may not be influenced strongly by the change of two eigenfrequencies due to noise. If the maximum is determined by superposition of many eigenfrequencies, this 'collective' maximum may be strongly influenced by changes in component frequencies and with noises, it may be difficult to attain the maximum which is possible without noises. In Figs.5 and 6, the spectrum of the initial state is shown for various values of a in *Models I and III*. We observe that the spectrum becomes narrow with the decrease of the coupling at the ends.

In addition, when a is sufficiently small, the inner part of the Hamiltonian in *Model III* of $(n-2) * (n-2)$ dimensions has the equal spacing structure of eigenvalues. The components of the initial state transferred to $(n-2)$ -dimensional space, may thus have the property of perfect transfer. This is expected to give an advantages to *Model III* compared with *Model I*.

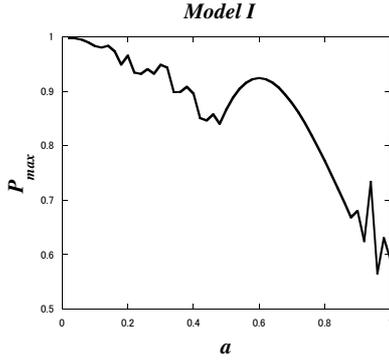


FIG. 1: Maximum probability vs. value of a in *Model I* without noise.

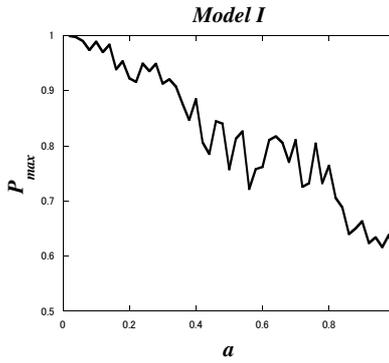


FIG. 2: Maximum probability vs. value of a in *Model I* under noise with relative amplitude 0.2.

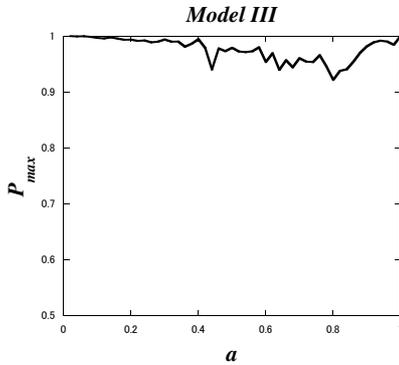


FIG. 3: Maximum probability vs. value of a in *Model III* without noise.

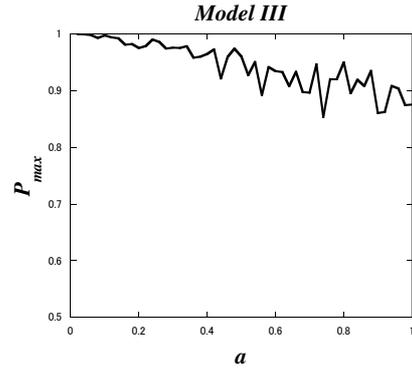


FIG. 4: Maximum probability vs. value of a in *Model III* under noise with relative amplitude 0.2.

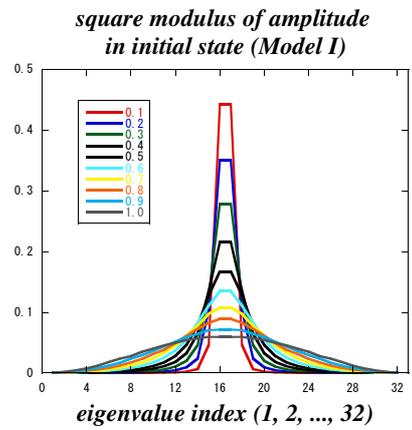


FIG. 5: Dependence of initial state spectrum on a in *Model I*.

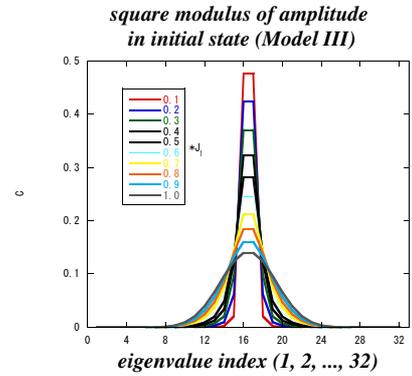


FIG. 6: Dependence of initial state spectrum on a in *Model III*.

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