

## **Determination of Crossarm Installation in Fill Dams by Back Analysis**

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**ABSTRACT:** This paper discusses a numerical model that can be used to optimize the installation in a zoned type and a homogeneous type fill dam. Before installation in a real dam to evaluate dam behavior, numerical model described in this paper is carried out on a prototype dam to check the optimum installation, using cross arm measurements.

Three cross arm installations at the upstream, the core and downstream to measure displacements are considered. The installation options considered are three cross arm combinations for best installation to verify the safety of dams and to reduce cost. Finite element method is used for generating the displacement field in a linear elastic numerical model. The generated data is used as an input data in the back analysis to check the adequacy of each installation option.

### **1. INTRODUCTION.**

In Fill dams provisions are made to install piezometers, settlement gauges, extensometers at strategic positions such as the core, the shoulders and other sensitive areas of the dam as described by Dunnicliff(1988). Those instrumentation gives many information about the behavior of the dam especially using the numerical methods such as back analysis. Optimization of the system can be made by abandoning some instruments or installing others as the real behavior becomes better understood(ICOLD, 1989).

The successful implementation of back analysis requires a proper selection of stress-strain equation, because most soils show essentially nonlinear stress-strain behavior, and because it is not possible to back analysis all types of stress-strain parameters from field measurements. The simplest and the most stable method from the viewpoint of numerical analysis, is to assume the linear elastic behavior of soil stratum and to back calculate the elastic constants from monitored displacements[1]. Insufficient instrumentation would make the behavior of the dam not well understood and also monitoring very ineffective. There are many cases where back analysis using measured data from actual fill dams have failed

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because there is no sufficient data due to deficiency or wrong geometrical arrangement in the instrumentation.

This paper proposes to determine the optimum instrumentation, using a numerical model for a zone and a homogeneous type fill dam based on cross arm measurements during construction and during reservoir filling by numerical methods.

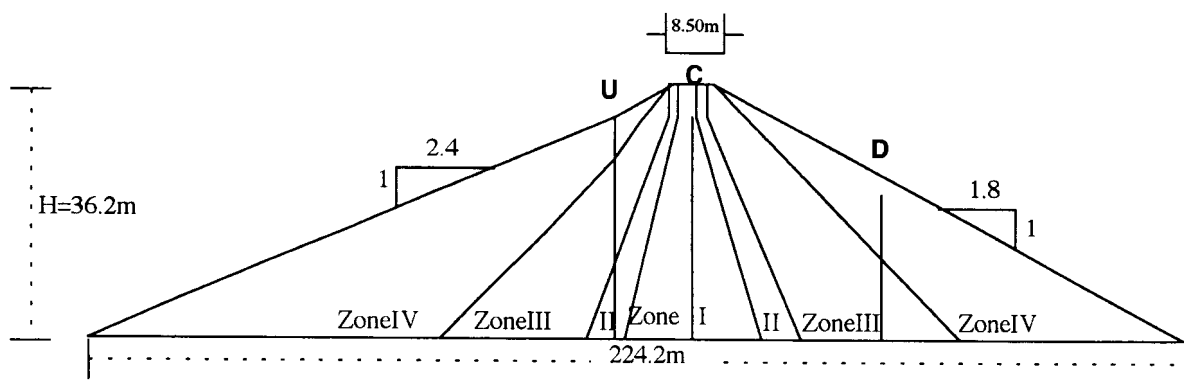
When employing back analysis procedures to determine soil elastic parameters such as Young's modulus and Poisson's ratio, displacements are better than the stress as the deformation measurements change as the elastic parameters under all loading. Therefore, a general measurements such as cross arm and surface monuments are considered in the analysis instead of stress measurements. This paper first considers only cross arm measurements. This device is used to measure vertical displacements of series of telescoping pipe connected to horizontal crossarms embedded several meters<sup>1</sup> intervals.

## 2. METHODS OF NUMERICAL ANALYSIS

### 2.1 Normal Analysis and its Generated Data

Two types of numerical model of fill dams are provided as zoned type with four zones and homogeneous type. Each is assumed to have three cross arms at the upstream, core and/or the downstream sections.

Fig.1 shows the numerical model of zone type dam with four zones such as core zones(zone 1), filter zone(zone 2), transition zone(zone 3) and rockfill zone(zone 4) Another numerical model of homogeneous dam is the same dimension as zone type dam but the same soil for its four zones. A finite element linear elastic method is used for generating the displacement in field(vertical and horizontal) numerically. The generated data from the



**Fig. 1 Numerical model of a zone typed fill dam;**  
**Position of Cross arm U:upstream, C: core; D: downstream**

normal analysis is used as input data in the back analysis and the results compared with the parameters used in generating the numerical data.

During construction, a linear elastic(Simplified Duncan's hyperbolic[5]) model is used to generate the data. It simulates the construction operation by addition of a layer at a time that is stage method.

During reservoir filling, the dam was assumed to be filled in a single stage and the water load was applied instantaneously as an external load to the upstream face of the dam.

**2.2 Back Analysis.**

**2.2.1 Methods of Back Analysis**

The method of back analysis involves the following:

- a) making an initial guess of E and  $\nu$ ,
- b) calculating the model response(axial and lateral displacements) by FEM linear elastic analysis,
- c) comparing measured and predicted displacements,
- d) adjusting the assumed parameters at each stage of the iteration to reduce the difference between the measured and predicted displacements,
- e) repeating (b) to (d) using the updated soil parameters until the error between the measured and the predicted displacements are within tolerable limits.

The following error function is adopted;

$$\epsilon = \sum (U^* - U(E, \nu))^2 \dots\dots\dots(1)$$

where  $U^*$  is the observed value,  $U(E, \nu)$  the predicted values and  $\epsilon$  the error function to be minimized.

**2.2.2 Minimization Algorithm**

The model is nonlinear in the parameters and it is calibrated by means of the Levenberg-Marquardt modification of the Gauss-Newton minimization algorithm implemented in the subroutine LEVMAR[1].

The procedure involves computing a parameter correction vector  $\Delta p_k$  such that the new estimate

$$p_{k+1} = p_k + \Delta p_k \dots\dots\dots(2)$$

reduces the objective function  $\epsilon(p_{k+1}) < \epsilon(p_k)$  at each iteration k.

$$\Delta p_k = (J_k^T J_k + \mu D)^{-1} J_k^T f_k \dots\dots\dots(3)$$

where f is the vector of residuals, D is a diagonal matrix of order equal to the number of parameters. The elements of the matrix D is equivalent to the diagonal elements of the matrix  $J^T J$ ,  $\mu$  is a scalar known as Levenberg-Marquardt parameter, and J is the Jacobian

matrix with elements given by a forward finite difference approximation:

$$J_{ij} = \frac{\partial U(p)_i}{\partial p_j} \approx \frac{U(p_j + \delta p_j) - U_i(p_j)}{\delta p_j} \dots\dots\dots(4)$$

with the difference increment:

$$\delta p_j = \alpha \cdot p_j$$

where  $\alpha$  is a user specified parameter and it is set at 0.001 in this paper.

**Table 1**  
**Possible cross arm installation.**

CASE	U	C	D	Total
1-U-Z	6			6
1-U-X	6			6
1-U-ZX	6X2			12
1-C-X		6		6
1-C-Z		6		6
1-C-ZX		6X2		12
1-D-Z			4	4
1-D-X			4	4
1-D-ZX			4X2	8
2-UD-Z	6		4	10
2-UD-X	6		4	10
2-UD-ZX	6X2		4X2	20
2-CU-Z	6	6		12
2-CU-X	6	6		12
2-CU-ZX	6X2	6X2		24
3-CUD-Z	6	6	4	16
3-CUD-X	6	6	4	16
3-CUD-ZX	6X2	6X2	4X2	32

**Table 2** Parameters for numerical model  
**of homogeneous dam.**

	E(kN/m <sup>2</sup> )	v
Construction	3500	0.475
Filling	3500	0.475

**Table 3** Parameters for numerical model of zoned type dam

	Construction		Filling	
	E(kN/m <sup>2</sup> )	v	E(kN/m <sup>2</sup> )	v
	Actual	Actual	Actual	Actual
I	1000.00	0.475	1000.00	0.475
II	3000.00	0.450	3000.00	0.450
III	5000.00	0.400	5000.00	0.400
IV	10000.00	0.400	10000.00	0.400

In each back analysis calculation, the solution being looked for consists of minimizing an error function and also obtaining a set of identified parameters that will coincide with those used in generating the numerical data.

### **2.2.3 Generated Data and Crossarm Installation.**

For the purpose of optimum instrumentations, several combinations of installation options are considered and some of them used as input data in the back analysis.

Generally, the installation of crossarm at a cross section of a dam consists of one the three as follows;

TYPE 1 : only one crossarm ; most popular case , installed at the center of dam positions to measure vertical displacement, at C in Fig 1,

TYPE 2: two cross arms; usually one installed at the center of dam and another at positions downstream or upstream at C and D , or C and U in Fig1,

TYPE 3: three crossarm; one is at the center and the others are at the upstream and downstream of dam at C, U and D in Fig1.

The crossarm measures vertical displacement of Z direction or settlements in an embankment usually about ten measuring points or so. However in this analysis, the cross arm has function to be able to measure not only vertical displacement but also horizontal displacement or the displacement in the X direction just the same as an inclinometer. Table 1 shows the possible combination of each crossarm in current use. Table 2 and Table 3 show the parameters for generating numerical data in the homogeneous dam and the zoned type dam, respectively.

## **3.GENERATED DATA BY NORMAL ANALYSIS.**

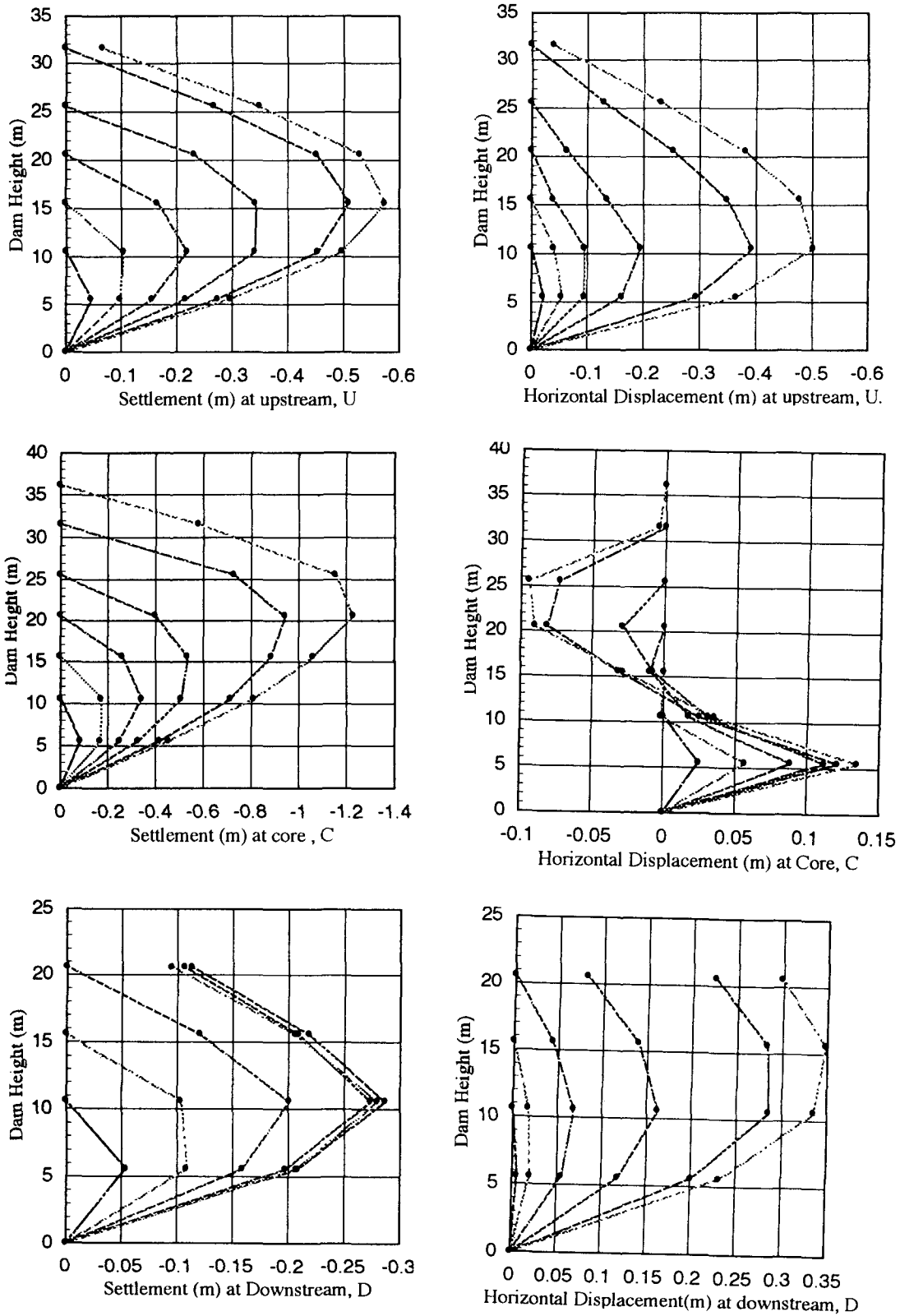
### **3.1 During Construction**

Generated data for back analysis are gotten by normal analysis.

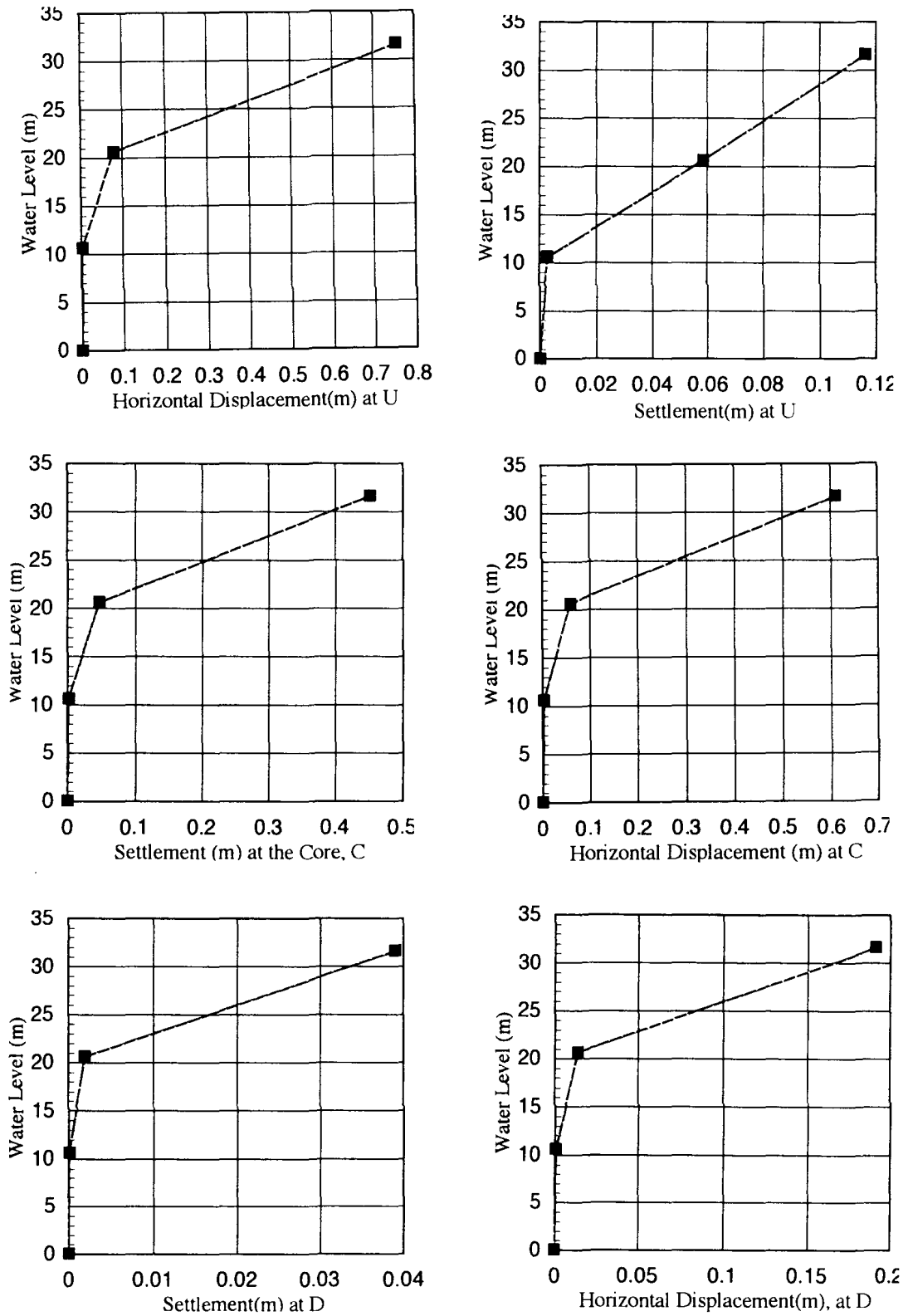
Fig.2 shows an example of the results of normal analysis of three cases of TYPE 3 in Table 1 that is three crossarms in zone type dam. As the height of embankment, the maximum settlements occur at middle height of embankment on each sets of crossarm. However, the horizontal movement of center of dam are minimum at near intermediate height.

### **3.2 During Water Filling**

Fig.3 shows an example of the results of normal analysis of three cases of TYPE 1 during water filling at U, C and D. The displacements are suddenly largest after about two third of full water level. This phenomenon occurs in the actual dam.



**Fig.2 Example of the results of normal analysis of three cases of TYPE 1**



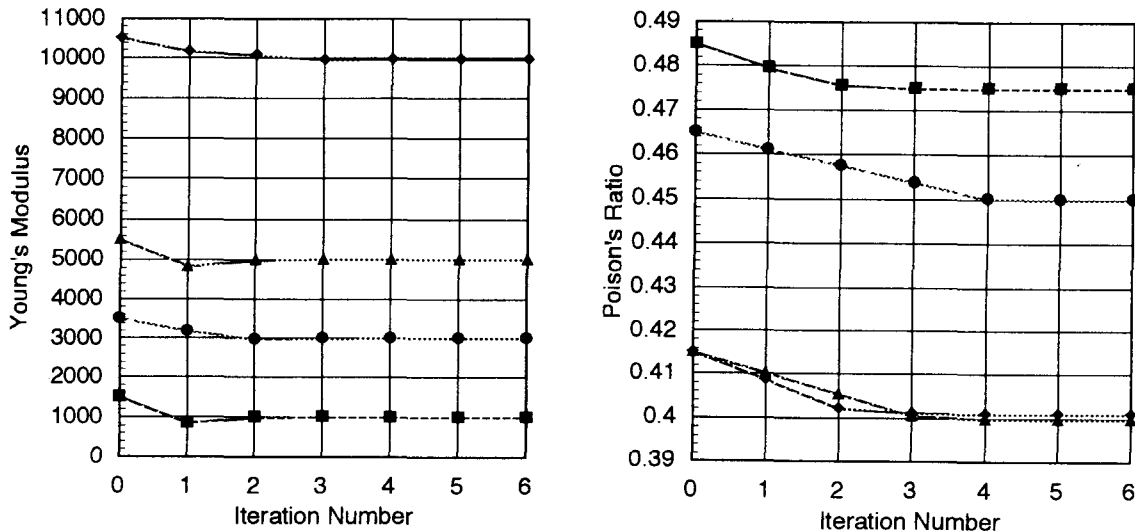
**Fig.3 Example of the normal analysis of three cases during water filling**

**4. RESULTS AND DISCUSSIONS OF BACK ANALYSIS**

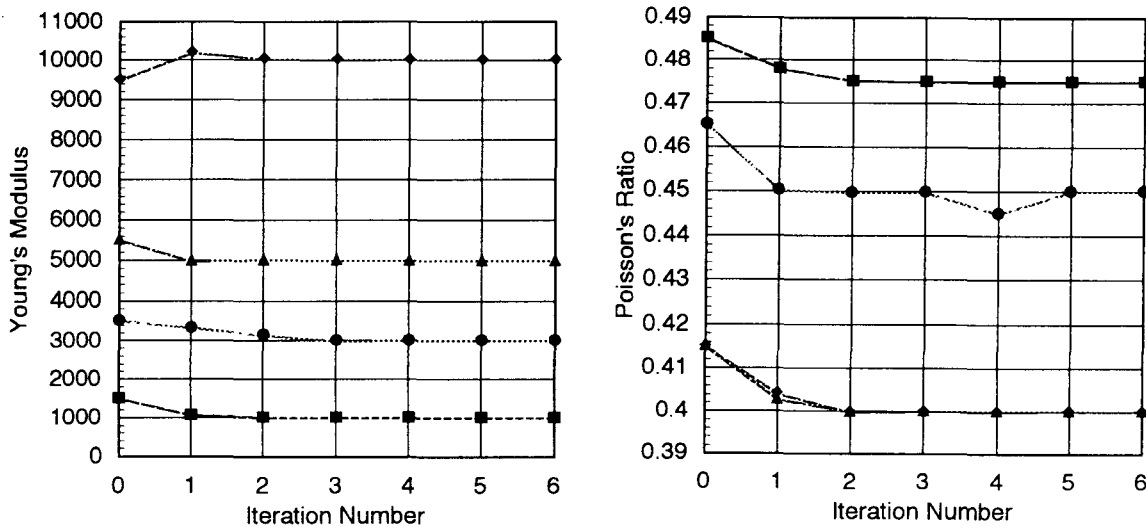
**4.1 Convergence and definition of error index**

**4.1.1 Convergence of the results of back analysis**

The solution of back analysis converges to a certain value after several iteration. In order to check the validity of the method of back analysis, here shows CASE 3-CUD during construction and also during reservoir filling as shown in Fig.4 and Fig.5. From these figures, the solution converges to a stable value after the third iteration.



**A during construction**



**B during filling**

**Fig. 4 Results of back analysis for data set 3-CUD-Z**



**4.1.2 Error index**

The selection of the optimum instrumentation is based on results of the back analysis and on the values of the error indices 'EI' defined in (5) and (6) respectively to judge for better installation.

$$\text{ErrorIndex} = \frac{(\text{Identified Parameter} - \text{Actual Parameter}) * 100}{\text{Actual Parameter}} \dots\dots\dots(5)$$

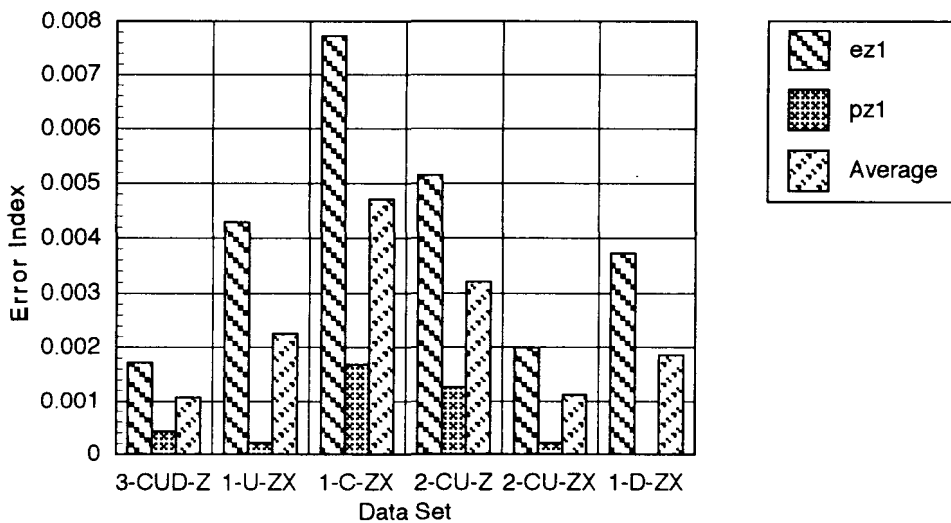
$$\text{Averaged Error Index} = \frac{\sum_{i=1}^N (\text{Error Index})}{N} \dots\dots\dots(6)$$

where N is the number of parameters.

**4.2 Better Installation judging from Error Index**

**4.2.1 Homogeneous type dam**

**1) During construction** The combination of three crossarm is shown in Table 1. Here shows several examples of the results back analysis such installation as CASE 1-U-ZX, CASE 1-C-ZX, CASE 1-D-ZX , CASE 2-CU-Z, CASE 2-CU-ZX, CASE 3-CUD-Z. Figure 5 is shown Error Index and Averaged Error Index of Young's modulus and Poisson's Ratio as ez1 and pz1, respectively. The highest Error Index in Fig. 5 is about 0.0046 and it occurs on CASE 1-C-ZX. And it takes order as CASE 2-CU-ZX, CASE 1-D-ZX, CASE 1-U-ZX, downward. The lowest value occurs at CASE 3-CUD-Z. This figure shows that the error of Young's modulus is larger than that of Poisson's ratio in all the cases considered. This means that instllation of the crossarm in center of dam is not recommended. This figure also show that the error of Young's modulus is larger than Posson's ratio.



**Fig.5 Error Index of Homogeneous dam during construction**

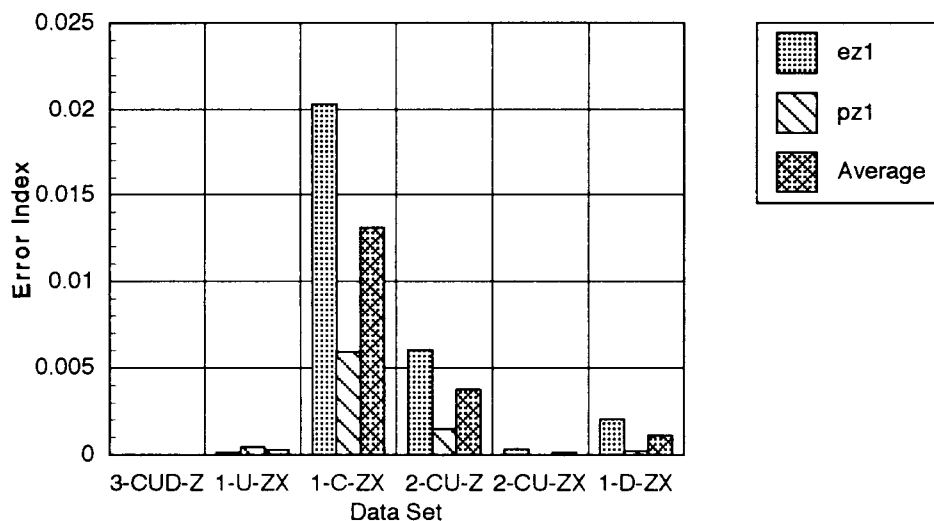
**2) During filling.** Figure 6 is shown Error Index and Averaged Error Index of Young's modulus and Poisson's Ratio Results of the back analysis during water filling at the same condition as the previous section.

In figure 6, the highest averaged error index is about 0.013 and it occurs at the installation CASE 1-C-ZX . And it continues as CASE 2-CU-Z, CASE 1-D-ZX, CASE 1-U-ZX, CASE 2-CU-ZX downward. The lowest averaged error index occurs at the installation options 3-CUD-Z and 2-CU-ZX about 0.00.

**3) Better installation of homogeneous dam.** Fig.5 and Fig. 6 show that installation of three crossarms is the best combination as CASE 3-CUD-Z even if only Z-direction. And the next is two crossarm CASE 2-CU-ZX and continues as CASE 1-U-ZX, CASE 1-D-ZX, CASE 2-CU-Z, CASE 1-C-ZX.

Therefore, one can say as follow about installation in the homogeneous dam.

- (1) If one crossarm is installed, it should be at the upstream of dam but not at the center.
- (2) If one crossarm is installed, it should measure displacements in X and Z directions. That is the crossarm should have the function as an inclinometer.
- (3) If two crossarms are to be installed, the displacement of Z(settlement) and X direction(horizontal displacement) should be measured.
- (4) One crossarm that measures displacements in Z and X directions is much better than two crossarms that measures only settlement( Z direction).
- (5) In the case of one crossarm, the upstream of dam is better than downstream.
- (6) There are not so much difference between one/two crossarm with function to measure Z and X direction in the upstream case.



**Fig.6 Error Index of Homogeneous dam during water filling**

(7) In selecting combinations of crossarm, it is best installation with three crossarms but not so much difference for one crossarm measuring displacements in Z and X direction as far as only settlement is to be measured.

### 4.2.2 Zoned type dam

1) **During Construction.** Several examples of the results back analysis are shown such installation as the same as homogeneous dam. They are CASE 1-U-ZX, CASE 1-C-ZX, CASE 1-D-ZX, CASE 2-CU-Z, CASE 2-CU-ZX, CASE 3-CUD-Z in Table 1.

Fig.7 shows the Error Index and the Averaged Error Index of Young's modulus

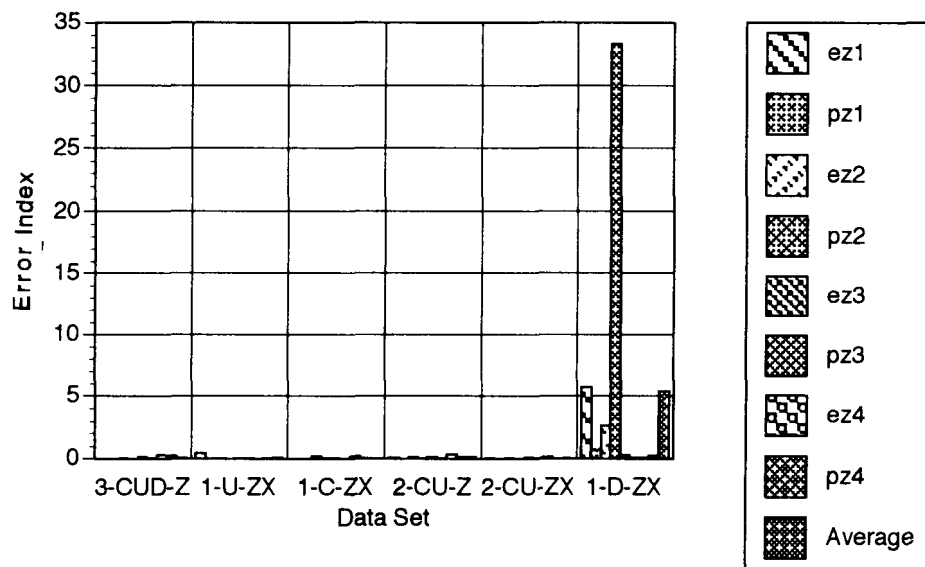


Fig.7 Error Index of zone type dam during construction

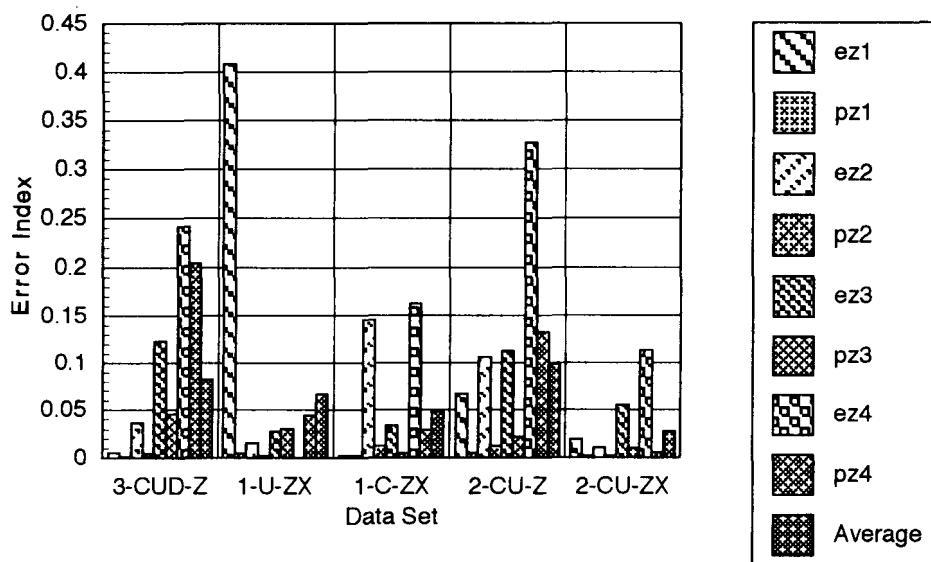


Fig.8 Error Index of zone type dam during construction ( large scale)

and Poisson's Ratio as ez1 to ez4 and pz1 to pz4 according to four zones in zone type dam in Fig.1. Fig.7 shows both error index and averaged error index. On the other hand, Fig.8 shows only the Error Index and the Averaged Error Index to clarify of Fig7 excluding 1-D-ZX.

The averaged Error Index in Figure 7 shows that the highest value is about 5.2 % in CASE 1-D-ZX whereas the other cases are less than 0.5 %. And error index of zone 2 is extremely large. This shows that CASE 1-D-ZX is not satisfactory according to our selection criteria. The cases except CASE 1-D-ZX shows in Fig. 8.

Fig. 8 shows that the highest Averaged Error Index is except CASE 1-D-ZX is 0.098 in CASE2-CU-Z. Then it follows the order as CASE 3-CUD-Z, CASE 1-U-ZX, CASE 1-C-ZX downward.

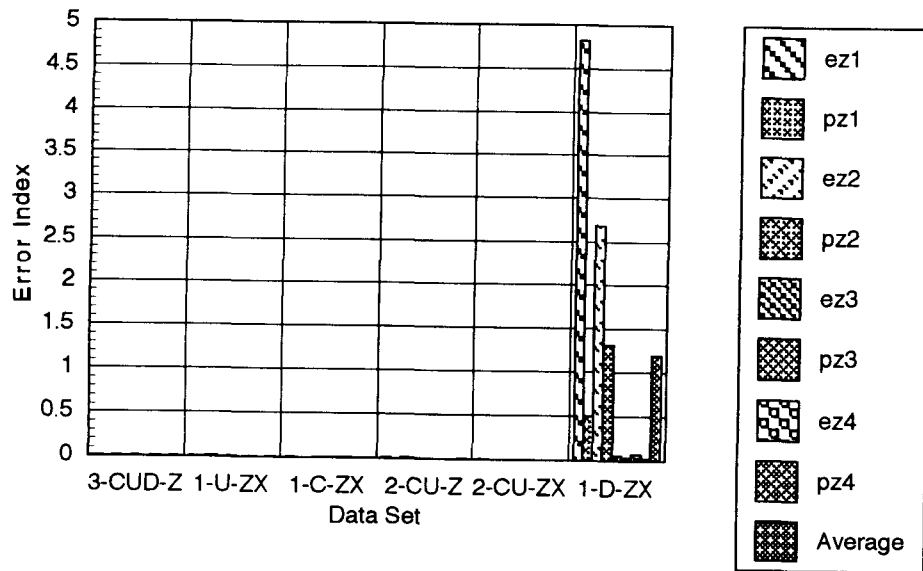


Fig.9 Error Index of zone type dam during water filling

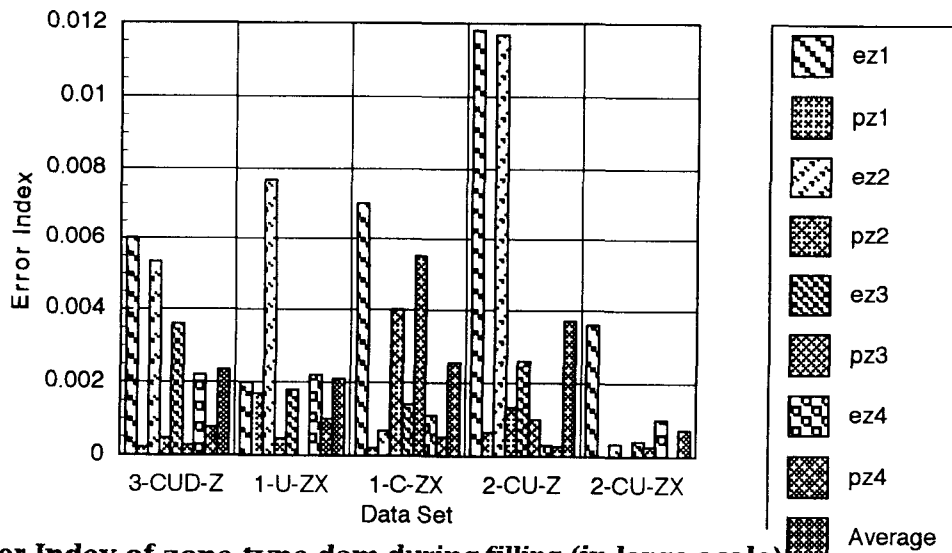


Fig.10 Error Index of zone type dam during filling (in large scale)

Fig.7 shows the Error Index and the Averaged Error Index of Young's modulus. The lowest Averaged Error Index is 0.028 at CASE 2-CU-ZX.

**2) During filling.** Results of the back analysis during reservoir filling for the same cases as in the previous section are shown in Fig.9 and Fig.10 as before.

The results of Error Index and Averaged Error Index is shown in Fig.9. The highest averaged Error Index is about 1.2 at the CASE 1-D-ZX. The Averaged Error Index for the other cases are less than 0.1. This shows that CASE 1-D-ZX is unsatisfactory, according to the selection criteria. The cases except CASE 1-D-ZX is shown in Fig. 10. The highest(except CASE 1-D-ZX) is 0.0036 as shown in Fig.10 and it occurs at CASE 2-CU-Z. Then it is ordered as CASE 1-C-ZX followed by CASE 3-CUD-Z. The lowest Averaged Error Index is about 0.0007 and it occurs at CASE 2-CU-ZX.

**3) Better installation of zone type dam.** Considering the above results, the installation of crossarm in zone type dam can be said as follows:

1) One crossarm at downstream should not be adopted even if it measures displacements in Z and X directions because it is extremely large in error compared with other installations.

(2) If one crossarm is to be installed, it should be at the upstream or at the center.

(3) If two crossarms are to be installed, the displacement of Z(settlement) and X directions(horizontal displacement) should be measured.

(4) One crossarm measuring displacements in Z and X direction is much better than two crossarms measuring only settlement( Z direction).

(5) In the case of one crossarm, the upstream of dam is better than downstream during water filling but is worse during construction.

(6) If crossarm is to be installed under the same conditions, upstream installation is better than the other cases.

(7) Data during water filling is much better than one during construction if they are to be used to identify the parameters by back analysis.

(8) Error Index of Young's modulus are much larger than that of Poisson's ratio.

(9) Comparing Error Index of each zone, zone 1(core zone) or zone 2(filter zone) is larger than the other zones.

(10) Error Index of zone1(core zone) is larger than Error Index in zone 2(filter zone) except the case where one crossarm is installed at upstream.

(11) Data during water filling gives better results than the data during construction.

(12) The results in Zone 2 are not so good because its area might be too thin.

(13) Only one crossarm in downstream zone is a worse installation even if it measures displacements in Z and X directions.

(14) Selecting best combination of crossarm, three crossarms is best but not so much difference as compared with one crossarm measuring Z and X directions.

## 5. CONCLUSION.

Combination of instrumentation must consider not only about best estimation of dam behavior during construction but also during reservoir water filling. This paper has considered above, a numerical model before adopting for actual dam.

Needless to say, crossarms give the more accurate estimate of dam behavior if each crossarm is functioned to measure at the same time directions such as Z and/or X direction. Measuring data of two directions(Z and X direction) give much better information. On the other hand the error of estimated values of zone type dam are much larger than ones of homogeneous dam. However the data gotten during water filling give much better estimate than during construction.

If one crossarm is installed, it should be at upstream of dam and should measure displacements in X and Z directions. One crossarm at downstream give extremely large error compared to other installation even if measuring Z and X direction.

One crossarm measuring Z and X direction is much better than two crossarm measuring only settlement( Z direction). There are not so much differences between one or two crossarms with function to measure displacements in Z and X direction in all cases including the upstream one. The results from the data of three crossarms' settlement are not so much different from one crossarm measuring Z and X direction.

Error Index of Young's modulus are much larger than that of the Poisson's ratio.

These methods can be suggested for the optimum installation in fill dams.

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