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A Numerical Analysis of Unsteady Flow in Kojima Lake

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We study unsteady flow that models lake flow in Kojima Lake. We assume that a lake is connected to another area with gates, and that those gates are opened when the water level outside the gates is lower than the water level of the lake. We study unsteady flow due to the opening of the gates. We analyze the problem with the finite element method, and take detailed structures of the gates into account.

Keywords : unsteady flow, lake flow, finite element method

1. INTRODUCTION

Kojima Lake is separated from Kojima Bay with a bank that is approximately 1 km long. There are six gates, each of which is 24 m wide, installed on the bank in order to control the water level of the lake. Rivers, including Sasagase River and Kurashiki River, flow into the lake, and a frequent discharge of water becomes necessary to maintain a constant water level of the lake. In such a case, the gates are opened when the water level outside the lake is lower. Then the discharge through the gates causes a disturbance to steady lake water. As a result, unsteady lake flow occurs. Such unsteady flow in Kojima Lake has been studied numerically. In [5], the problem is analyzed with the difference equations in space-staggered grid. Here we apply the finite element method to governing partial differential equations of lake flow [2, 4]. We show that detailed structures of the gates can be taken into account. We illustrate how the spatial domain of the partial differential equations are divided into triangular elements, and introduce some numerical results obtained with the finite element method.

A finite element analysis of the unsteady flow in Kojima Lake is found in [7]. In section 2, we show how the method used in [7] can be applied. A spatial region of the partial differential equations governing lake flow is shown. The part of the boundary corresponding to the location of the gates are illustrated. In section 3, we introduce some numerical results. An experimental method, in which the global positioning system is utilized, and some experimental results are also introduced in [7].

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2. APPLICATION OF THE FINITE ELEMENT METHOD TO LAKE FLOW ANALYSIS

We consider the following system of partial differential equations as a model for unsteady lake flow [1, 3].

$$
\begin{aligned}
\frac{\partial M}{\partial t} &= -g\,(h+\zeta)\,\frac{\partial \zeta}{\partial x} + A\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right) - \frac{\gamma^2\sqrt{M^2+N^2}}{(h+\zeta)^2}M,\\
\frac{\partial N}{\partial t} &= -g\,(h+\zeta)\,\frac{\partial \zeta}{\partial y} + A\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) - \frac{\gamma^2\sqrt{M^2+N^2}}{(h+\zeta)^2}N,\\
&\frac{\partial \zeta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0.
\end{aligned}
\tag{1}
$$

Here x (m) and y (m) are spatial variables. The positive directions of x axis and y axis correspond to the east and the north, respectively. t (s) is a temporal variable. M (m^2/s) and N (m^2/s) denote discharge fluxes in the x direction and in the y direction, respectively. g (m/s^2) represents the gravitational acceleration. A and γ are constants. $\zeta\,(x,y,t)$ and $h\,(x,y)$ are functions that represents the surface of water and the bottom of water, respectively. Suppose that z axis is taken vertically with the positive direction pointing upward. Then $z = \zeta\,(x,y,t)$ represents the surface of water at the point (x,y) and at the time t. Similarly, $z = h\,(x,y)$ represents the bottom of water at the point (x,y). The components, $\overline{u}$ and $\overline{v}$, of the average velocity taken over the interval in the z axis,

$[-h(x,y), \zeta(x,y,t)]$, are given by

$$\overline{u}(x,y,t) = \frac{M(x,y,t)}{\zeta(x,y,t)+h(x,y)}$$

$$\overline{v}(x,y,t) = \frac{N(x,y,t)}{\zeta(x,y,t)+h(x,y)}$$

In [7], the system at (1) is transformed to a system that is suitable for application of the finite element method. Figure 1 shows the spatial domain of the system obtained after the transformation. Figure 1 (a) shows the entire domain, and Figure 1 (b) shows details in the vicinity of the gates. As is done in [7], we apply the finite element method to the unsteady lake flow problem. Figure 2 shows the division of the spatial domain into the triangular elements. Figure 2 (a) shows the division of the entire domain, There are 1000 nodes and 1812 elements in the division. Figure 2 (b) shows the division of in the vicinity of the gates. The values of h set at the nodes are based on the data in [6]. We also assumed that h is identically equal to 3 (m) in the vicinity of the gates

3. RESULTS FROM FINITE ELEMENT ANALYSIS OF UNSTEADY LAKE FLOW

On September 6, 1999, the gates were opened approximately at 12:30 P. M., and they were kept open approximately for two hours. The water level decreased approximately by 0.2 m over the time interval. This information leads to boundary conditions concerning the water level at the gates. Using these conditions as some of the boundary conditions, we analyzed the unsteady lake flow problem numerically with the finite element method.

Based on information shared by the Kojima Bay Closing Bank Central Administration Office, Section of Land Improvement in Kojima Bay Area, we assumed that the initial water level at the gates was identically equal to 0.9867 (m) and that it decreased linearly by 0.22 (m) over the time interval between $t=0\,(s)$ and $t=7200\,(s)$. We also assumed that the initial water level ζ at the lake was identically equal to 0.9933 (m) The values of the remaining parameters such as those which appear in the system at (1) and other scaling parameters are the same as those used in [7].

Some of the results are shown in Figures 3 - 5. Figures 3, 4, and 5 show the velocity vectors at $t=600\ (s)$, at $t=3600\ (s)$, and at $t=7200\ (s)$, respectively.

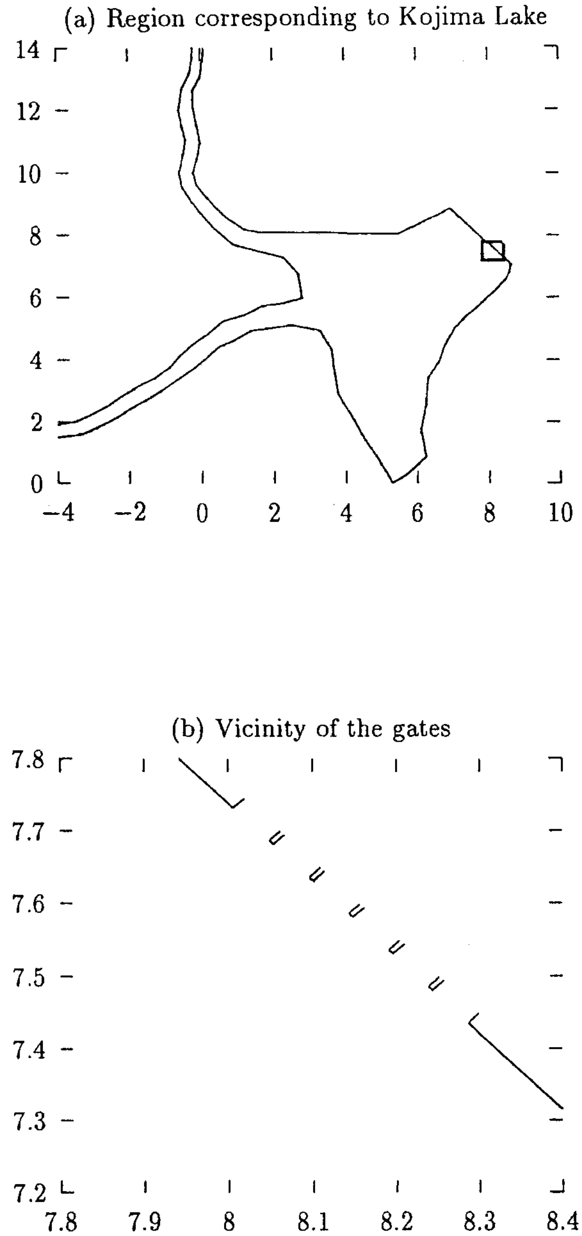

FIGURE 1. The spatial domain of the partial differential equations obtained from the system at (1) is shown. Figure (a) shows the entire domain . The part indicated by the box contains the gates, and details in the part are shown in Figure (b). The openings in the boundary corresponds to the gates, each of which is 24 m long.

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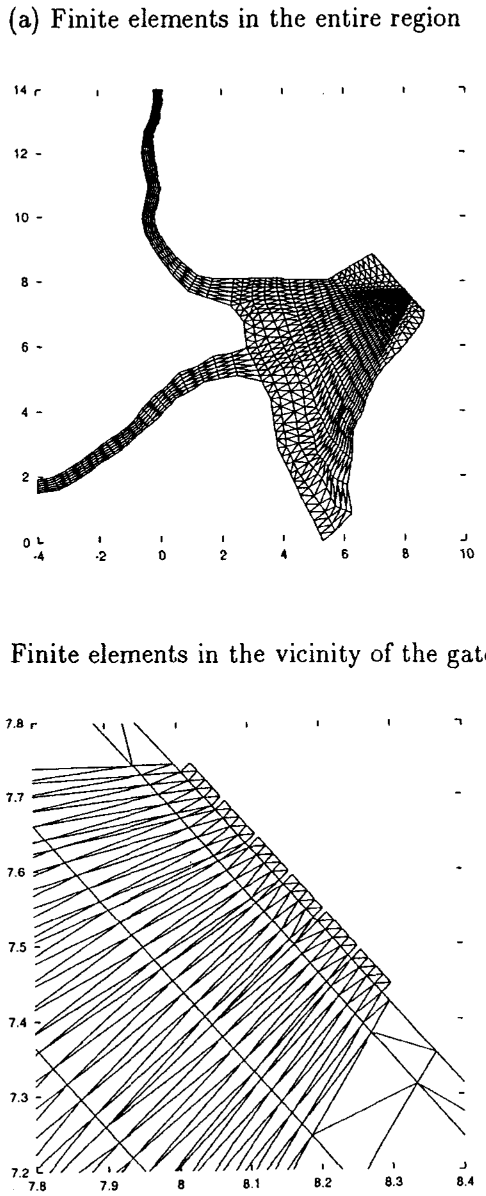

FIGURE 2. The division of the domain into triangular elements is illustrated. Figure (a) shows the division of the entire domain. Figure (b) shows the division in the part of the domain illustrated in Figure 1 (b).

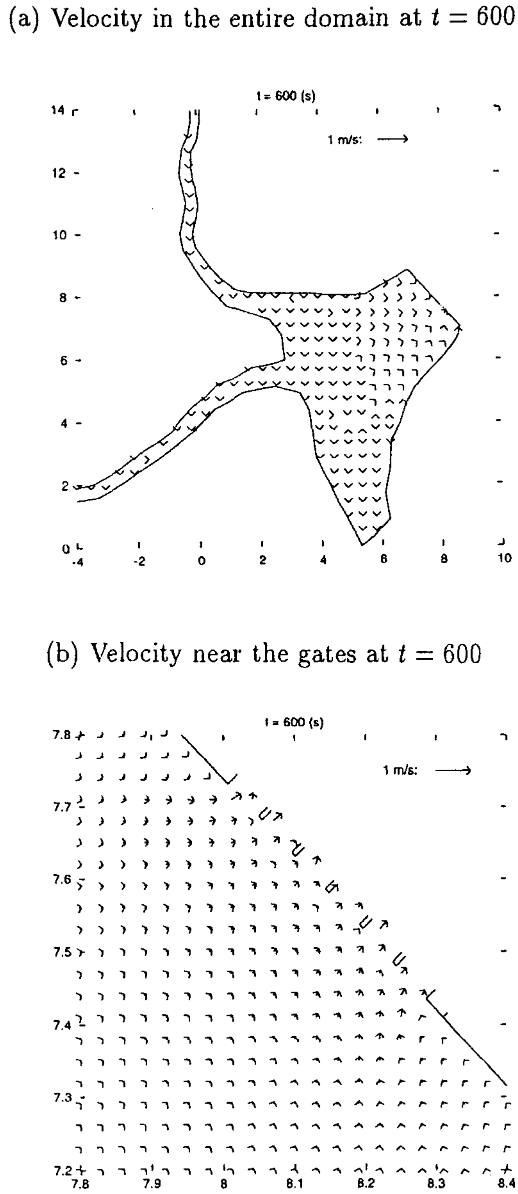

FIGURE 3. The velocity vectors $(\overline{u}, \overline{v})$ at $t = 600\,(s)$ are shown. Figure (a) shows the velocity vectors in the entire domain. Figure (b) shows the velocity vectors in the part of the domain illustrated in Figure 1 (b).

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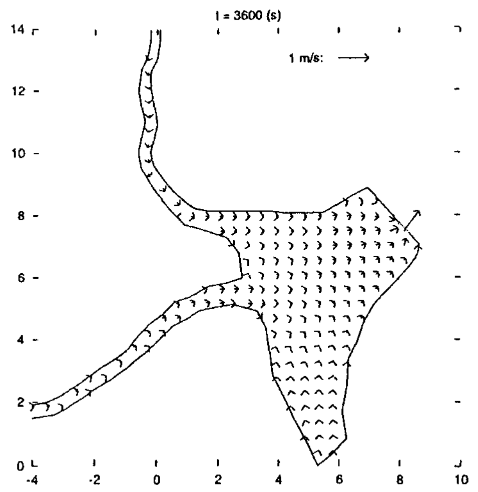
(a) Velocity in the entire domain at $t = 3600$



(b) Velocity near the gates at $t = 3600$

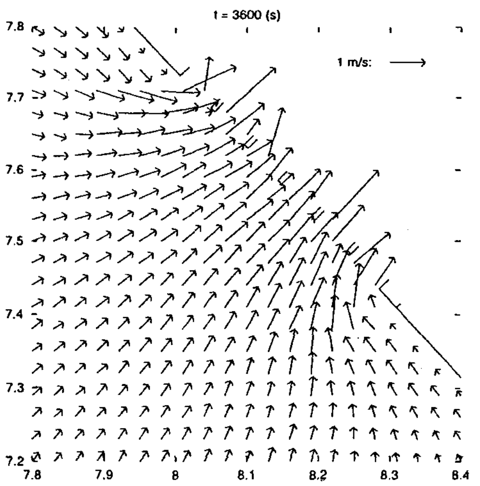
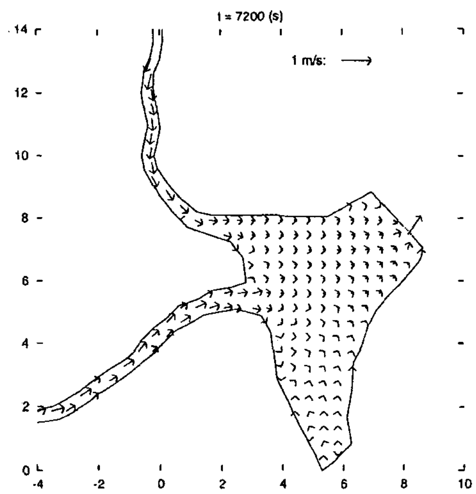


FIGURE 4. The velocity vectors $(\overline{u}, \overline{v})$ at $t = 3600\,(s)$ are shown. Figure (a) shows the velocity vectors in the entire domain. Figure (b) shows the velocity vectors in the part of the domain illustrated in Figure 1 (b).

(a) Velocity in the entire domain at $t = 7200$



(b) Velocity near the gates at $t = 7200$

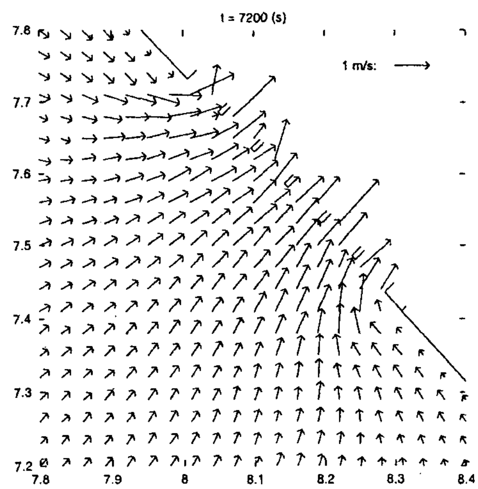


FIGURE 5. The velocity vectors $(\overline{u}, \overline{v})$ at $t = 7200\,(s)$ are shown. Figure (a) shows the velocity vectors in the entire domain. Figure (b) shows the velocity vectors in the part of the domain illustrated in Figure 1 (b).