Fraction and decimal teaching

—By comparison between Japanese and English textbooks—

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ABSTRACT; The teaching of fraction and decimals is compared between Japanese and English textbooks for elementary arithmetic. Definition of a fraction and the use of the decimals are focused for comparison.

1. Introduction

Fraction should be one of the topics which are hard to grasp their concepts for the pupils. In order to know how it is handled in the textbook as well as the decimals which relate intimately with fractions, we take a series of textbooks from United Kingdom as the textbook in such country that has a different teaching method. We compare these two series of textbooks to pick up several differences.

2. Teaching fraction and decimal number in UK and Japan

As an example, I take "peak mathematics", one of the textbooks of primary schools in UK. This series consist of eight books, book 0 to 7, corresponding to the ages as follows:

Book 0	age 5 to 7	Book 5	age 9 to 10
Book 1 and Book 2	age 7 to 8	Book 6	age 10 to 11
Book 3 and Book 4	age 8 to 9	Book 7	age 11 to 12

For an example from Japan, I take "Shintei sansu (Newly revised arithmetic)", which consists of six books for the first to sixth school year.

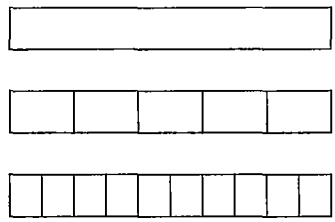
Fraction

Fractions firstly appear in the book I with an introduction :
"(Showing the picture of squared paper foleded into two parts) Each part is called a half. Half is written like this : $\frac{1}{2}$ " In the book 2 the squared paper or area of shapes are used to explain " $\frac{1}{2} + \frac{1}{2} = 1$, $\frac{3}{4} + \frac{1}{4} = 1$ "etc. In the book 3 both the continuous quantity (such as cake, chocolate, and area of shapes) and the discrete quantity (such as the number of the button). Equivalent fractions are explained in the book 4 to 5:

(book 4)
"(Showing the part of chocolate bar) $\frac{1}{2}$ or $\frac{\square}{8}$ "
(book 5)
"Complete these :

1. $\frac{1}{5} = \frac{*}{10}$ 2. $\frac{2}{5} = \frac{*}{10}$

3. $\frac{3}{5} = \frac{*}{10}$ 4. $\frac{4}{5} = \frac{*}{10}$ "



In the book 5, $\frac{2}{3}$ and $\frac{3}{4}$ are changed to the same denominator. There also is a question on their ordering:

"Put these fractions in order, largest first. Change them to the same denominator first.

$\frac{1}{3}, \frac{1}{4}, \frac{1}{2}$ $\frac{3}{4}, \frac{7}{8}, \frac{5}{12}$ etc. "

The number line is not found in peak maths, so fractions and decimals are never put on the number line for the order comparison with integers. There is no definite statement"....is called a fraction."for defining the fraction.

In case of Japanese textbooks, in the grade 3, we explain"one half", "one third" by comparing two strings,and " $\frac{1}{2}$ m" is introduced by cutting 1 meter of tape into three equal parts. The definition of the fraction is "Numbers such as $\frac{1}{3}$ and $\frac{2}{3}$ are called fractions." This follows the ordering of fractions using "equality sign"and "inequality sign". In the grade 4, various fractions are put on the number line with the explanation of "proper fraction","improper fraction", and "mixed fraction", There is also an illustration of equivalent fractions; "$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$, and $\frac{5}{10}$ are all equivalent fractions."

All the materials for fractions till the grade 5 are from the continuous quantity such as length, volume, time, and distance but nothing from the discrete quantity.

Decimals

In the peak maths the section "decimals" first appears in the book 5. Decimals are introduced as follows: "1 made 10 times smaller is $\frac{1}{10}$. "," $\frac{1}{10}$ is written 0.1". We have also the term "decimal point" here. Still from the eralier stage we find the numbers with "decimal point" and their operation. At the section "length" in the book 2, there is a statement: "The height of this man is 1 m 79 cm. You can write this as 1.79 m." There is also a calculation by writing such as :

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   m
  3.15
 +2.86
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At the section "length" in the book 3, we also find the calculation by writing for addition and subtraction, and at the section "money" we have a multiplication such as :

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   £
  1.48
 ×  4
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At the section "addition" and "length" respectively,we have

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   £              m
  15.72
 +14.68        5) 1.50
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But it is important that we have seen such numbers that are not the "decimals" but the

numbers with the units. The "decimals" appears at the section "decimals" in the book 5 as mentioned above. Here the decimal number is defined for the first time, then we have numbers like 1.4 without the units £ or m. Four arithmetic operaitons are already practiced, as mentioned, in a unit-attached style. So here we have only exercises of decimal + decimal, decimal - decimal, decimal × integer, decimal ÷ integer.

In Japanese case, on the contrary, we have at the section "decimals" in the grade 3 the introduction of the decimal number:
"one tenth of 1 $d\ell$ is called 0.1 $d\ell$. 0.1 $=\frac{1}{10}d\ell$."
There are also the definition of "integer" , "decimal" , and "decimal point". The decimal notation first appears here and starts as an decimal in a unit-fallen style. Position on the number line is discussed in the same section. The next section "addition and subtraction of decimals" has an explanation:
"We have 0.3 liter of oil and 0.5 liter of oil. How much is the sum ?
$0.3 + 0.5 = 0.8$ <u>0.8 liter.</u> "
Only decimals are used for calculation and only the answer has takes the unit. At the section "decimal" in the grade 4, there is a conversion such as "280 cm is 2.8 m."

Relation between decimals and fractions

In the book 6 of the peak maths we have :

" 0.7 can be written as $\frac{7}{10}$
0.07 can be written as $\frac{7}{100}$ "
"$\frac{1}{4}$ can be written as $\frac{25}{100} = 0.25$ "
"Write these as decimals. $1\frac{3}{4}$, $4\frac{1}{2}$kg "
In the book 7 : "Change $\frac{3}{4}$ into a decimal. "
"Some fractions cannot be changed accurately into decimals. The decimal numbers form a repeating pattern."
$\frac{2}{11}$ is taken as an example for showing that a part of the fractions are written as decimals.

In Japanese case, we have at the section "integers, decimals and fractions" in the grade 5 :
"Integer is a fraction with the denominator 1. "
"Decimals are special fractions with denominator 10, 100, 1000 etc. "
"$0.12 = \frac{3}{25}$ $0.008 = \frac{1}{125}$ "
Fractions corresponding to periodic are not mentioned here.

3 . Final remarks

Followings are the typical differences between two series of textbooks.

(1) In the peak maths the sections "fractions" and "decimals" are partitioned into several parts (spiral method) but in Japanese textbooks each topic tends to stay in one spot.

(2) The number line is not used in the peak maths. The Japanese textbook uses the

number line for explaining the order of fractions and decimals or the relation with integers.

(3) The distinction between fractions and fracturers as seen in Germany is not found in both two series of textbooks.

(4) Decimals are used from an early stage in the peak maths in the unit-attached style. The operations are the same as the case of integers. Definition of a decimal comes later. Similar situation is reported inFrance. In Japanese case the definition of a decimal comes first and then come the explanation ofthe number with decimal point and their operations. Therefore the notation and the operation without the unit are introduced comparatively earlier.

(5) For the relation between fractions and decimals, the peak maths seems to show more clearly that decimals are a part of fractions.

References

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2) Keirinkan "Shintei Sansuu (Newly revised arithmetic) grade 1 to 6" 1987

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(Received July 15, 1987)